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MOTION OF AN INSULATING SOLID PARTICLE NEAR A PLANE BOUNDARY UNDER THE ACTION OF UNIFORM AMBIENT ELECTRIC AND MAGNETIC FIELDS

A. Sellier

LadHyX. Ecole polytechnique, 91128 Palaiseau Cédex, France
e-Mail: sellier@ladhyx.polytechnique.fr

This work addresses the wall-particle interactions between a solid and insulating particle immersed above a plane wall in a conducting liquid when subject to ambient uniform electric and magnetic fields. Both for a perfectly conducting or for an insulating wall, attention is paid to the net electromagnetic force and torque exerted on the particle when it is *held fixed* and also to the particle rigid-body migration when it is freely suspended in the liquid. A new boundary approach is proposed to obtain those quantities whatever the particle shape and location. The advocated procedure actually reduces the task to the determination of a few surface quantities by inverting seven boundary-integral equations on the particle boundary. The entire procedure is then asymptotically worked out in the case of a perfectly conducting wall for a distant particle, i.e. when the wall–particle gap is much larger than the particle length scale. The derived asmptotic results reveal that the magnitude of the wall–particle interactions deeply depends on the distant particle geometry. In that sense, the results previously obtained by other authors for a distant sphere interacting with an insulating plane wall are far from being general.

Introduction. As known both theoretically [1] and experimentally [2], an insulating solid particle freely suspended in a Newtonian and conducting liquid with uniform viscosity μ and conductivity $\sigma > 0$ migrates when subject to uniform ambient electric $\mathbf{E}$ and magnetic $\mathbf{B}$ fields. This particle rigid-body motion, with a translational velocity $\mathbf{U}$ (here the velocity of the particle center-of-volume) and an angular velocity $\mathbf{\Omega}$, deeply depends upon (σ, μ), the particle's geometry and on $(\mathbf{E}, \mathbf{B})$. For example [1], an insulating sphere with a radius a translates without rotating at the velocity $\mathbf{U} = -a^2\sigma[\mathbf{E} \wedge \mathbf{B}]/(6\mu)$, whereas insulating non-spherical particles, in general, both rotate and translate [3, 4].

Of course, the migration predicted for the unbounded liquid is affected by particle–boundary interactions when the liquid is bounded. Such interactions have been handled in the literature using different approaches for two different plane boundaries (see also section 1.1): the insulating wall [5,6] and the perfectly conducting wall [7]. Note that [6] presents a boundary method for a conducting or insulating arbitrary shaped particle, but the associated numerical implementation is so involved that only the case of the fields $\mathbf{E}$ and $\mathbf{B}$ normal to the insulating wall has been investigated. In contrast, [5] allows an arbitrary magnetic field $\mathbf{B}$, but it is confined to a spherical particle because of the employed bipolar coordinates technique. Finally, [7] asymptotically approximated the sphere rigid-body motion for an insulating *sphere distant* from an insulating plane wall when freely-suspended and also the electromagnetic force $\mathbf{F}_e$ and the torque $\mathbf{C}_e$ (about the sphere center) acting on the sphere when *it is held fixed.* For a sphere with a radius a and a wall-center distance h such that $\epsilon = a/h \ll 1$, those authors found that

$$\mathbf{F}_e = \sigma a^3[1 + \epsilon^3/8 + O(\epsilon^4)]\mathbf{B} \wedge \mathbf{E} \text{ and } \mathbf{C}_e = \sigma a^4|\mathbf{E}||\mathbf{B}|O(\epsilon^2) \text{ if } \mathbf{B} \wedge \mathbf{e}_3 = \mathbf{0}, \quad (1)$$

$$\mathbf{F}_e = \sigma a^3[1 + \epsilon^3/16 + O(\epsilon^4)]\mathbf{B} \wedge \mathbf{E} \text{ and } \mathbf{C}_e = \mathbf{0} \text{ if } \mathbf{B}.\mathbf{e}_3 = 0. \quad (2)$$

According to Eqs. (1)–(2), the *electromagnetic* sphere–wall interactions are very weak (here of the order $O(\epsilon^3)$ or at the most $O(\epsilon^2)$ for the force or torque,

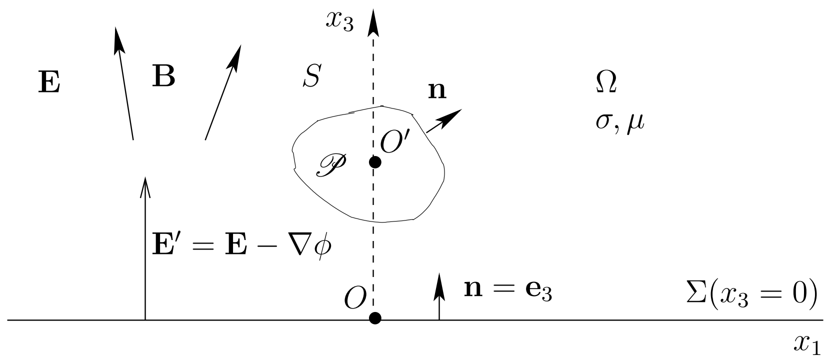

Fig. 1. A solid insulating particle immersed in a conducting Newtonian liquid above a perfectly conducting $x_3 = 0$ solid plane wall Σ (Case 2).

respectively). However, because a sphere exhibits strong symmetries (isotropic particle), such conclusions are not necessary true any more for a non-spherical distant particle! Thus, dealing with a distant and non-spherical particle is still a challenging issue. Therefore, this work, for the two types of walls, presents a new boundary formulation (valid whatever the insulating particle geometry and location) more suitable for a numerical implementation than the one given in [6] for the insulating wall. This procedure is also asymptotically worked out for a distant but arbitrary-shaped particle in the case of a perfectly conducting wall.

1. Governing problems and flow decomposition. This section presents the governing problems and the adopted flow decomposition.

1.1. Assumptions and governing electric and hydrodynamic problems. As illustrated in Fig. 1, we consider a solid and insulating particle $\mathscr{P}$ immersed above the solid and motionless $x_3 = 0$ plane wall Σ in a conducting Newtonian liquid having a uniform viscosity μ and a conductivity $\sigma > 0$. The particle has the center of volume O' and a smooth boundary S. Moreover, on $S \cup \Sigma$ the unit normal directed into the liquid domain Ω is denoted by $\mathbf{n}$.

We use Cartesian coordinates (O, x_1, x_2, x_3) attached to the wall with $\mathbf{x} = \mathbf{OM}$ and $x_i = \mathbf{x}.\mathbf{e}_i$. While the uniform ambient electric $\mathbf{E}$ and magnetic $\mathbf{B}$ fields dominate far from the particle, the electric field reads $\mathbf{E}' = \mathbf{E} - \nabla\phi$ in the liquid domain Ω, with ϕ being the potential disturbance. At the insulating particle boundary one requires $\mathbf{E}'.\mathbf{n} = 0$ and two different types of wall are considered:

(i) Case 1: the insulating wall with $\mathbf{E}'$ parallel to Σ;

(ii) Case 2: the perfectly conducting wall with $\mathbf{E}$ and $\mathbf{E}'$ normal to the wall Σ.

