ELECTROMAGNETIC INTERACTION OF A SMALL MAGNET AND LIQUID METAL FLOW IN A PIPE WITH INSULATING OR CONDUCTING WALLS

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We study the effects of electrically conducting walls on the interaction between a permanent magnet and a liquid-metal flow in a cylindrical pipe using experiments and numerical simulation. The problem is motivated by Lorentz force velocimetry, where the drag force on the magnet due to the induced eddy currents in the flow is used for flow measurement. Compared with insulating walls, the conducting walls lead to an increased drag force on the magnet. Except for low distances, the experimental results are satisfactorily reproduced in simulations using two different approximations of the magnetic field distribution.

Introduction. In recent years, several flow measurement methods based on electromagnetic induction have been developed. They can be applied for measurements in conducting liquids, which are typically hot and chemically aggressive. The main advantage of these methods is that they are contactless. In the present work, we are concerned with Lorentz force velocimetry (LFV) [1]. In this method, a magnet placed next to a moving conducting fluid causes induction of eddy currents, which give rise to a braking Lorentz force on the flow. An equal but opposite force acts on the magnet, which can be measured. The force depends on the conductivity and on the distribution of velocity and magnetic fields in the flow domain.

Typically, a permanent magnet system is used in LFV since the force measurement is the simplest in this case. When LFV is applied in high-temperature metallurgical processes, such a permanent magnet system requires thermal shielding. Another difficulty is due to the conductivity of the liquid, which is typically not very accurately known. Moreover, the force measurement itself is challenging because the forces are typically much smaller than the weight of the magnet system. Vibrations and other disturbances from the environment are, therefore, to be minimized. The temporal resolution of such a system is also limited by the mass of the magnet system.

The Lorentz force on the magnet system represents a weighted integral of the velocity distribution. The weight function depends on the distribution of the magnetic field and conductivity. LFV can, therefore, be used for global flow rate measurements when the magnetic field pervades the whole cross-section of a pipe or a duct. It can also be used for local velocity measurement when the magnetic field is suitably localized. Several studies of LFV have, therefore, examined the influence of the distance between a small magnet and the liquid as well as the effects of field distribution [2–4]. In those previous studies, eddy currents were confined to the moving conductor. We are now interested in configurations where eddy currents also spread to conducting walls confining a conducting liquid. When compared with insulating walls, eddy currents are closer to the magnet, in this case,

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provided that the distance of the magnet to the liquid remains the same. This may lead to a higher Lorentz force which could be advantageous for the application of LFV.

To study this effect, we carried out LFV experiments with a small permanent magnet and cylindrical pipes with insulating and electrically conducting walls. In addition, we carried out a numerical study with the commercial FEM software and a kinematic approach to verify that the effect could be predicted successfully. In the following, we describe the experiments and the numerical setup. After that we present a detailed comparison between experimental and numerical results for different flow rates and different magnet positions relative to the pipes. Finally, conclusions and suggestions for the future work are given.

1. Experimental installation and measurements. Our test facility depicted in Fig. 1 is a liquid metal loop [5] filled with the eutectic alloy GaInSn which is liquid at room temperature. The lower part of the loop comprises a circular stainless steel tube with an inner diameter of d = 27 mm and a wall thickness of 2.6 mm. The upper part consists of three independent, valve-controlled, 400 mm long circular test sections made of brass, copper and PVC, respectively. All three test sections have the same cross-section and wall thickness as the steel tube, and only one test section is active at a given time of an experiment. Therefore, the volumetric flow rate in the test sections can be controlled with a commercial inductive flowmeter (CopaXL DN25) from ABB that is located at the steel tube section. The flow is driven by an electromagnetic pump. The maximum flow rate corresponds to an average velocity u = 1.4 m/s.



