MAGNETOHYDRODYNAMICS Vol. 51 (2015), No. 3, pp. 543–550

INFLUENCE OF FEEDBACK LOOP CHARATERISTICS ON THE PERFORMANCE OF A TRAVELLING WAVE THERMOACOUSTIC ENGINE

J.H. Goh[1], *Y.A. Abakr*[1], *D.B. Hann*[2]

[1] *Department of Mechanical, Manufacturing and Materials Engineering, The University of Nottingham Malaysia Campus, Jalan Broga, 43500 Semenyih, Selangor Darul Ehsan, Malaysia*
E-mail: yousif.abakr@nottingham.edu.my
[2] *Department of Mechanical, Manufacturing and Materials Engineering, The University of Nottingham, University Park, Nottingham, NG7 2RD, UK*

A method to find experimentally the equivalent effective length of the Thermoacoustic (TA) core of a TA Engine (TAE) is proposed. The TAE is first operated with various Feedback Loop (FL) length configurations. For every configuration, the frequencies of sound waves were recorded to determine the wavelengths. These wavelengths are compared with the measured FL lengths and a constant deviation was observed, which gives the effective length of the TA Core as 55 cm. This effective length is used in the calculation of the FL ratio. The peak pressure in the TA cores and the onset Temperature Difference (TD) of the TAE with various FL ratios were compared. A normalized intensity of the sound wave produced by the TAE was also compared for different FL ratios. The optimum FL ratio was found to be 3.37.

Introduction. In 1979, Ceperley explained the fundamental principles of a travelling wave TAE and its potential of achieving higher efficiency compared to standing wave TAE [1]. However, only 20 years later this potential was significantly demonstrated by Backhaus and Swift [2]. By placing the regenerator in a toroid tube connected to a resonance tube, the travelling wave TAE designed by Backhaus *et al.* operated at 41% of Carnot efficiency, which was at least 50% higher than the best standing wave TAE at that time. Despite demonstrating that high efficiency is achievable, the design and operating conditions of this TAE is not feasible for many real world applications. Biwa *et al.* has shown that increasing the number of regenerators in the toroid is able to reduce the onset temperature of the TAE, thus increasing the possibility of different heat sources to the TAE [3]. Also, De Blok suggested that the regenerator in the toroid shape had to overcome large acoustic losses when connected to the resonance tube [4]. To overcome this, the resonance tube is removed entirely and the regenerators are connected in a complete loop, which is denoted as the FL. De Bloks design involves 4 regenerators positioned at quarter wavelengths apart so that the impedance of the TA core matches the impedance of the FL in a similar manner as a quarter wave impedance transformer, thus eliminating reflection in the FL [5]. The minimum onset temperature difference achieved by this engine is 30 K [5]. These works have encouraged developments of multi-stage travelling wave TAEs.

At the University of Nottingham (UoN) and University of Nottingham Malaysia Campus (UNMC), together with the collaboration of the SCORE project [6], a two stage travelling wave TAE has been designed and manufactured by Baiman *et al.* [7, 8]. This TAE has two identical TA cores connected by the FL. Thus, the FL is separated into two sections. The lengths of these two FL sections are intended to be a quarter wavelength and three quarter wavelength to match the impedance of the TA core and the FL. However, the dimensions of the parts in the TA core also contribute to the length of the FL. Along with the complicated