Accordingly, the function ϕ satisfies the well-posed problem

$$\nabla^2\phi = 0 \text{ in } \Omega, \ \nabla\phi \to \mathbf{0} \text{ as } r = |\mathbf{x}| \to \infty, \tag{3}$$

$$\nabla\phi.\mathbf{n} = \mathbf{E}.\mathbf{n} \text{ on } S, \ \nabla\phi.\mathbf{e}_3 = 0 \text{ on } \Sigma \text{ in Case 1}, \ \phi = 0 \text{ on } \Sigma \text{ in Case 2}. \tag{4}$$

The Lorentz body force drives both a liquid flow with a pressure Q and velocity $\mathbf{u}$ and a particle rigid-body motion with an unknown translational velocity $\mathbf{U}$ (here the particle center-of-volume O' velocity) and angular velocity $\mathbf{\Omega}$. Denoting by $V > 0$ the typical magnitude of $\mathbf{u}$ and by a the particle length scale, we assume (as in [1-7]) vanishing Reynolds number $\mathrm{Re} = \rho V a/\mu$ and Hartmann and magnetic Reynolds numbers. Consequently, the magnetic field $\mathbf{B}$ is not disturbed and $(\mathbf{u}, Q)$ is a quasi-steady Stokes flow driven by the non-uniform Lorentz body force $\mathbf{f} = \sigma(\mathbf{E} - \nabla\phi) \wedge \mathbf{B}$. Setting $Q = P + \sigma(\mathbf{E} \wedge \mathbf{B}).\mathbf{x}$, the flow $(\mathbf{u}, P)$ obeys the creeping flow problem

$$\nabla.\mathbf{u} = 0 \text{ and } \mu\nabla^2\mathbf{u} = \nabla P + \sigma\nabla\phi \wedge \mathbf{B} \ \text{ in } \Omega, \tag{5}$$

$$\mathbf{u} = \mathbf{U} + \mathbf{\Omega} \wedge \mathbf{O'M} \text{ on } S, \ \mathbf{u} = \mathbf{0} \text{ on } \Sigma, \ (\mathbf{u}, P) \to (\mathbf{0}, 0) \text{ as } |\mathbf{x}| \to \infty. \tag{6}$$

For a freely-suspended particle with negligible inertia, the force $\mathbf{F}$ and the torque $\mathbf{C}$ (about O') exerted by the flow $(\mathbf{u}, Q)$ on the particle with the volume $\mathscr{V}_{\mathscr{P}}$ vanish. If $(\mathbf{u}, P)$ has a stress tensor $\boldsymbol{\sigma}$, one then gets, setting $\mathbf{x}' = \mathbf{O'M}$,

$$\mathbf{F} := \int_S \boldsymbol{\sigma}.\mathbf{n}\mathrm{d}S - \sigma\mathscr{V}_{\mathscr{P}}(\mathbf{E} \wedge \mathbf{B}) = \mathbf{0}, \ \mathbf{C} := \int_S \mathbf{x}' \wedge \boldsymbol{\sigma}.\mathbf{n}\mathrm{d}S = \mathbf{0}. \tag{7}$$

In summary, one has first to solve Eqs. (3)–(4) to obtain ϕ and then solve Eqs. (5)–(7) to obtain $(\mathbf{u}, P)$ and $(\mathbf{U}, \mathbf{\Omega})$.

1.2. Flow decomposition and auxiliary Stokes flows. When solving Eqs. (5)–(7), it is fruitful to exploit the following decompositions $\mathbf{u} = \mathbf{u}_h + \mathbf{w} + \mathbf{v}$ and $P = p_h + p$ with the governing problems

$$\nabla.\mathbf{u}_h = 0 \text{ and } \mu\nabla^2\mathbf{u}_h = \nabla p_h \ \text{ in } \Omega, \tag{8}$$

$$\mathbf{u}_h = \mathbf{U} + \mathbf{\Omega} \wedge \mathbf{x}' \text{ on } S, \ \mathbf{u}_h = \mathbf{0} \text{ on } \Sigma, \ (\mathbf{u}_h, p_h) \to (\mathbf{0}, 0) \text{ as } |\mathbf{x}| \to \infty, \tag{9}$$

$$\nabla.\mathbf{w} = 0 \text{ and } \mu\nabla^2\mathbf{w} = \sigma\nabla\phi \wedge \mathbf{B} \ \text{ in } \Omega, \mathbf{w} \to \mathbf{0} \ \text{ as } \ |\mathbf{x}| \to \infty, \tag{10}$$

$$\nabla.\mathbf{v} = 0 \text{ and } \mu\nabla^2\mathbf{v} = \nabla p \ \text{ in } \Omega, \tag{11}$$

$$\mathbf{v} = -\mathbf{w} \text{ on } S, \ \mathbf{v} = -\mathbf{w} \text{ on } \Sigma, \ (\mathbf{v}, p) \to (\mathbf{0}, 0) \ \text{ as } \ |\mathbf{x}| \to \infty. \tag{12}$$

Clearly, the flow $(\mathbf{u}_h, p_h)$ is the one produced by the particle when moving in an insulating ($\sigma = 0$) liquid. By linearity, it thus exerts on the particle a force $\mathbf{F}_h$ and a torque $\mathbf{C}_h$ (about O') reading

$$\mathbf{F}_h = -\mu\{\mathbf{K}.\mathbf{U} + \mathbf{V}.\mathbf{\Omega}\}, \ \ \mathbf{C}_h = -\mu\{\mathbf{D}.\mathbf{U} + \mathbf{W}.\mathbf{\Omega}\} \tag{13}$$

with $\mathbf{K}, \mathbf{W}, \mathbf{V}$ and $\mathbf{D}$ being the usual so-called second-rank resistance tensors. The Cartesian components of those tensors are obtained by considering six auxiliary Stokes flows $(\mathbf{u}_L^{(i)}, p_L^{(i)})$ with $i = 1, 2, 3$ and $L = t$ or $L = r$ for the particle translation or rotation, respectively. Such flows obey Eqs. (8)–(9) except for the boundary condition on S, which is replaced by $\mathbf{u}_t^{(i)} = \mathbf{e}_i$ or $\mathbf{u}_r^{(i)} = \mathbf{e}_i \wedge \mathbf{x}'$ (using t and r for particle translation or rotation, respectively). Inspecting the relations (13) and denoting by $\mathbf{f}_L^{(i)}$ the surface traction exerted on S by the flow $(\mathbf{u}_L^{(i)}, p_L^{(i)})$, one finally arrives at the desired Cartesian components

$$K_{ij} = -\left[\int_S \mathbf{e}_i.\mathbf{f}_t^{(j)}\mathrm{d}S\right]/\mu, \ W_{ij} = -\left[\int_S (\mathbf{e}_i \wedge \mathbf{x}').\mathbf{f}_r^{(j)}\mathrm{d}S\right]/\mu, \tag{14}$$

$$D_{ij} = -\left[\int_S (\mathbf{e}_i \wedge \mathbf{x}').\mathbf{f}_t^{(j)}\mathrm{d}S\right]/\mu, \ V_{ij} = -\left[\int_S \mathbf{e}_i.\mathbf{f}_r^{(j)}\mathrm{d}S\right]/\mu. \tag{15}$$

From Eq. (10), the flow $\mathbf{w}$, with zero pressure and stress tensor $\boldsymbol{\sigma}_{\mathbf{w}}$, is driven by the non-uniform body force $\sigma\mathbf{B} \wedge \nabla\phi$. [1] This flow exerts on the particle the following force and torque

$$\mathbf{F}_{\mathbf{w}} = \int_S \boldsymbol{\sigma}_{\mathbf{w}}.\mathbf{n}\mathrm{d}S, \ \mathbf{C}_{\mathbf{w}} = \int_S \mathbf{x}' \wedge \boldsymbol{\sigma}_{\mathbf{w}}.\mathbf{n}\mathrm{d}S. \tag{16}$$

[1] Note that $\mathbf{w}$ is not unique because it is free from boundary conditions on $S \cup \Sigma$. In contrast, the flow $(\mathbf{w} + \mathbf{v}, p)$ is unique.

