DEFORMATION OF A DOUBLE-LAYER DROP IN AN ALTERNATING ELECTRIC FIELD

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Abstract: In the present work, the problem of deformation of a double-layer drop, which is surrounded by other fluid, in an alternating electric field is solved. Permittivity and conductivity of all fluids are considered as constants. Fluids are assumed to be incompressible and viscous. The problem is solved by the method of expansion on small parameter which is proportional to the amplitude of an electric field squared. At the solution capillary forces are taken into account, the gravity is not taken into account. Average shape of the surface of the double-layer drop is found.

1. Introduction

The deformation of a simple liquid drop, immersed in other conducting fluid, which is subjected to an electric field, was investigated by many authors [1-5]. The most interesting researches deal with the form's dependency on the frequency of the harmonic electric field [2, 3, 5]. But there is one common thing in these papers: it is not possible for a prolate drop to become oblate while the frequency of electric field increases. In [5] the convective transport of a surface charge was taken into account, but it makes drops only more prolate. The purpose of this work is to investigate the form in the framework of double-layer liquid drop, and compare results with well-known ones in the framework of simple drop [2].

2. Presentation of the problem

To describe the task, an electrohydrodynamic model is used. Electrohydrodynamics is a branch of fluid mechanics concerning electrical force effects, at the same time neglecting magnetic force effects. More specifically this model is described in [1].

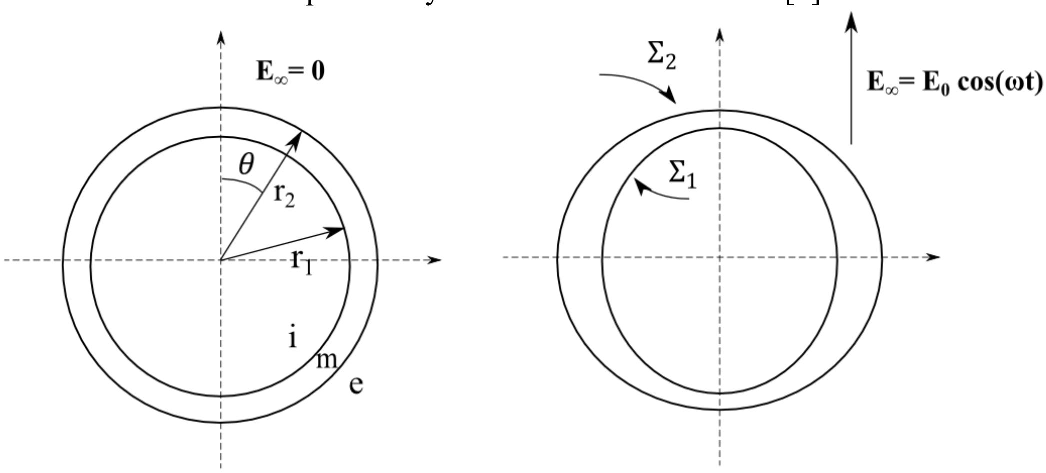

Figure 1: Double-layered drop in the harmonic electric field

Now let us consider an axisymmetric double-layer liquid drop, immersed in another liquid, and subject it to a harmonic electric field E_∞ (Fig. 1). If the intensity of the field equals zero, surfaces of the double-layer drop are spherical with radii $r=r_1$, $r=r_2$. Liquids are assumed to be dielectric with small electric conductivity. The dielectric permeability ε and conductivity λ of the liquids are considered as constants. Also, all liquids are incompressible and significantly viscous (μ = const). Moreover, the velocity is expected to be small, so it leads to a small Reynolds number. Because of it, the Stokes approximation can be used. Considering all these assumptions, fluid motion equations can be written as follows:

$$\Delta\phi = 0, \qquad \boldsymbol{E} = -\nabla\phi,$$

$$\rho\frac{\partial \boldsymbol{v}}{\partial t} = -\nabla p + \mu\Delta\boldsymbol{v}, \qquad \operatorname{div}\boldsymbol{v} = 0$$

Boundary conditions at all the interfaces $r = f_i(\theta, t), i = 1,2$:

$$[\phi] = 0, \qquad \boldsymbol{n}\cdot(\varepsilon_0\varepsilon\boldsymbol{E}) = \sigma,$$

$$\boldsymbol{n}\cdot[\boldsymbol{j}] + \nabla_\Sigma\cdot\boldsymbol{j}_\Sigma + \frac{\partial(\sigma\sqrt{g})}{\sqrt{g}\partial t} = 0,$$

$$[p_{ij}n^j\boldsymbol{e}^i] = \pm 2TK\boldsymbol{n},$$

$$[\boldsymbol{v}] = 0, \qquad {}^{df}/_{dt} = v_n$$

and the boundary conditions at the infinity:

$$\boldsymbol{v} \to \infty, \qquad \phi \to \phi_\infty, \qquad E_\infty = -\nabla\phi_\infty, \qquad (r \to \infty).$$