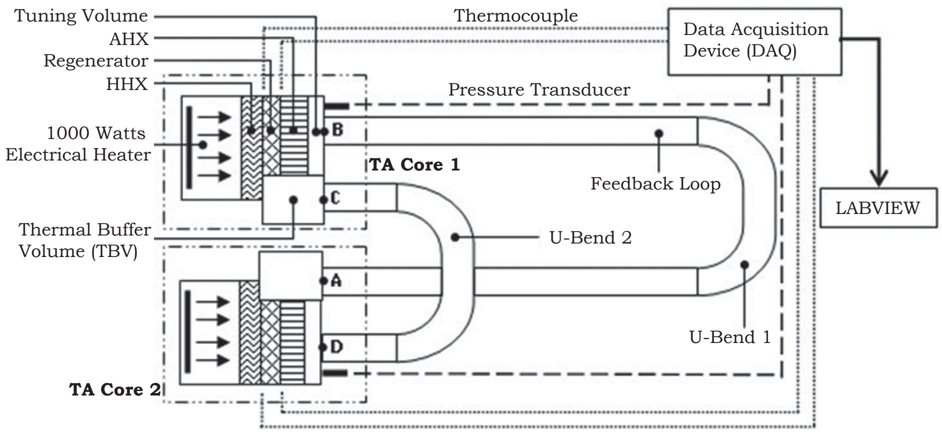

Fig. 1. Schematic diagram of the two-stage travelling wave TAE. Both TA cores are identical. Illustration is not to scale.

design in the TA core, it was found to be very challenging to obtain the optimum length configuration of the FL to achieve the quarter and three quarter wavelengths configuration.

This paper describes an experimental investigation conducted to obtain an equivalent length contributed by the TA core and subsequently obtain the optimum length configuration of the FL. An offset between the theoretical and measured length of the FL indicates the equivalent length of the TA core. It is found that the TA core of the TAE has an equivalent length of 55 cm. This 55 cm is included in the calculations of the FL ratio and the optimum FL ratio is 3.4. Modifications were made to the experimental set up as part of the future work.

1. Experimental setup. The two-stage travelling wave TAE mentioned above was used in this experiment. The schematic diagram of the TAE is shown in Fig. 1 and a detailed description of the TAE is presented in this paper [7, 8]. Referring to Fig. 1, the FL connects both TA cores to form a closed loop. There are two U-bends in the loop. The straight sections of the FL are made by connecting PVC pipes of various lengths until the desired length is achieved. This allows the FL length to be varied and so does the operating frequency. During the experiment, the following data were measured and recorded:

the frequency f of the oscillation;

the peak pressures at TA cores 1 and 2;

the temperatures at the hot heat exchangers (HHX) and ambient heat exchangers (AHX);

the difference between both heat exchangers temperatures will be denoted as temperature difference (TD) in this paper;

the length of section AB and section CD in Fig. 1.

The HHX is heated from room temperature until 400°C, allowing the onset TD of the engine to be recorded. The AHX is kept at room temperature through water cooling. K-type thermocouples are attached to both sides of the regenerator in both TA cores. The pressure transducers used are Kulite HKL-375 (M) Series. A self-written Labview Program is used to record the experimental data.

2. Experimental results. The lack of moving parts in a TAE allows the TA core to be designed to facilitate the performance of the heat exchangers. However, this would cause the TA core to be designed with complex geometries, as evident in the TAE used in this work. Due to this, it is very challenging to measure

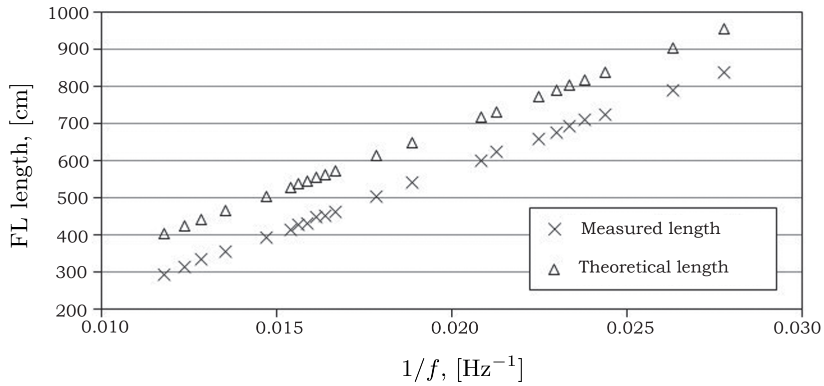

Fig. 2. Measured and theoretical FL length at various frequencies f.