The flow $(\mathbf{v}, p)$ is free from body force. It has a stress tensor $\boldsymbol{\sigma}_{\mathbf{v}}$ and exerts on the particle a force $\mathbf{F}_{\mathbf{v}}$ and a torque $\mathbf{C}_{\mathbf{v}}$. These vectors are here expressed by applying the reciprocal identity [8] to two Stokes flows free from body force: the flow $(\mathbf{v}, p)$ and successively each auxiliary Stokes flow. One then easily arrives at (with summations over i=1,2,3 in Eqs. (17)–(18))

$$\mathbf{F}_{\mathbf{v}} = \int_S \boldsymbol{\sigma}_{\mathbf{v}}.\mathbf{n} \mathrm{d}S = -\left[\int_S \mathbf{w}.\mathbf{f}_t^{(i)} \mathrm{d}S + \int_\Sigma \mathbf{w}.\boldsymbol{\sigma}_t^{(i)}.\mathbf{e}_3 \mathrm{d}S\right] \mathbf{e}_i, \tag{17}$$

$$\mathbf{C}_{\mathbf{v}} = \int_S \mathbf{x}' \wedge \boldsymbol{\sigma}_{\mathbf{v}}.\mathbf{n} \mathrm{d}S = -\left[\int_S \mathbf{w}.\mathbf{f}_r^{(i)} \mathrm{d}S + \int_\Sigma \mathbf{w}.\boldsymbol{\sigma}_r^{(i)}.\mathbf{e}_3 \mathrm{d}S\right] \mathbf{e}_i. \tag{18}$$

Clearly, the electromagnetic force $\mathbf{F}_e$ and the torque $\mathbf{C}_e$ experienced by the particle when *held fixed* (see also the introduction) obey

$$\mathbf{F}_e = \mathbf{F}_{\mathbf{w}} + \mathbf{F}_{\mathbf{v}} - \sigma \mathscr{V}_{\mathscr{P}}(\mathbf{E} \wedge \mathbf{B}), \ \mathbf{C}_e = \mathbf{C}_{\mathbf{w}} + \mathbf{C}_{\mathbf{v}}. \tag{19}$$

1.3. Key linear system for the particle rigid-body motion and needed surface quantities. Recalling the definitions (7) of $\mathbf{F}$ and $\mathbf{C}$ and relations (13) and (19), one has $\mathbf{F} = \mathbf{F}_e + \mathbf{F}_h$ and $\mathbf{C} = \mathbf{C}_e + \mathbf{C}_h$. Accordingly, the freely suspended particle rigid-body motion $(\mathbf{U}, \boldsymbol{\Omega})$ fulfills the linear system

$$\mathbf{K}.\mathbf{U} + \mathbf{V}.\boldsymbol{\Omega} = \{\mathbf{F}_{\mathbf{w}} + \mathbf{F}_{\mathbf{v}} - \sigma \mathscr{V}_{\mathscr{P}}(\mathbf{E} \wedge \mathbf{B})\}/\mu, \ \mathbf{D}.\mathbf{U} + \mathbf{W}.\boldsymbol{\Omega} = \{\mathbf{C}_{\mathbf{w}} + \mathbf{C}_{\mathbf{v}}\}/\mu. \tag{20}$$

As shown in [6], the system (20) is well-posed, i.e. admits a unique solution $(\mathbf{U}, \boldsymbol{\Omega})$. In view of Eqs. (14)–(20), one can thus gain the quantities $\mathbf{F}_e, \mathbf{C}_e$ and $(\mathbf{U}, \boldsymbol{\Omega})$ by solely determining the velocity $\mathbf{w}$ and the surface tractions $\boldsymbol{\sigma}_{\mathbf{w}}.\mathbf{n}$ and $\boldsymbol{\sigma}_L^{(i)}.\mathbf{n}$ at the entire boundary $S \cup \Sigma$.

2. Advocated boundary approach and boundary-integral equations. This section reduces our problem to the determination of seven surface quantities at the particle boundary S only (i.e. not any more on the unbounded surface $S \cup \Sigma$!).

2.1. Solution for ϕ and for the velocity field $\mathbf{w}$. A surface charge density q arises on the insulating particle when the ambient uniform electric field $\mathbf{E}$ is imposed. One actually has the usual integral representation

$$\phi(\mathbf{x}) = \frac{1}{4\pi} \int_S q(\mathbf{y}) \left\{ \frac{1}{|\mathbf{x} - \mathbf{y}|} - \frac{(-1)^l}{|\mathbf{x} - \mathbf{y}_s|} \right\} \mathrm{d}S(\mathbf{y}) \ \text{ for } \mathbf{x} \text{ in } \Omega \ (\text{Case } l = 1, 2) \tag{21}$$

where $\mathbf{y}_s$ designates the symmetry of the point $\mathbf{y}$ with respect to the $x_3 = 0$ plane wall Σ. Using Eq. (21), *one* analytical solution to Eq. (10) is

$$\mathbf{w}(\mathbf{x}) = \left(\frac{\sigma \mathbf{B}}{8\pi\mu}\right) \wedge \left[\int_S q(\mathbf{y}) \left\{ (-1)^l \frac{\mathbf{x} - \mathbf{y}_s}{|\mathbf{x} - \mathbf{y}_s|} - \frac{\mathbf{x} - \mathbf{y}}{|\mathbf{x} - \mathbf{y}|} \right\} \mathrm{d}S(\mathbf{y})\right], \mathbf{x} \text{ in } \Omega \cup S \cup \Sigma. \tag{22}$$

The resulting surface traction $\mathbf{f}_{\mathbf{w}} = \boldsymbol{\sigma}_{\mathbf{w}}.\mathbf{n}$ on the particle surface S then reads

$$\mathbf{f}_{\mathbf{w}}(\mathbf{x}) = -\frac{\sigma}{8\pi} \int_S q(\mathbf{y}) \left[\frac{(\mathbf{x} - \mathbf{y}).\mathbf{n}(\mathbf{x})(\mathbf{x} - \mathbf{y}) \wedge \mathbf{B} + \mathbf{n}(\mathbf{x}).[(\mathbf{x} - \mathbf{y}) \wedge \mathbf{B}](\mathbf{x} - \mathbf{y})}{|\mathbf{x} - \mathbf{y}|^3} \right.$$
$$\left. -(-1)^l \frac{(\mathbf{x} - \mathbf{y}_s).\mathbf{n}(\mathbf{x})(\mathbf{x} - \mathbf{y}_s) \wedge \mathbf{B} + \mathbf{n}(\mathbf{x}).[(\mathbf{x} - \mathbf{y}_s) \wedge \mathbf{B}](\mathbf{x} - \mathbf{y}_s)}{|\mathbf{x} - \mathbf{y}_s|^3} \right] \mathrm{d}S(\mathbf{y}). \tag{23}$$