Fig. 1. Experimental setup: (a) photo of the GaInSn loop. Lower section: steel loop with an electromagnetic pump and an inductive flowmeter. Upper section: three test sections of brass, copper, PVC, all operated independently. The force measurement system is behind the middle test section. (b) Close-up of the copper test section with the cubic permanent magnet of the L2F2.

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We used a local Lorentz force flowmeter (L2F2) [2] to generate and measure the Lorentz forces generated inside the GaInSn. The L2F2 comprises a cubic magnet of 1 cm side length that is attached to a force measurement system. The whole flowmeter can be positioned in the range of up to 5 cm both in the vertical (y-axis) and in the horizontal (z-axis) plane. The positioning is repeatable to within 0.05 mm. Fig. 2 shows the coordinates. Compared with Fig. 1b, this sketch is rotated by $\pi/2$ about the pipe axis. The magnet has a magnetic flux density of 475 mT on the surface. Inside the liquid metal, the maximum flux density is 300 mT (magnet at y = z = 0). The measurement system determines the axial component of the Lorentz force. It reaches a typical resolution of 0.3 μ N at a sampling rate of 6.3 Hz.

Before the start of a measurement, the L2F2 is positioned such that the magnet center is at the same height as the test section at its greatest width, i.e. at y = 0, and as close as possible to the test section without mechanical contact between the magnet and the section wall, as depicted in Fig. 1b. The measurement procedure is the same for all three test sections.

1.1. Measurement of F(u). The magnet is left at its original position. At the beginning of each measurement, the force on the magnet is recorded for one minute as a reference force. After one minute, the electromagnetic pump is turned on to the desired speed and the force is being recorded for at least another 100 s. The flow develops within a few seconds. The force reported here is the average of the final 300 recorded force values (roughly 50 s).

1.2. Measurement of F(y). After the horizontal control positioning, the magnet is put to its upmost position. There, the reference force is recorded for about 50 s. Afterwards, the flow velocity is set to 1 m/s, and the magnet is moved downwards in steps of 0.5 mm each 50 s. The total covered distance is 30 mm, where the start and end points are not symmetric about the mid-height of the test section. The reported forces are the averages over the final 40 s of each measurement step, as the first 10 s are omitted to ignore effects simply due to the movement of the magnet or to the change in pump speed at the beginning of a measurement.

1.3. Measurement of F(z). After horizontal positioning, the magnet is placed at 40 mm distance from the test section, where the reference force is recorded. After the flow velocity is set to 1 m/s, the magnet is moved towards the test section in 40 steps of 1 mm and in intervals of 50 s. Afterwards, the magnet is moved back to z = 1 cm, and the measurement is repeated with steps of 0.1 mm until the magnet reaches the wall of the test section.

2. Mathematical model. In our mathematical model, we employ two basic assumptions about the interaction of the flow with the magnetic field. The first one is that the induced field associated with eddy currents is small compared with the field **B** due to the magnet system. This assumption allows us to use the quasi static approximation of the induction equation. It is justified when the magnetic diffusivity. The second assumption is that the velocity field **u** is not affected by the induced Lorentz forces in the liquid. We can, therefore, consider the flow field as purely hydrodynamic. With the parameters of the experiment, it is fully turbulent. We shall employ a simple mixing-length formula for the mean flow. Fluctuations of the flow field will be ignored.

The geometry of the problem and the coordinate system are shown in Fig. 2, and the material properties and specific geometry parameters are given in Table 1. For the computations of eddy currents, we use a different coordinate system, where the x-axis coincides with the cylinder axis. The coordinate system in Fig. 2 is



Fig. 2. Problem geometry with the coordinate system for magnet position.

property	symbol	value	unit
density of GaInSn	ho	6492	kg/m^3
kinematic viscosity of GaInSn	u	$3.4 \cdot 10^{-7}$	m^2/s
conductivity of GaInSn	σ_1	$3.4\cdot 10^6$	S/m
conductivity of copper	σ_2	$58 \cdot 10^6$	S/m
conductivity of brass	σ_3	$16 \cdot 10^6$	S/m
inner pipe radius	R_1	$1.35 \cdot 10^{-2}$	m
wall thickness	$R_2 - R_1$	$2.7\cdot 10^{-3}$	m

Table 1. Material and geometry parameters.

used to specify the location of the magnet and has already been introduced in the previous section. Its origin is on the outer surface of the pipe, i.e. z = 0 corresponds to the magnet touching the pipe wall.