There g is a determinant of metrical tensor, K is a mean curvature, T is an interfacial tension, σ is a charge on the surface, $r = f_j(\theta, t), j = 1,2$ is a form of the interfaces Σ_1 and Σ_2, ∇_Σ is a surface Nabla. All the parameters of liquids inside, in the middle, and outside the droplet hereinafter will be marked “i”, “m” and “e” consequently. Also the properties of interfaces Σ_1 and Σ_2 will be marked “1” and “2” consequently. [A] is a jump of A across the interface.

3. Solution of the problem

Before solving the problem, let us introduce some dimensionless parameters:

$$\alpha = \frac{\varepsilon_0\varepsilon E_0^2 r_2}{T_2}, v_c = \frac{\varepsilon_0\varepsilon E_0^2 r_2}{\mu_e}, \gamma = \frac{\varepsilon_0\varepsilon T_2}{\lambda_e\mu_e r_2}, R = \frac{\rho v_c r_2}{\mu_e}, l_0 = \frac{r_1}{r_2},$$

$$\Lambda_i = \frac{\lambda_i}{\lambda_e}, \Lambda_m = \frac{\lambda_m}{\lambda_e}, \Lambda_e = \frac{\lambda_e}{\lambda_e} = 1, \Lambda_{S1} = \frac{\lambda_{\Sigma 1}}{\lambda_e r_2}, \Lambda_{S2} = \frac{\lambda_{\Sigma 2}}{\lambda_e r_2},$$

$$\varepsilon_i^* = \frac{\varepsilon_i}{\varepsilon_e}, \varepsilon_m^* = \frac{\varepsilon_m}{\varepsilon_e}, \varepsilon_e^* = \frac{\varepsilon_e}{\varepsilon_e} = 1, M_i = \frac{\mu_i}{\mu_e}, M_m = \frac{\mu_m}{\mu_e}, M_e = \frac{\mu_e}{\mu_e} = 1.$$

and some dimensionless variables:

$$r^* = \frac{r}{r_2}, p^* = \frac{pr_2}{T}, v^* = \frac{v}{v_c}, E^* = \frac{E}{E_0}, \phi^* = \frac{\phi}{E_0 r_2}, \sigma^* = \frac{\sigma}{\varepsilon_0\varepsilon_e E_0}, t^* = \frac{tv_c}{r_2}$$

From this moment all calculations will be performed in a dimensionless way. So, to simplify the notation we will not write asterisks anymore. The equations dimensionless form can be written as follows (hereinafter k takes values {i, m, e}):

$$\Delta\phi_k = 0,$$

$$\alpha R\frac{\partial \boldsymbol{v}_k}{\partial t} = -\nabla p_k + \alpha M_k\Delta\boldsymbol{v}_k, \qquad \operatorname{div}\boldsymbol{v}_k = 0$$

The boundary conditions on the inside surface Σ_1 for $r = f_1(\theta, t)$:

$$[\phi]_i^m = 0, \qquad \varepsilon_m E_{nm} - \varepsilon_i E_{ni} = \sigma_1,$$

$$\Lambda_m E_{nm} - \Lambda_i E_{ni} + \Lambda_{S1}\frac{\partial\left(E_\theta\sqrt{g_{1\varphi\varphi}}\right)}{\sqrt{g_1}\partial\theta} + \frac{\alpha\gamma}{\sqrt{g_1}}\left(\frac{\partial(\sigma_1\sqrt{g_1})}{\partial t} + \frac{\partial\left(\sigma_1 v_\theta\sqrt{g_{1\varphi\varphi}}\right)}{\partial\theta}\right) = 0,$$

$$[p]_i^m\boldsymbol{n} - \alpha[M\Pi_1]_i^m = -2\frac{T_1}{T_2}K_1\boldsymbol{n} + \alpha\boldsymbol{F}_1,$$

$$\boldsymbol{F}_1 = \left(\varepsilon_m\left(E_{nm}\boldsymbol{E}_m - \frac{1}{2}E_m^2\boldsymbol{n}\right) - \varepsilon_i\left(E_{ni}\boldsymbol{E}_i - \frac{1}{2}E_i^2\boldsymbol{n}\right)\right),$$