From Eqs. (17)–(18) one also needs to evaluate $\mathbf{w}$ for $\mathbf{x}$ on Σ. Since $|\mathbf{x}-\mathbf{y}_s| = |\mathbf{x}-\mathbf{y}|$ for $x_3 = 0$, using the representation (22) gives

$$\mathbf{w}(\mathbf{x}) = \frac{\sigma}{4\pi\mu}\left\{\left[\int_S \frac{q(\mathbf{y})}{|\mathbf{x}-\mathbf{y}|}\mathrm{d}S(\mathbf{y})\right]\mathbf{x} - \left[\int_S \frac{q(\mathbf{y})\mathbf{y}.\mathbf{e}_1}{|\mathbf{x}-\mathbf{y}|}\mathrm{d}S(\mathbf{y})\right]\mathbf{e}_1 \right.$$
$$\left. - \left[\int_S \frac{q(\mathbf{y})\mathbf{y}.\mathbf{e}_2}{|\mathbf{x}-\mathbf{y}|}\mathrm{d}S(\mathbf{y})\right]\mathbf{e}_2\right\}\wedge\mathbf{B} \text{ for } \mathbf{x} \text{ on } \Sigma \text{ (Case 1)}, \quad (24)$$

$$\mathbf{w}(\mathbf{x}) = -\frac{\sigma}{4\pi\mu}\left[\int_S \frac{q(\mathbf{y})\mathbf{y}.\mathbf{e}_3}{|\mathbf{x}-\mathbf{y}|}\mathrm{d}S(\mathbf{y})\right](\mathbf{e}_3\wedge\mathbf{B}) \text{ for } \mathbf{x} \text{ on } \Sigma \text{ (Case 2)}. \quad (25)$$

2.2. Key integral representations. Henceforth, we adopt the usual tensor summation convention with, for instance, $\mathbf{u} = u_j\mathbf{e}_j$. As explained in [9], for each auxiliary Stokes flow one has the key integral velocity representation

$$\left[\mathbf{u}_L^{(i)}.\mathbf{e}_j\right](\mathbf{x}) = -\frac{1}{8\pi\mu}\int_S G_{jk}(\mathbf{x},\mathbf{y})\left[\mathbf{f}_L^{(i)}.\mathbf{e}_k\right](\mathbf{y})\mathrm{d}S(\mathbf{y}) \text{ for } \mathbf{x} \text{ in } \Omega\cup S\cup\Sigma\,, \quad (26)$$

where $G_{jk}(\mathbf{x},\mathbf{y})\mathbf{e}_j/(8\pi\mu)$ is the Stokes velocity produced at the point $\mathbf{x}$ by a unit force with strength $\mathbf{e}_k$ placed at the point $\mathbf{y}$, which vanishes on the wall Σ. For a distant wall (unbounded liquid), $G_{jk}(\mathbf{x},\mathbf{y}) = \mathscr{S}_{jk}(\mathbf{x}-\mathbf{y})$ with $\mathscr{S}_{jk}(\mathbf{X}) = \delta_{jk}/|\mathbf{X}| + (\mathbf{X}.\mathbf{e}_j)(\mathbf{X}.\mathbf{e}_k)/|\mathbf{X}|^3$. In the presence of the wall, one obtains [10]

$$G_{jk}(\mathbf{x},\mathbf{y}) = \mathscr{S}_{jk}(\mathbf{x}-\mathbf{y}) - \mathscr{S}_{jk}(\mathbf{x}-\mathbf{y}_s) - \frac{2c_j(\mathbf{y}.\mathbf{e}_3)}{|\mathbf{x}-\mathbf{y}_s|^3}\left\{\delta_{k3}(\mathbf{y}_s-\mathbf{x}).\mathbf{e}_j \right.$$
$$\left. -\delta_{j3}(\mathbf{y}_s-\mathbf{x}).\mathbf{e}_k + (\mathbf{x}.\mathbf{e}_3)\left[\delta_{jk} - \frac{3[(\mathbf{y}_s-\mathbf{x}).\mathbf{e}_j][(\mathbf{y}_s-\mathbf{x}).\mathbf{e}_k]}{|\mathbf{y}_s-\mathbf{x}|^2}\right]\right\} \quad (27)$$

where δ_{jk} denotes the Kronecker delta, $\mathbf{y}_s$ is the symmetry of $\mathbf{y}$ with respect to Σ, and $c_1 = c_2 = 1, c_3 = -1$.

By virtue of Eqs. (17)–(18), it is also necessary to get the traction $\boldsymbol{\sigma}_L^{(i)}.\mathbf{n}$ on the $x_3 = 0$ wall Σ. This is achieved by using the single-layer representation (26) and employing the stress tensor associated with the fundamental Stokes velocity field $G_{jk}(\mathbf{x},\mathbf{y})\mathbf{e}_j/(8\pi\mu)$ and given in [9,10]. One then arrives at

$$\left[\boldsymbol{\sigma}_L^{(i)}.\mathbf{n}\right](\mathbf{x}) = \frac{1}{8\pi}\int_S T_{jk3}(\mathbf{x},\mathbf{y})\left[\mathbf{f}_L^{(i)}.\mathbf{e}_k\right](\mathbf{y})\mathrm{d}S(\mathbf{y}) \;\text{ for } \mathbf{x} \text{ on } \Sigma \quad (28)$$

with $x_3 = 0$, the identity $\mathbf{n} = \mathbf{e}_3$ and then the relation (to be substituted in Eq. (28))

$$T_{jk3}(\mathbf{x},\mathbf{y}) = \frac{6(\mathbf{y}.\mathbf{e}_3)}{|\mathbf{y}-\mathbf{x}|^5}\big\{(\mathbf{y}-\mathbf{x}).\mathbf{e}_j(\mathbf{y}-\mathbf{x}).\mathbf{e}_k + (\mathbf{y}_s-\mathbf{x}).\mathbf{e}_j(\mathbf{y}_s-\mathbf{x}).\mathbf{e}_k$$
$$+2c_k(\mathbf{y}.\mathbf{e}_3)[\delta_{j3}(\mathbf{y}_s-\mathbf{x}).\mathbf{e}_k - \delta_{k3}(\mathbf{y}_s-\mathbf{x}).\mathbf{e}_j]\big\}. \quad (29)$$