Since the conductivities of the wall and liquid are different, they require separate computational domains. In the quasi static formulation of the induction equation, the induced currents are given by the Ohm's law with the induced electric field represented by the negative electric potential gradient, i.e.

$$\mathbf{j}_1 = \sigma_1 \left(-\nabla \phi + \mathbf{u} \times \mathbf{B} \right) \tag{1}$$

in the liquid. In the stationary conducting wall there is no electromotive force, i.e. the currents are simply caused by the gradient of the electric potential:

$$\mathbf{j}_2 = -\sigma_2 \nabla \phi. \tag{2}$$

Due to the high conductivity, both the liquid and the walls remain electrically neutral, and the charge conservation then reduces to $\nabla \cdot \mathbf{j}_i = 0$, i.e. one has to solve a Poisson equation

$$\nabla^2 \phi = \nabla \cdot (\mathbf{u} \times \mathbf{B}) \tag{3}$$

for the potential in the liquid, and the Laplace equation

$$\nabla^2 \phi = 0 \tag{4}$$

in the solid wall. Boundary conditions on the inner boundary (liquid to solid) are imposed by charge conservation and continuity of the tangential electric field, i.e. ϕ and the normal component j_n of the electric current are continuous. When the wall is insulating, the condition simplifies to $j_n = 0$. On the outer boundary (wall to air), the normal component $j_n = 0$.

For a complete specification of the problem, the velocity and the magnetic fields have to be given. The mean velocity is purely axial, i.e. $\mathbf{u} = (u_x(r), 0, 0)$ in



Fig. 3. Velocity profile in the pipe for u = 1 m/s. The friction velocity is $u_{\tau} = 0.074 \text{ m/s}$.

the coordinate system with the x-axis as the pipe axis. According to [6],

$$u_x(r) = \frac{u_\tau}{\kappa} \ln\left(1 + \kappa \frac{u_\tau R_1}{\nu} \frac{1}{2} \left(1 - \frac{r^2}{R_1^2}\right)\right).$$
 (5)

In this equation, u_{τ} is the friction velocity and $\kappa = 0.42$ is the von Karman constant. u_{τ} has to be chosen such that the desired mean velocity u is obtained from the integration of Eq. (5) over the cross-section. A typical profile is shown in Fig. 3.

The magnetic fields in the experimental setup are generated by a cubic permanent magnet with a side length L = 10 mm. To describe its magnetic field distribution, we use two different approaches. The first is to approximate the magnet by a point dipole located at the center of the permanent magnet. In this dipole approximation, the magnetic field at the position **r** relative to the center of the magnet is

$$\mathbf{B}\left(\mathbf{r}\right) = \frac{\mu_{0}}{4\pi} \left(3\frac{\mathbf{m}\cdot\mathbf{r}}{r^{5}}\mathbf{r} - \frac{\mathbf{m}}{r^{3}}\right). \tag{6}$$

The magnetic moment **m** can be calculated from the measured distribution [2] of the magnetic field **B** at larger distances according to Eq. (6). The numerical value is m = 1.1 Am².