$$v_{\tau i} = v_{\tau m}, \qquad v_{ni} = v_{nm} = D_1$$

The boundary conditions on the outside surface Σ_2 for $r = f_2(\theta, t)$ look the same:

$$[\phi]_m^e = 0, \qquad \varepsilon_e E_{ne} - \varepsilon_m E_{nm} = \sigma_2,$$

$$\Lambda_e E_{ne} - \Lambda_m E_{nm} + \Lambda_{S2} \frac{\partial\left(E_\theta \sqrt{g_{2\varphi\varphi}}\right)}{\sqrt{g_2}\partial\theta} + \frac{\alpha\gamma}{\sqrt{g_2}}\left(\frac{\partial(\sigma_2\sqrt{g_2})}{\partial t} + \frac{\partial\left(\sigma_2 v_\theta \sqrt{g_{2\varphi\varphi}}\right)}{\partial\theta}\right) = 0,$$

$$[p]_m^e \boldsymbol{n} - \alpha[M\Pi_2]_m^e = -2K_2\boldsymbol{n} + \alpha \boldsymbol{F}_2,$$

$$\boldsymbol{F}_2 = \left(\varepsilon_e\left(E_{ne}\boldsymbol{E}_e - \frac{1}{2}E_e^2\boldsymbol{n}\right) - \varepsilon_m\left(E_{nm}\boldsymbol{E}_m - \frac{1}{2}E_m^2\boldsymbol{n}\right)\right),$$

$$v_{\tau m} = v_{\tau e}, \qquad v_{nm} = v_{ne} = D_2$$

and, finally, the boundary conditions at the infinity:

$$\boldsymbol{v} \to \infty, \qquad \phi \to -r\cos\theta\, e^{i\omega t}, \qquad (r \to \infty).$$

In should be noted that: $g_{j\varphi\varphi} = f_j^2 \sin^2\theta,\ g_{j\theta\theta} = f_j^2 + \left(\frac{\partial f_j}{\partial\theta}\right)^2,\ g_j = g_{j\varphi\varphi}g_{j\theta\theta},\ j = 1, 2$.

Let us assume that α parameter is small, and use the asymptotic method - expand all functions (for example, $\boldsymbol{v}$, p, σ) by this small parameter in the following way:

$\Phi(r,\theta,t) = \Phi^{(0)}(r,\theta,t) + \alpha\Phi^{(1)}(r,\theta,t) + \alpha^2\Phi^{(2)}(r,\theta,t) + \cdots$

The zero approximation of the small parameter corresponds to the absence of electric field. So, in this case, the drop's form remains spherical, no flows appear. But there is a pressure jump on the interfaces because of interfacial tension.

$$p_e^{(0)} = 0, \qquad p_m^{(0)} = 2, \qquad p_i^{(0)} = 2 + 2\,{}^{T_1}\!/_{T_2}$$

The first approximation by α for simple drop theory was presented in papers [2-5], and according to them, in this paper the same method is used. The whole system of equations splits into two parts. The first part is Laplace equation for electrodynamic potential and its boundary conditions, and the second part is the hydrodynamic equations and boundary conditions. As the first part does not depend on the second part, it is possible to solve it separately, and then substitute the variables found to the second part.

The solution for electric potential:

$$\phi_i^{(0)} = \frac{9}{(L_1+2)(L_2+2) + 2(L_1-1)(L_3-1)l_0^3}\, r\cos\theta\, e^{i\omega t},$$

$$\phi_m^{(0)} = 3\frac{(L_1+2)r + (L_1-1)l_0^3 r^{-2}}{(L_1+2)(L_2+2) + 2(L_1-1)(L_3-1)l_0^3}\cos\theta\, e^{i\omega t},$$

$$\phi_m^{(0)} = \left(-r + \frac{(L_1+2)(L_2-1) + (L_1-1)(2L_3+1)l_0^3}{(L_1+2)(L_2+2) + 2(L_1-1)(L_3-1)l_0^3}r^{-2}\right)\cos\theta\, e^{i\omega t},$$

$$L_1 = \frac{\Lambda_i + i\alpha\gamma\omega\varepsilon_i + 2\Lambda_{S1}}{\Lambda_m + i\alpha\gamma\omega\varepsilon_m}, \qquad L_2 = \frac{\Lambda_m + i\alpha\gamma\omega\varepsilon_m + 2\Lambda_{S2}}{\Lambda_e + i\alpha\gamma\omega\varepsilon_e}, \qquad L_3 = \frac{\Lambda_m + i\alpha\gamma\omega\varepsilon_m - \Lambda_{S2}}{\Lambda_e + i\alpha\gamma\omega\varepsilon_e}$$

Knowing these formulas, it is possible to find the surface charge distribution.