2.3. Relevant boundary-integral equations on the particle surface. The surface charge density q is obtained by enforcing the boundary condition (4) on the particle surface. This provides the following well-posed Fredholm boundary-integral equation of the second kind

$$\frac{q(\mathbf{x})}{2} + \frac{1}{4\pi}\int_S\left\{\frac{\mathbf{x}-\mathbf{y}}{|\mathbf{x}-\mathbf{y}|^3} - (-1)^l\frac{\mathbf{x}-\mathbf{y}_s}{|\mathbf{x}-\mathbf{y}_s|^3}\right\}.\mathbf{n}(\mathbf{x})q(\mathbf{y})\mathrm{d}S(\mathbf{y})$$
$$= -[\mathbf{E}.\mathbf{n}](\mathbf{x}) \;\text{ for } \mathbf{x} \text{ on } S \text{ in Case } l = 1, 2. \quad (30)$$

In addition, because the integral representation (26) holds on the surface S, one obtains each traction $\mathbf{f}_L^{(i)}$ from the Fredholm boundary-integral equation of the first kind

$$-\frac{1}{8\pi\mu}\int_S G_{jk}(\mathbf{x},\mathbf{y})\left[\mathbf{f}_L^{(i)}.\mathbf{e}_k\right](\mathbf{y})\mathrm{d}S(\mathbf{y}) = \left[\mathbf{u}_L^{(i)}.\mathbf{e}_j\right](\mathbf{x}) \text{ for } \mathbf{x} \text{ on } S. \tag{31}$$

Summing up, the proposed new boundary formulation consists in inverting seven boundary-integral equations (30)–(31) on the particle surface to gain there q and $\mathbf{f}_L^{(i)}$. The knowledge of those surface quantities is sufficient to determine the electromagnetic force $\mathbf{F}_e = \mathbf{F_w} + \mathbf{F_v} - \sigma\mathscr{V}_{\mathscr{P}}(\mathbf{E}\wedge\mathbf{B})$ and the torque $\mathbf{C}_e = \mathbf{C_w} + \mathbf{C_v}$ by invoking Eqs. (16)–(18), (23)–(25) and (28)–(29) and subsequently the rigid-body motion $(\mathbf{U},\mathbf{\Omega})$ by using Eqs. (14)–(15) and (20).

3. Asymptotic analysis for a distant particle. As outlined in the introduction, the asymptotic results (1)–(2) for a distant insulating sphere (in Case 1) might be quite different for a non-spherical particle or for an arbitrary-shaped particle in Case 2. Such a basic issue is investigated in this section by asymptotically working out the previously proposed boundary approach for the perfectly conducting wall (Case 2).

3.1. First-order estimates for $\mathbf{F}_h$ *and* $\mathbf{C}_h$. The particle has the length scale a and the center-of-volume O' with $\mathbf{OO'} = h\mathbf{e}_3$ and $h > 0$. Henceforth, it is sufficiently distant from the wall Σ in the sense that $\epsilon = a/h \ll 1$. As shown in [11], for both $\mathbf{x}$ and $\mathbf{y}$ located on the distant surface S one then gets

$$G_{jk}(\mathbf{x},\mathbf{y}) = \mathscr{S}_{jk}(\mathbf{x}-\mathbf{y}) + \epsilon G^1_{jk} + O(\epsilon^2) \text{ with } G^1_{jk} = -3\delta_{jk}[1+\delta_{j3}]/(4a). \tag{32}$$

By linearity, the traction $\mathbf{f}^0$ arising on the particle surface when it translates at the velocity $\mathbf{U}$ in the absence of wall reads $\mathbf{f}^0 = \mu\mathbf{T}_0.\mathbf{U}$, with $\mathbf{T}_0$ being a second-rank tensor solely depending upon the particle geometry. Noting that the second-rank tensor $\mathbf{G}^1 = G^1_{jk}\mathbf{e}_j\otimes\mathbf{e}_k$ is uniform and substituting Eq. (32) into the boundary-integral equation (31) then easily shows that

$$\mathbf{f}_L^{(i)} = \mathbf{f}_L^{(i),0} + \epsilon\mathbf{f}_L^{(i),1} + O(\epsilon^2), \tag{33}$$

with $\mathbf{f}_L^{(i),0}$ being the surface traction prevailing *in the absence of wall*, and

$$\mathbf{f}_L^{(i),1} = \mathbf{T}_0.\left(\mathbf{G}^1.\mathbf{F}_L^{(i),0}\right)/(8\pi),\ \mathbf{F}_L^{(i),0} = \int_S \mathbf{f}_L^{(i),0}\mathrm{d}S. \tag{34}$$

From the approximation (33) it immediately follows that the particle second-rank resistance tensors admit the estimates

$$\mathbf{K} \sim \mathbf{K}_0 + \epsilon\mathbf{K}_1,\ \mathbf{V} \sim \mathbf{V}_0 + \epsilon\mathbf{V}_1,\ \mathbf{D} \sim \mathbf{D}_0 + \epsilon\mathbf{D}_1,\ \mathbf{W} \sim \mathbf{W}_0 + \epsilon\mathbf{W}_1, \tag{35}$$

with the Cartesian components of $\mathbf{K}_m, \mathbf{V}_m, \mathbf{D}_m$ and $\mathbf{W}_m$ for $m = 0.1$ obtained by replacing in Eqs. (14)–(15) the vector $\mathbf{f}_L^{(i)}$ with $\mathbf{f}_L^{(i),m}$. Combining Eq. (13) with Eq. (35) then gives the first-order approximations of the hydrodynamic force $\mathbf{F}_h$ and $\mathbf{C}_h$ for a given particle migration $(\mathbf{U},\mathbf{\Omega})$.

3.2. Estimates for $q, \mathbf{F_w}$ *and* $\mathbf{C_w}$. The boundary-integral equation (30) for q is now asymptotically inverted by using the approximation $(\mathbf{x}-\mathbf{y}_s)/|\mathbf{x}-\mathbf{y}_s|^3 \sim \epsilon^2 a^2\mathbf{e}_3/4$. One then immediately gets

$$q = q_0 + O(\epsilon^3) \text{ with } \textstyle\int_S q_0(\mathbf{y})\mathrm{d}S(\mathbf{y}) = 0, \tag{36}$$

where the charge density q_0 prevails when the liquid domain is unbounded. This density is thus solution to the Fredholm boundary-integral equation

$$\frac{q_0(\mathbf{x})}{2}+\frac{1}{4\pi}\int_S\left[\frac{\mathbf{x}-\mathbf{y}}{|\mathbf{x}-\mathbf{y}|^3}\right].\mathbf{n}(\mathbf{x})q_0(\mathbf{y})\mathrm{d}S(\mathbf{y})=-[\mathbf{E}.\mathbf{n}](\mathbf{x})\text{ for }\mathbf{x}\text{ on }S. \tag{37}$$

From Eq. (23) it easily turns out that $\boldsymbol{\sigma}_{\mathbf{w}}.\mathbf{n}=\boldsymbol{\sigma}_{\mathbf{w}_0}.\mathbf{n}+O(\epsilon^2)$ on S with

$$[\boldsymbol{\sigma}_{\mathbf{w}_0}.\mathbf{n}](\mathbf{x})=-\frac{\sigma}{8\pi}\times$$
$$\int_S q_0(\mathbf{y})\left[\frac{(\mathbf{x}-\mathbf{y}).\mathbf{n}(\mathbf{x})(\mathbf{x}-\mathbf{y})\wedge\mathbf{B}+\mathbf{n}(\mathbf{x}).[(\mathbf{x}-\mathbf{y})\wedge\mathbf{B}](\mathbf{x}-\mathbf{y})}{|\mathbf{x}-\mathbf{y}|^3}\right]\mathrm{d}S(\mathbf{y}). \tag{38}$$