In the second approach, the magnet is finite, i.e. a cube with uniform magnetic moment density (magnetization) parallel to one of the edges. The field is obtained via the magnetostatic potential, which is the solution of the Poisson equation with the divergence of the magnetization as a source term. This divergence is concentrated as an effective magnetic surface charge density on the two faces of the cube, whose normal is aligned with the magnetization. If we assume the cube to be aligned with the coordinate axes and the magnetization is along z, then the magnetic field at x, y, z is

$$\mathbf{B} = \frac{\mu_0 M_{\rm S}}{4\pi} \sum_{k=1}^2 (-1)^k \int_{y_1}^{y_2} \int_{x_1}^{x_2} \frac{\left[(x - \tilde{x}) \,\mathbf{e}_x + (y - \tilde{y}) \,\mathbf{e}_y + (z - z_k) \,\mathbf{e}_z \right]}{\left[(x - \tilde{x})^2 + (y - \tilde{y})^2 + (z - z_k)^2 \right]^{3/2}} \mathrm{d}\tilde{x} \mathrm{d}\tilde{y}. \tag{7}$$

In this equation, the points (x_i, y_j, z_k) denote the eight corners of the cube, and $M_{\rm S}$ is the magnetization. The integration can be carried out analytically and provides somewhat lengthy expressions of elementary functions [7]. The magnetization is obtained from the magnetic moment: $M_{\rm S} = m/L^3 = 1.1 \cdot 10^6$ A/m.



Fig. 4. Ratio of the components B_z of the finite magnet and dipole on the z-axis as a function of the distance from the surface of the finite magnet. Distance is measured in units of the side length L. The magnetization is along the z-axis.

Differences between the fields of the finite magnet and the point dipole are expected when the distance to the magnet is small. Fig. 4 shows that these differences are significant when the distance is comparable to or smaller than the side length of the magnet. The singular behavior of the dipole field at the origin leads to a significant overestimation. For distances twice larger than the side length, there is a good agreement.

The Lorentz force on the magnet is opposite to the total Lorentz force exerted on the current distribution in the solid wall and in the liquid. We compute the latter force,

$$\mathbf{F} = \int \mathbf{j} \times \mathbf{B} \, dV \tag{8}$$

once the current density \mathbf{j} due to the magnetic field \mathbf{B} is known from the solution of Eqs. (3, 4). We consider only the component of the force in the axial direction of the pipe. The other components are zero.

3. Numerical solution. For the numerical solution of the Poisson equations (3), (4) for the electric potential, we used the PDE module in the commercial finite-element software package Comsol Multiphysics. The velocity distribution and the primary field were specified directly by Eqs. (5) and (6) or (7). After that, the current densities in the liquid and in the walls were computed from Eqs. (1, 2) and used to determine the Lorentz force from Eq. (8). These computations have been also performed within Comsol. The computational domains are a full inner cylinder for the liquid and a surrounding hollow cylinder for the solid walls. Both have a finite length L_x . On the axial faces of the cylinders, insulating boundary conditions are applied.

We used non-uniform structured meshes of second-order hexahedral elements similar to those used in [8]. They provided a finer meshing close to the magnet. An example is displayed in Fig. 5. The central part of the inner cylinder crosssection is covered with a deformed mesh that conforms to a circular boundary of radius $R_1/4$. This was done to simplify the meshing close to the magnet. The surrounding annular part has non-equidistant grid lines along the radial direction. In the azimuthal direction, the grid lines are non-equidistant in the upper half of the annular region. The mesh for the hollow outer cylinder has a similar layout. In the axial direction, the grid lines also have non-uniform spacing. The magnet is located in the mid-plane, where the density of the grid lines is high.



Fig. 5. Typical structure of the Comsol meshes: (a) view of the cross-section, (b) top view. The inner boundary between the liquid and the wall is apparent in (a) from the high density of the radial grid lines.

Table 2. Parameters of different meshes. N_a , N_r and N_l are the numbers of azimuthal, radial and axial grid lines.