To find velocities and pressure let us introduce a stream function as follows, k = e, m, i:

$$\psi^k = \left(K_1^k r^{-2} + K_2^k r^3 + K_3^k + K_4^k r^5\right)\sin^2\theta\cos\theta$$

There K_j^k are constants, which can be found from boundary conditions. There are no formulas for K_j^k in this article because of their big size. So, velocities and pressure expressed in terms of these constants can be written in the following way:

$$v_i^{(0)} = \left[2\left(K_2^i r + K_4^i r^3\right)P_2, -\left(3K_2^i r + 5K_4^i r^3\right)\sin\theta\cos\theta, 0\right],$$

$$v_m^{(0)} = \left[2\left(K_1^m r^{-4} + K_2^m r + K_3^m r^{-2} + K_4^m r^3\right)P_2, \left(2K_1^m r^{-4} - 3K_2^m r - 5K_4^m r^3\right)\sin\theta\cos\theta, 0\right],$$

$$v_e^{(0)} = \left[2\left(K_1^e r^{-4} + K_3^e r^{-2}\right)P_2, 2K_1^e \sin\theta\cos\theta, 0\right],$$

$$p_i^{(1)} = 14M_i P_2 K_4^i r^2 + C_1,$$

$$p_m^{(1)} = 4M_m P_2\left(K_3^m r^{-3} + \frac{7}{2}K_4^m r^2\right) + C_2,$$

$$p_e^{(1)} = 4M_e P_2 K_3^e r^{-3} + C_3$$

The main purpose of this work is to find deformation, so here are some formulas, which allow finding the form or the drop.

$$f_2^{(1)} = 2XP_2, \qquad P_2 = \frac{1}{2}(3\cos^2\theta + 1),$$

$$X = \frac{1}{4}\left((8K_1^m - 2K_2^m + 6K_3^m + K_4^m)M_m - 2(4K_1^e + 3K_2^e)M_e + ([F]_2 \cdot n)\right)$$

Let us define a new parameter as:

$$D = \frac{f_2(\theta = 0) - f_2\left(\theta = \frac{\pi}{2}\right)}{2} = \frac{3\alpha}{2}X$$

In the next section, we will show the dependence of parameter *D* on properties of liquids.

4. Results

In this section, the dependence of drop deformation *D* on the frequency of electric field *ω* is shown. The parameters of liquids are in the Table 1.

	μ, Pa*s	ε	λ, 1/(Om*m)	T, N/m	$l_0=r_1/r_2$
Magnetic fluid	0.015	5.2	10^(-6)	0.028	0.9
Oil	0.03	2.2	1.4*10^(-12)	0.027	
Liquid X	0.003	2.46	10^(-16)	0.00325	

Table 1. Parameters of the liquids.

Let us consider the case where the magnetic fluid is inside the drop, the oil is outside the drop, and the "liquid X" is in the middle. This liquid is almost like oleic acid, but with much smaller viscosity.

In Fig. 2, you can see the difference between simple drop of magnetic fluid in oil and double-layer drop, including "liquid X" in the middle. As you can see the double-layer drop can change the prolate form *(D>0)* to oblate form (*D<0*) while the frequency increases, instead of the simple drop.