Accordingly, $\mathbf{F}_{\mathbf{w}}=\mathbf{F}_{\mathbf{w}_0}+O(\epsilon^2)$ and $\mathbf{C}_{\mathbf{w}}=\mathbf{C}_{\mathbf{w}_0}+O(\epsilon^2)$ with the definitions

$$\mathbf{F}_{\mathbf{w}_0}=\int_S\boldsymbol{\sigma}_{\mathbf{w}_0}.\mathbf{n}\mathrm{d}S,\ \mathbf{C}_{\mathbf{w}_0}=\int_S\mathbf{x}'\wedge\boldsymbol{\sigma}_{\mathbf{w}_0}.\mathbf{n}\mathrm{d}S. \tag{39}$$

3.3. First-order estimates for $\mathbf{F}_{\mathbf{v}}$ *and* $\mathbf{C}_{\mathbf{v}}$. The results derived in sections 3.1 and 3.2 hold whatever the wall nature (insulating in Case 1 or perfectly conducting in Case 2). For conciseness, we, however, now restrict our attention to Case 2 of the perfectly conducting wall (as shown in Fig. 1).

From Eqs. (17)–(18) and our approximation (33), the asymptotic estimates of $\mathbf{F}_{\mathbf{v}}$ and $\mathbf{C}_{\mathbf{v}}$ are obtained by expanding the velocity $\mathbf{w}$ on $S\cup\Sigma$ and the tractions $\boldsymbol{\sigma}_L^{(i)}.\mathbf{e}_3$ on Σ. Appealing to Eq. (22) first shows that on the particle surface S

$$\mathbf{w}\sim\mathbf{w}_0+\epsilon\mathbf{w}_1,\mathbf{w}_0(\mathbf{x})=\left(\frac{\sigma}{8\pi\mu}\right)\left[\int_S q_0(\mathbf{y})\frac{\mathbf{x}-\mathbf{y}}{|\mathbf{x}-\mathbf{y}|}\mathrm{d}S(\mathbf{y})\right]\wedge\mathbf{B}, \tag{40}$$

$$\mathbf{w}_1=\left(\frac{\sigma}{16\pi\mu a}\right)[\mathbf{P}_0-(\mathbf{P}_0.\mathbf{e}_3)\mathbf{e}_3]\wedge\mathbf{B}\text{ and }\mathbf{P}_0=\int_S q_0(\mathbf{y})\mathbf{y}'\mathrm{d}S(\mathbf{y}), \tag{41}$$

where it is recalled that $\mathbf{y}'=\mathbf{y}-\mathbf{OO}'$. Note that $\mathbf{w}_1$ is constant.

To approximate $\mathbf{w}(\mathbf{x})$ for $\mathbf{x}$ on Σ, this time we use Eq. (25) which shows that $\mathbf{w}=\mathbf{0}$ if $\mathbf{B}\wedge\mathbf{e}_3=\mathbf{0}$ and also that, in general,

$$\mathbf{w}(\mathbf{x})\sim-\frac{\sigma}{4\pi\mu}\left[\frac{(\mathbf{P}_0.\mathbf{e}_3)}{|\mathbf{x}'|}-\frac{h^2(\mathbf{P}_0.\mathbf{e}_3)}{|\mathbf{x}'^3|}+\frac{\mathbf{P}_0.\mathbf{x}}{|\mathbf{x}'^3|}\right](\mathbf{e}_3\wedge\mathbf{B})\text{ for }\mathbf{x}\text{ on }\Sigma. \tag{42}$$

When deriving Eq. (42), we actually noted that with $\mathbf{x}-\mathbf{y}=\mathbf{x}'-\mathbf{y}'$ and for $\mathbf{y}$ located on the distant surface S the property $|\mathbf{y}'|=O(a)\ll h\leq|\mathbf{x}'|$, and for $\mathbf{x}$ on Σ the identities $x_3=0$ and $\mathbf{x}'.\mathbf{e}_3=-h$.

The leading approximation of $[\boldsymbol{\sigma}_L^{(i)}.\mathbf{n}](\mathbf{x})$ for $\mathbf{x}$ on Σ is derived from Eqs. (28)–(29) by replacing $\mathbf{f}_L^{(i)}$ with $\mathbf{f}_L^{(i),0}$ and setting $\mathbf{y}=-\mathbf{y}_s=\mathbf{OO}'=h\mathbf{e}_3$. Recalling that $x_3=0$ for $\mathbf{x}$ on Σ, Eq. (29) then becomes

$$T_{jk3}(\mathbf{x},\mathbf{y})\sim\frac{12h}{|\mathbf{x}'^5|}\Big\{x_jx_k+\delta_{j3}\delta_{k3}h^2+hc_k(\delta_{k3}x_j-\delta_{j3}x_k)\Big\}, \tag{43}$$

with $\mathbf{x}'=x_1\mathbf{e}_1+x_2\mathbf{e}_2-h\mathbf{e}_3$ for $\mathbf{x}$ on Σ. From Eq. (43) one gets on Σ the estimate

$$\left[\boldsymbol{\sigma}_L^{(i)}.\mathbf{n}\right](\mathbf{x})\sim\frac{3h}{2\pi|\mathbf{x}'^5|}\Big\{\Big(\mathbf{F}_L^{(i),0}.\mathbf{x}\Big)\mathbf{x}-h\mathbf{F}_L^{(i),0}.\mathbf{e}_3+h\left[h\mathbf{F}_L^{(i),0}.\mathbf{e}_3-\mathbf{F}_L^{(i),0}.\mathbf{x}\right]\mathbf{e}_3\Big\} \tag{44}$$

with $\mathbf{F}_L^{(i),0}$ already defined in Eq. (34). When integrating $\mathbf{w}.\boldsymbol{\sigma}_L^{(i)}.\mathbf{n}$ over Σ in Eqs. (17)–(18), we use Eqs. (42) and (44). For symmetry reasons, some contributions vanish. Retaining the non-zero terms finally yields the key approximation

$$\int_\Sigma \mathbf{w}.\boldsymbol{\sigma}_L^{(i)}.\mathbf{n}\mathrm{d}S \sim -\frac{3\sigma h}{8\pi^2\mu}(\mathbf{e}_3 \wedge \mathbf{B}).\int_\Sigma \left[\frac{(\mathbf{R}.\mathbf{x})\mathbf{x}}{|\mathbf{x}'^6|} - h^2\frac{(\mathbf{A}.\mathbf{x})\mathbf{x}}{|\mathbf{x}'^8|}\right]\mathrm{d}S, \quad (45)$$

$$\mathbf{A} = \left(\mathbf{F}_L^{(i),0}.\mathbf{e}_3\right)\mathbf{P}_0 + (\mathbf{P}_0.\mathbf{e}_3)\mathbf{F}_L^{(i),0},\ \mathbf{R} = (\mathbf{P}_0.\mathbf{e}_3)\mathbf{F}_L^{(i),0}. \quad (46)$$