the effective length of the components in the TA core, which contributes to the overall length of the FL. Thus, an experimental investigation was carried out to estimate the effective length of the TA core. The TAE was first tested at various FL configurations to obtain a range of operating frequencies. Then, by using the following relation,

$$v = f\lambda, \tag{1}$$

where v is the speed of sound (343 m/s) and f is the frequency of the wave in the engine, the wavelength λ can be obtained. This wavelength is the theoretical length of the FL. Fig. 2 shows the measured and theoretical FL length at various frequencies. From Eq. (1), it is apparent that the wavelength and $1/f$ have a linear relationship, thus the FL lengths are plotted against $1/f$. The measured length refers to the physically measured length. The lengths of both U-bend centerlines are measured physically as 57 cm. From Fig. 2, it is observed that there is an offset of 1.1 m between the measured and theoretical lengths for every frequency, which reveals the length contributed by both TA cores. Thus, ignoring manufacturing tolerances, a single TA core contributes 0.55 m.

By adding 0.55 m to the length of sections AB and CD, the FL ratio of every FL configuration was computed by

$$\text{FL ratio} = \frac{0.55\,m + \text{length AB}}{0.55\,m + \text{length CD}} \tag{2}$$

The TAE performance at various FL ratios is compared in Fig. 3. The average onset TD between both TA cores (left vertical axis) and peak pressures at a TD of 300°C (right vertical axis) are plotted against the FL ratio. When the TD reaches 300°C, the corresponding peak pressures in both TA cores are recorded to give a common point for comparison. The desired performance is for a low onset TD and high peak pressure, which was observed at an FL ratio of 3.4. Also note that at smaller FL ratios, the peak pressure in the TA core 1 is lower than that in the TA core 2, but at larger FL ratios the peak pressure in the TA core 1 is higher. The FL ratio that gives the highest peak pressures at the TD 300°C occurs when the peak pressure at both TA cores is similar, indicating that both TA cores are performing at their optimum at this FL ratio. At the FL ratio 3.4, the lengths AB and CD are 5.51 m and 1.23 m, respectively. The onset TD recorded for the TA cores 1 and 2 are 167.61°C and 136.66°C, respectively.

However, to achieve the quarter wavelength and three quarter wavelength configuration, an FL ratio of 3.0 is desired. The TAE performs at its optimum

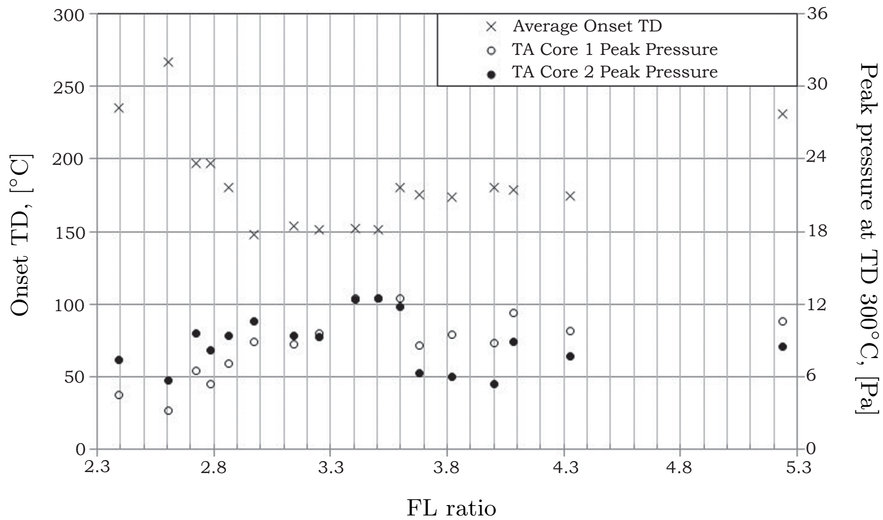

Fig. 3. Comparison of the TAE performances at various FL ratios.

at the FL ratio 3.4 and indicates that a slight impedance mismatch between the FL and the TA core is desired. A slight impedance mismatch will introduce a minor standing wave component to the sound wave which has shown to improve the performance of the TAE [9].