$N_{\rm a}$	N_1	$N_{\rm r}$	Number of elements	Degrees of freedom
40	60	10	41340	$1.7\cdot 10^5$
55	75	25	159300	$8.1\cdot 10^5$
70	90	40	378810	$1.8\cdot 10^6$
85	105	55	729120	$2.19\cdot 10^6$
100	120	70	1239480	$4.14 \cdot 10^6$

Different values of the length L_x were tried to ensure convergence to the limit of infinite L_x . For the range of magnet distances considered, the computed force did not change once L_x exceeded $5R_2$. Grid convergence has been tested with different meshes listed in Table 2. The result is illustrated in Fig. 6. For the actual computations, the mesh with the largest number of elements was consistently used.



Fig. 6. Grid convergence for copper walls at u = 1 m/s and for the magnet at y = z = 0.



Fig. 7. Experimental results at the fixed velocity u = 1 m/s.

4. Results and discussion. We first consider the effect of the magnet position. As can be seen in Fig. 7, there is a fairly rapid decrease in Lorentz force when the magnet is moved in the transverse z-direction in the experiment. A change in tangential y-direction leads to a more gradual reduction. Fig. 7 also shows that the conducting walls provide consistently larger forces for the same magnet position. Compared with the insulating PVC walls, the force for brass is typically about 10% higher. For copper, the force can be increased by about 40%. This observation can be attributed to the effectively larger distance between the eddy currents and the field source for insulating walls. The currents in the walls are exposed to a higher magnetic induction because the field decreases strongly with the distance. For this reason, it is plausible that the conducting walls amplify the Lorentz force.



Fig. 8. Comparison of the measured and calculated Lorentz forces at u = 1 m/s.

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Fig. 9. Ratio of theoretical and experimental Lorentz forces (finite magnet).

In Fig. 8 these results are compared with the numerical solutions obtained with the dipole field and the finite magnet. The magnetization is aligned with the z-axis. Typically, the forces are overestimated by the computations with both field models. The disagreement between theory and experiment is significant at small distances and decreases with y and z. Unsurprisingly, the agreement with the experiment is generally better for the finite magnet, but this is pronounced only at small z and y. For distances larger than the side length L of the magnet, there is not much difference between the finite magnet and the dipole field.

The relative error between experiment and theory is further illustrated in Fig. 9, which shows the ratio between the theoretical and experimental Lorentz force values as a function of the distance for PVC and copper walls. The simulation used the finite magnet model. For both displacements, the error was as large as 40% at small distances and dropped to about 2% or less at the largest distances. For the insulating pipe, good agreement was found at smaller distances than for the copper pipe. The result is somewhat disappointing because the field of the finite magnet should be fairly realistic also at small distances. Nevertheless, better agreement at larger distances may be in part due to the inaccurate representation of the near field. Another cause of the differences could be the flow modification by the Lorentz force in the experiment. This would likely reduce the experimental values since the flow would tend to avoid the regions of strong field. Such a reduction is in line with Fig. 9b. However, the apparent initial increase in error with the distance in Fig. 9a does not fit this explanation.

The velocity dependence is shown in Fig. 10. Both the experimental and the theoretical forces appear to satisfy a linear relation $F \sim u$ to a good degree.



Fig. 10. Velocity dependence (magnet at y = z = 0, finite magnet model).

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Stronger deviations from this linear relation might have been expected since the velocity distribution of a turbulent flow changes with the Reynolds number. However, this happens mainly near the wall. It seems to have only a small effect on the Lorentz force as the magnetic field does not rapidly vary near the inner wall due to the minimum distance between the pipe and the magnet. The quantitative agreement between the simulation and the experiment is comparable with Fig. 9.

5. Conclusions. We have performed experiments and kinematic simulations of the electromagnetic interaction between a small magnet and a liquid-metal pipe flow with insulating and conducting walls. In the envisaged application of such a setup for Lorentz force velocimetry, the conducting walls are beneficial because they increase the measured force. For the Lorentz force, the agreement between the simulations and the experiments is generally good except for small distances. The differences may be caused by dynamic effects, i.e. the flow modification by the Lorentz forces. This should be addressed in future dynamic simulations.

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