The integral over the wall Σ in Eq. (45) is explicitly calculated using the relation $|\mathbf{x}'|^2 = x_1^2 + x_2^2 + h^2$ and the change of the variables $x_1/h = t\cos\theta, x_2/h = t\sin\theta$ with θ in $[0.2\pi]$ and $t \geq 0$. Because $\epsilon = a/h$, one then gets

$$\int_\Sigma \mathbf{w}.\boldsymbol{\sigma}_L^{(i)}.\mathbf{n}\mathrm{d}S \sim \epsilon\mathbf{L}\left[\mathbf{F}_L^{(i),0}\right].(\mathbf{e}_3 \wedge \mathbf{B}),\ \mathbf{L}[\mathbf{f}] = \frac{\sigma}{16\pi\mu a}\left[\frac{(\mathbf{f}.\mathbf{e}_3)}{2}\mathbf{P}_0 - (\mathbf{P}_0.\mathbf{e}_3)\mathbf{f}\right]. \quad (47)$$

Substituting Eqs. (40) and (47) into Eqs. (17)–(18) yields the following desired first-order behaviours

$$\mathbf{F}_\mathbf{v} = \mathbf{F}_\mathbf{v}^{(0)} + \epsilon\mathbf{F}_\mathbf{v}^{(1)} + O(\epsilon^2),\ \mathbf{C}_\mathbf{v} = \mathbf{C}_\mathbf{v}^{(0)} + \epsilon\mathbf{C}_\mathbf{v}^{(1)} + O(\epsilon^2), \quad (48)$$

with, as values prevailing in the absence of wall (unbounded liquid),

$$\mathbf{F}_\mathbf{v}^{(0)} = -\left[\int_S \mathbf{w}_0.\mathbf{f}_t^{(i),0}\mathrm{d}S\right]\mathbf{e}_i,\ \mathbf{C}_\mathbf{v}^{(0)} = -\left[\int_S \mathbf{w}_0.\mathbf{f}_r^{(i),0}\mathrm{d}S\right]\mathbf{e}_i \quad (49)$$

and also the formulae

$$\mathbf{F}_\mathbf{v}^{(1)} = -\left\{\int_S \mathbf{w}_0.\mathbf{f}_t^{(i),1}\mathrm{d}S + \mathbf{w}_1.\mathbf{F}_t^{(i),0} + \mathbf{L}\left[\mathbf{F}_t^{(i),0}\right].(\mathbf{e}_3 \wedge \mathbf{B})\right\}\mathbf{e}_i, \quad (50)$$

$$\mathbf{C}_\mathbf{v}^{(1)} = -\left\{\int_S \mathbf{w}_0.\mathbf{f}_r^{(i),1}\mathrm{d}S + \mathbf{w}_1.\mathbf{F}_r^{(i),0} + \mathbf{L}\left[\mathbf{F}_r^{(i),0}\right].(\mathbf{e}_3 \wedge \mathbf{B})\right\}\mathbf{e}_i. \quad (51)$$

Let us denote by $\mathbf{F}_e^{(0)}$ and $\mathbf{C}_e^{(0)}$ the electromagnetic force and the torque experienced by a particle *held fixed* in an unbounded liquid. Those vectors satisfy

$$\mathbf{F}_e^{(0)} = \mathbf{F}_{\mathbf{w}_0} + \mathbf{F}_\mathbf{v}^{(0)} - \sigma\mathscr{V}_\mathscr{P}(\mathbf{E} \wedge \mathbf{B}),\ \mathbf{C}_e^{(0)} = \mathbf{C}_{\mathbf{w}_0} + \mathbf{C}_\mathbf{v}^{(0)}. \quad (52)$$

In view of Eq. (48), the electromagnetic force and the torque exerted on a distant and *fixed* particle admit the expansions

$$\mathbf{F}_e = \mathbf{F}_\mathbf{v} + \mathbf{F}_\mathbf{w} = \mathbf{F}_e^{(0)} + \epsilon\mathbf{F}_\mathbf{v}^{(1)} + O(\epsilon^2), \quad (53)$$

$$\mathbf{C}_e = \mathbf{C}_\mathbf{v} + \mathbf{C}_\mathbf{w} = \mathbf{C}_e^{(0)} + \epsilon\mathbf{C}_\mathbf{v}^{(1)} + O(\epsilon^2). \quad (54)$$

3.4. First-order approximation of the particle migration. Now substituting the results (35) and (53)–(54) into Eq. (20) provides for a freely suspended distant particle rigid-body motion $(\mathbf{U}, \boldsymbol{\Omega})$ the following estimates

$$\mathbf{U} = \mathbf{U}^{(0)} + \epsilon\mathbf{U}^{(1)} + O(\epsilon^2),\ \boldsymbol{\Omega} = \boldsymbol{\Omega}^{(0)} + \epsilon\boldsymbol{\Omega}^{(1)} + O(\epsilon^2) \quad (55)$$

with migrations $(\mathbf{U}^{(0)}, \boldsymbol{\Omega}^{(0)})$ and $(\mathbf{U}^{(1)}, \boldsymbol{\Omega}^{(1)})$ governed by the linear systems

$$\mathbf{K}_0.\mathbf{U}^{(0)} + \mathbf{V}_0.\boldsymbol{\Omega}^{(0)} = \mathbf{F}_e^{(0)}/\mu,\ \mathbf{D}_0.\mathbf{U}^{(0)} + \mathbf{W}_0.\boldsymbol{\Omega}^{(0)} = \mathbf{C}_e^{(0)}/\mu, \quad (56)$$

$$\mathbf{K}_0.\mathbf{U}^{(1)} + \mathbf{V}_0.\boldsymbol{\Omega}^{(1)} = -[\mathbf{K}_1.\mathbf{U}^{(0)} + \mathbf{V}_1.\boldsymbol{\Omega}^{(0)}] + \mathbf{F}_\mathbf{v}^{(1)}/\mu, \quad (57)$$

$$\mathbf{D}_0.\mathbf{U}^{(1)} + \mathbf{W}_0.\boldsymbol{\Omega}^{(1)} = -[\mathbf{D}_1.\mathbf{U}^{(0)} + \mathbf{W}_1.\boldsymbol{\Omega}^{(0)}] + \mathbf{C}_\mathbf{v}^{(1)}/\mu. \quad (58)$$

Table 1. Properties of the vectors $\mathbf{F}_{\mathbf{v}}^{(1)}$ and $\mathbf{U}^{(1)}$ for orthotropic particles weakly interacting with a perfectly conducting plane wall normal to both $\mathbf{e}_3$ and to the ambient uniform electric field $\mathbf{E}$. In the last line of the table, c and d are the real coefficients solely depending upon the axisymmetric and 'non-inclined' orthotropic particle, the acronym AO in the second line means "arbitrary orientation".