Subsequently, 16 pressure transducers are added to the FL to investigate the sound wave produced by the TAE. The pressure transducers used are Impress Sensors Type IMP-G0500-1A4-AAV-02-000. The pressure transducers are mounted to the FL in pairs and 15 cm apart in every pairings, as shown in Fig. 4. The peak pressures along the FL were recorded when the TD in both TA cores reached 270°C for three FL configurations, as shown in 5. The position and the peak pressure recorded by the pressure transducers are shown by the markers (△) in Fig. 5. Point A (from Fig. 1) is set as the starting position and the position of all the components in the FL are referenced to point A. From Fig. 5, it is observed

Fig. 4. The TAE with 8 pairs of pressure transducers mounted on the FL.

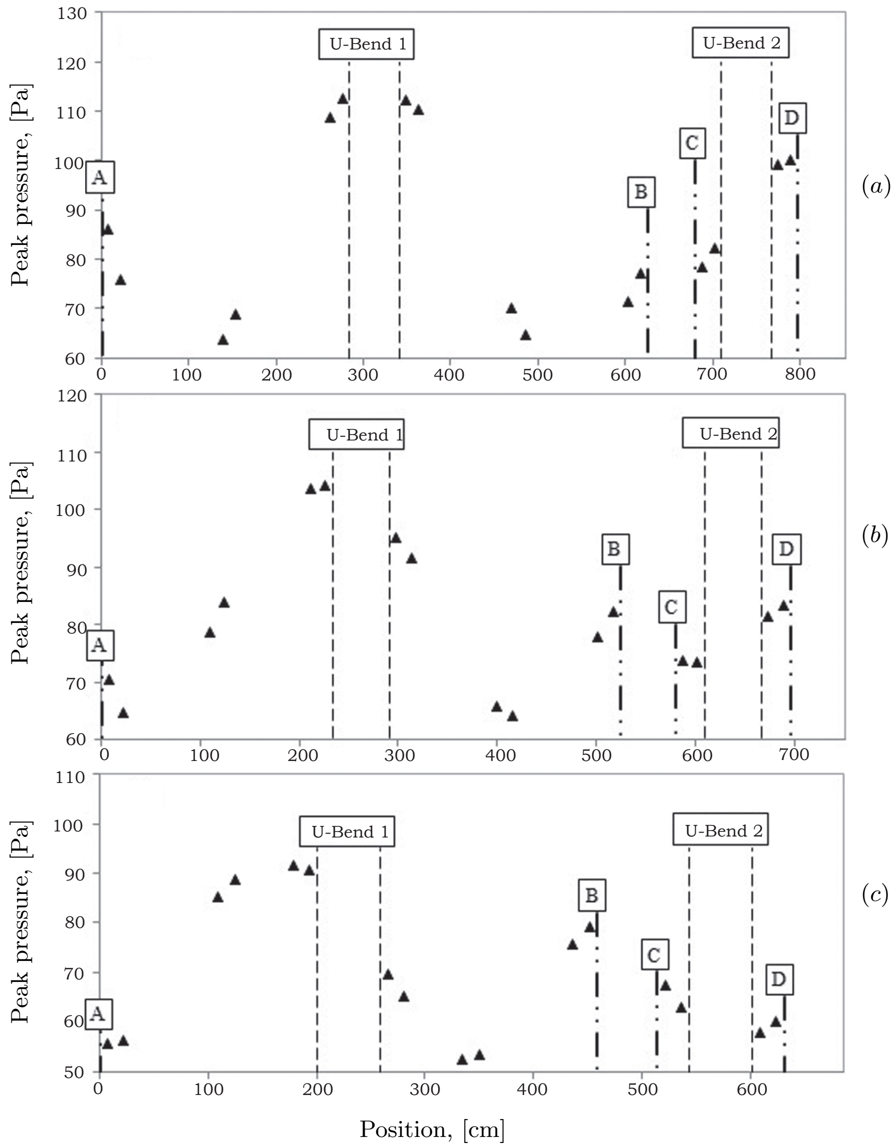

Fig. 5. Peak pressures along the FL for the TAE with FL ratios of (*a*) 3.95, (*b*) 3.37 and (*c*) 2.99.

that the sound wave produced by the TAE is amplitude-modulated, indicating a standing wave component is present in the sound wave.