Orthotropic particle nature	$\mathbf{F}_{\mathbf{v}}^{(1)}$	$\mathbf{U}^{(1)}$
inclined	AO	AO
non-inclined with $\mathbf{B} \wedge \mathbf{E} = \mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$
non-inclined with $\mathbf{B}.\mathbf{E} = 0$	$\mathbf{F}_{\mathbf{v}}^{(1)}.\mathbf{e}_3 = 0$	$\mathbf{U}^{(1)}.\mathbf{e}_3 = 0$
with axis of revolution parallel to $\mathbf{e}_3$	$c\sigma(\mathbf{E} \wedge \mathbf{B})$	$d\sigma(\mathbf{E} \wedge \mathbf{B})/\mu$

3.5. Discussion. The key vectors $\mathbf{F}_{\mathbf{v}}^{(1)}$ and $\mathbf{C}_{\mathbf{v}}^{(1)}$ characterize the $O(\epsilon)$ wall–particle electromagnetic interactions and deeply depend upon both the magnetic field $\mathbf{B}$ orientation and the distant particle shape. Those basic features are illustrated in this section by considering different types of particles exhibiting some symmetries.

(i) If the axis $(O', \mathbf{e}_3)$ belongs to one plane of symmetry of the particle, then $\mathbf{w}_1 = \mathbf{P}_0 \wedge \mathbf{e}_3 = \mathbf{0}$ because $\mathbf{E}$ is parallel to $\mathbf{e}_3$ (Case 2). If, in addition, $\mathbf{B} \wedge \mathbf{e}_3 = \mathbf{0}$, many simplifications occur in Eqs. (50)–(51).

(ii) If the particle is orthotropic, i.e. admits three orthogonal planes of symmetry intersecting at O' and normal to the unit vectors $\mathbf{e}'_1, \mathbf{e}'_2, \mathbf{e}'_3$ (with $\mathbf{e}'_j.\mathbf{e}'_k = \delta_{jk}$), some conclusions are easily drawn [2]. First, for such a particle both second-order tensors $\mathbf{V}_0$ and $\mathbf{D}_0$ vanish. Accordingly, $\mathbf{F}_r^{(i),0} = \mathbf{0}$ and, therefore, $\mathbf{f}_r^{(i),1} = \mathbf{0}$ and all the tensors $\mathbf{D}_1, \mathbf{V}_1$ and $\mathbf{W}_1$ vanish. From Eq. (51) one gets $\mathbf{C}_{\mathbf{v}}^1 = \mathbf{0}$, whereas we know (see [3]) that for an orthotropic particle $\mathbf{C}_e^0 = \mathbf{\Omega}^0 = \mathbf{0}$. It easily follows that, whatever the orientation of $\mathbf{B}$,

$$\mathbf{C}_e = O(\epsilon^2),\ \mathbf{\Omega} = O(\epsilon^2),\ \mathbf{U}^{(1)} = \{\mathbf{K}_0\}^{-1}.[\mathbf{F}_{\mathbf{v}}^{(1)}/\mu - \mathbf{K}_1.\mathbf{U}^{(0)}]. \tag{59}$$

Note that in general the velocity $\mathbf{U}^{(1)}$ is non-zero (see example (iii) below).

(iii) When the orthotropic particle has one plane of symmetry parallel to the plane wall Σ, it is termed 'non-inclined'. Since $\mathbf{E}$ is aligned with $\mathbf{e}_3$, one immediately gets, using the definition (41), the relations $\mathbf{P}_0 \wedge \mathbf{e}_3 = \mathbf{w}_1 = \mathbf{0}$. Accordingly, Eq. (50) reduces to

$$\mathbf{F}_{\mathbf{v}}^{(1)} = -\left\{ \int_S \mathbf{w}_0.\mathbf{f}_t^{(i),1} \mathrm{d}S + \mathbf{L}[\mathbf{F}_t^{(i),0}].(\mathbf{e}_3 \wedge \mathbf{B}) \right\} \mathbf{e}_i. \tag{60}$$

Using Eq. (60) and symmetries makes it possible to obtain additional properties for the vectors $\mathbf{F}_{\mathbf{v}}^{(1)}$ and $\mathbf{U}^{(1)}$. Those properties are listed in Table 1, in which the last reported case is the 'non-inclined' axisymmetric orthotropic particle with the axis of revolution normal to the wall Σ.

(iv) If the particle is spherical (with the radius a and center O'), the properties (i)–(iii) hold, and one can easily obtain the following analytical results (recall the introduction of the present paper, where $\mathbf{U}^{(0)}$ is given)

$$\mathbf{w}_0 = \frac{\sigma}{10}\{2a^2\mathbf{E} \wedge \mathbf{B} - (\mathbf{E}.\mathbf{x}')\mathbf{x}' \wedge \mathbf{B}\}, \mathbf{U}^{(0)} = -\frac{\sigma a^2}{6\mu}[\mathbf{E} \wedge \mathbf{B}], \tag{61}$$

$$\mathbf{P}_0 = -2\pi a^3 \mathbf{E},\ \mathbf{f}_t^{(i),0} = -\frac{3\mu}{2a}\mathbf{e}_i,\ \mathbf{f}_t^{(i),1} = \frac{9}{16}(1 + \delta_{i3})\mathbf{f}_t^{(i),0} \tag{62}$$

[2]Such conclusions will hold whatever the orthotropic particle, which may admit no plane of symmetry normal to $\mathbf{e}_3$ (the case of the inclined particle.)

without summation over indices i in Eq. (62). After elementary manipulations one then ends up with

$$\mathbf{F}_{\mathbf{v}}^{(1)} = \frac{15\sigma\pi a^3}{16}(\mathbf{E}\wedge\mathbf{B}),\ \mathbf{U}^{(1)} = -\frac{3}{2}\mathbf{U}^{(0)}. \tag{63}$$

The above results then show that $\mathbf{F}_{\mathbf{v}}^{(1)} = \mathbf{U}^{(1)} = \mathbf{0}$ when both $\mathbf{B}$ and $\mathbf{E}$ are normal to the perfectly conducting $x_3 = 0$ wall, while $\mathbf{F}_{\mathbf{v}}^{(1)}$ is non-zero and $\mathbf{U}^{(1)} = -3\mathbf{U}^{(0)}/2$ as soon as $\mathbf{B}\wedge\mathbf{e}_3 \neq \mathbf{0}$. In these latter circumstances, the sphere is found to experience $O(\epsilon)$ interactions with the distant perfectly conducting wall (in contrast to the conclusions drawn in [7] for the case of a plane insulating wall).

4. Conclusions. A new boundary approach has been proposed to determine the electromagnetic force and torque exerted on a non-conducting arbitray-shaped particle held fixed above a perfectly conducting or insulating plane wall and the incurred particle rigid-body motion when freely-suspended. The procedure reduces to the treatment of seven boundary integral equations on the particle surface for the occurring polarization surface charge density and to the surface traction arising for specific auxiliary Stokes flows.

The method has been asymptocally worked out for a distant particle in the case of a perfectly conducting wall. It has been then shown that in general the wall–particle electromagnetic interactions might be of larger magnitude than the ones predicted in [7] for an insulating sphere interacting with a distant and insulating wall. For conciseness, the case of a distant non-spherical particle interacting with an insulating plane wall (i.e. the extension of the analysis developed in [7]) has been postponed to a future study. Finally, one should note that such asymptotic results will be further employed to benchmark a numerical implementation of the advocated boundary treatment to be used for a particle located close to the wall. Such a key issue requires additional efforts and will be handled in another work.

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