By placing the pressure transducers in pairs, the intensity of the sound wave along the FL could be estimated [10, 11]. The intensity I of a wave is defined as the time averaged product of the pressure $p(t)$ and the particle velocity $u(t)$,

$$I = p(t)u(t), \tag{3}$$

The bar indicates time averaging. The particle velocity $u(t)$ is estimated from the pressure difference between two closely spaced pressure transducers by using the Eulers equation:

$$\frac{\partial u}{\partial t} = -\frac{1}{\rho}\frac{\partial p}{\partial x} \tag{4}$$

$$u(t) = -\frac{1}{\rho}\int \frac{\partial p}{\partial x}\mathrm{d}t, \tag{5}$$

where ρ is the density of the working gas. Then by using the finite difference estimation, the particle velocity in the centre of the pressure transducers is estimated as

$$u(t) = -\frac{1}{\rho}\int_{-\infty}^{t} \frac{p_2(\tau) - p_1(\tau)}{\partial \Delta x}\mathrm{d}\tau, \tag{6}$$

where $p_1(\tau)$ and $p_2(\tau)$ are the pressure readings from both pressure transducers and τ is a dummy time variable. The pressure at the centre of the pressure transducers is estimated by taking the average of the two transducers.

$$p(t) = \frac{p_1(t) + p_2(t)}{2}. \tag{7}$$

Sub-equations (6) and (7) into Eq. (3) yield

$$I = -\frac{1}{2\rho\Delta x}\left|p_1(t) + p_2(t)\right| \int_{-\infty}^{t} \frac{p_2(\tau) - p_1(\tau)}{\partial \Delta x}\mathrm{d}\tau, \tag{8}$$

Using Eq. (8), the intensity of the sound wave produced by the TAE was estimated. The intensity of the sound wave at two locations was of particular interest, which is at point A and point C. Point A and point C are the location, where the sound wave is "exiting" the TA cores 1 and 2, respectively. Thus, the intensity at the center of a pressure transducers pair nearest to point A and point C was estimated. The intensities of the FL configurations in Fig. 5 were estimated. To compare the intensities at different FL configurations, a normalized intensity was formulated. The intensity was first estimated at all FL configurations and then was normalized by dividing the average intensity of all FL configurations. Therefore, a normalized intensity of 1 indicates the TAE is operating at an average condition. The normalized intensity above unity indicates an above average performance and is thus desired. The normalized intensity of the wave "exiting" TA cores 1 and 2 for different FL ratios is shown in Fig. 6. From Fig. 6, it is observed that the TAE performs best at an FL ratio of 3.37, which is consistent with the trends in Fig. 3.

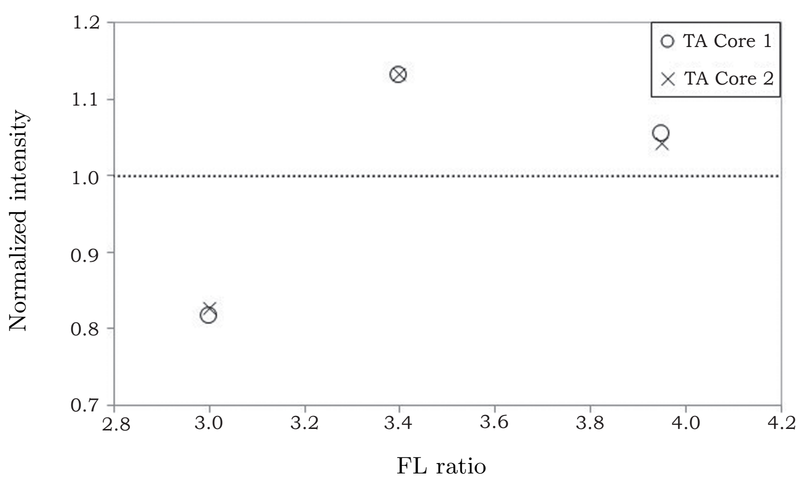

Fig. 6. Normalized intensity of the TAE at different FL ratios.

These will be the subject of the future work. The TAE used is still in its development stages and other features in the TA core are still not optimized. The work presented is useful in predicting the optimum length configuration of the FL of the TAE, especially when modifications are made to the TA core. It is also important to note that this experiment was conducted under the TAEs operating conditions.

3. Conclusion. In a two-stage travelling wave TAE developed by the SCORE project, the FL was designed such that the two TA cores are a quarter wavelength apart and three quarter wavelength apart for the remainder of the FL. However, due to the complexity of the TA core, the length of the FL is hard to estimate, as the effective length of the TA core has to be accounted for as well. An experimental investigation to obtain the effective length of the TA core was explained. The effective length was found to be 55 cm, and this is then used in the calculations of the FL ratio. A figure of the onset temperature difference and peak pressure versus the FL ratio was plotted to compare the TAE performance at various length configurations. It has been found that the TAE performs best at the FL ratio 3.4.

Subsequently, pressure transducers were added to the FL to measure the peak pressure along the FL and estimate the normalized intensity. The normalized intensity of the TAE at the FL ratio 3.4 is the highest, confirming that the FL ratio 3.4 is the optimum for the TAE.

REFERENCES

[1] P.H. Ceperley. A pistonless Stirling engine – the traveling wave heat engine. *J. Acoust. Soc. Am.*, vol. 66 (1979), no. 5, pp. 1508–1513.

[2] S. Backhaus and G. Swift. A thermoacoustic-Stirling heat engine: detailed study. *J. Acoust. Soc. Am.*, vol. 107 (2000), no. 6, pp. 3148–66.

[3] T. Biwa, D. Hasegawa, and T. Yazaki. Low temperature differential thermoacoustic stirling engine. *Appl. Phys. Lett.*, vol. 97 (2010), no. 3, p. 034102.

[4] K. De Blok. Low operating temperature integral thermo acoustic devices for solar cooling and waste heat recovery. *J. Acoust. Soc. Am.*, vol. 123 (2008), no. 5, p. 354.

[5] K. De Blok. Novel 4-stage travelling wave thermoacoustic power generator. *ASME 2010 3rd Jt. US-European*, 2010, pp. 1–8.

[6] P.H. Riley, C. Saha, and C.M. Johnson. Designing a low-cost, electricity-generating cooking stove. *IEEE Technol. Soc. Mag.*, vol. Summer (2010), pp. 47–53.

[7] B. Chen, P. Riley, Y. Abakr, K. Pullen, D. Hann, and C. Johnson. Design and development of a low-cost, electricity-generating cooking score-stove. *Proc. Inst. Mech. Eng. Part A J. Power Energy*, vol. 227 (2013), no. 7, pp. 803–813.

[8] B. Chen and Y. Abark. Development and assessment of a SCORETM DEMO2.1. Thermo-acoustic engine. *J. Eng.*, vol. 8 (2013), no. 2, pp. 253–263.

[9] G. Petculescu and L.A. Wilen. Travelling-wave amplification in a variable standing wave ratio device. *Acoust. Res. Lett. Online*, vol. 3 (2002), no. 2, p. 71.

[10] F. Fahy. *Sound Intensity* (2nd ed. Taylor & Francis, Inc., 1995).

[11] F. Jacobsen and H.-E. de Bree. A comparison of two different sound intensity measurement principles. *J. Acoust. Soc. Am.*, vol. 118 (2005), no. 3, p. 1510.

Received 31.10.2014