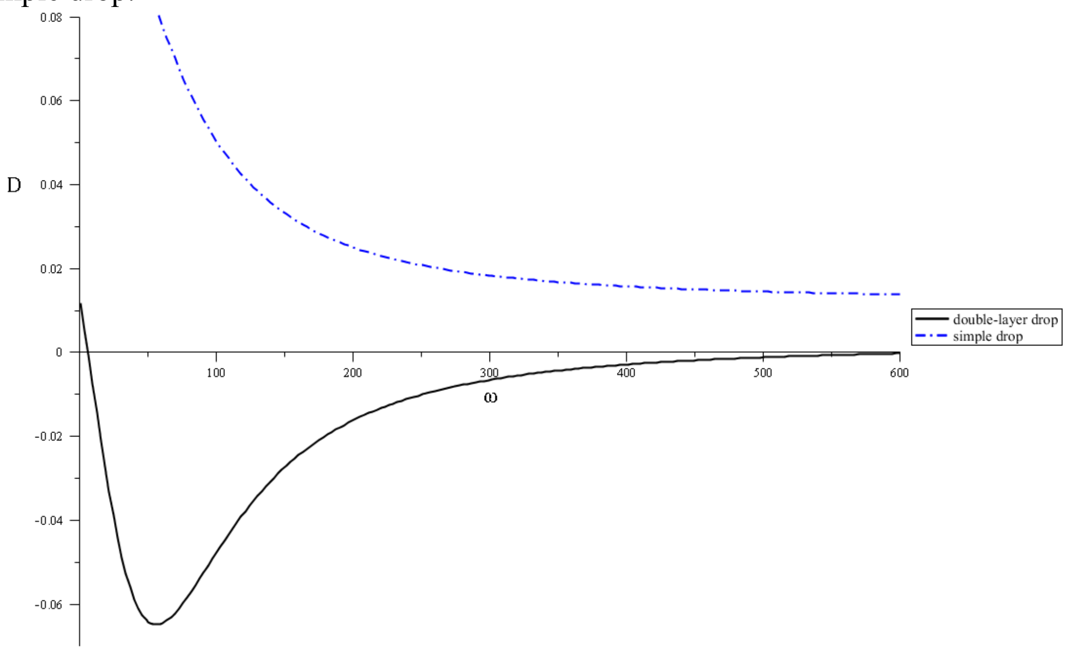

Figure 2: Deformation of magnetic drops in oil.

Now let us consider another case (Fig. 3) where the magnetic fluid and oil changes their places, and "liquid X" remains in the middle. Dependencies of *D* on the frequency *ω* for the double-layer drop and the simple drop of oil in magnetic fluid are also different.

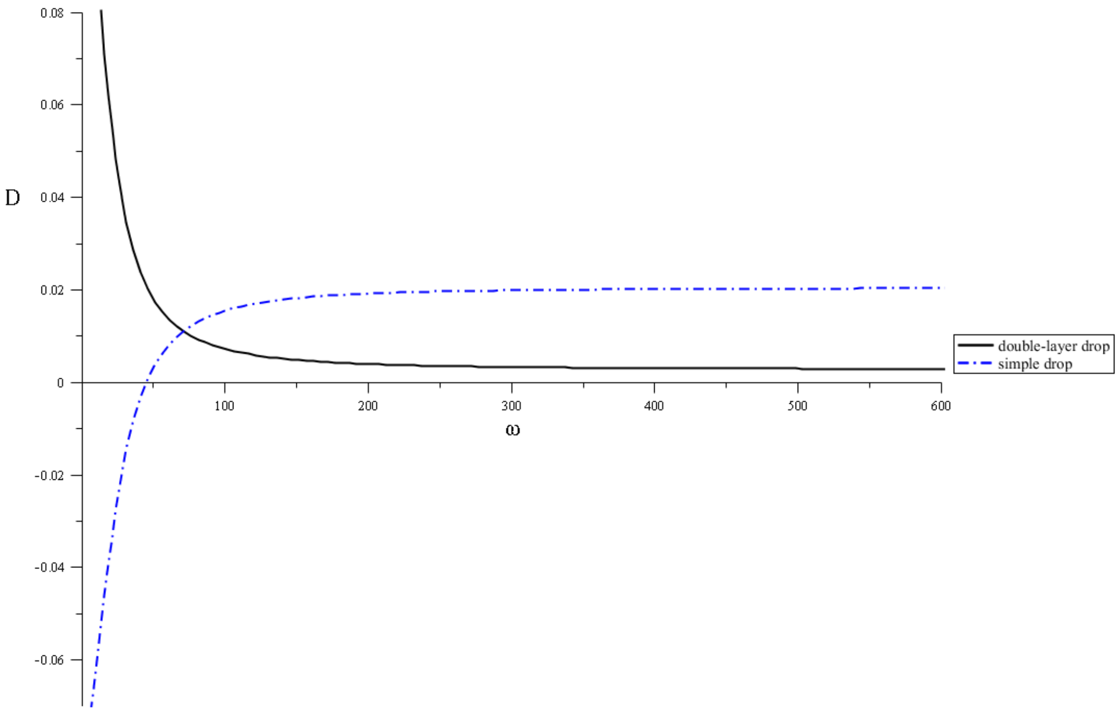

Figure 3: Deformation of oil drops in magnetic fluid.

5. Conclusion

So, in this work the double-layer liquid drop in harmonic electric field is considered. Many calculations are made to find the parameters of liquid "X" where the double-layer drop deformation differs considerably from simple drop deformation. It is possible due to the thin middle-layer of liquid "X" with very small viscosity and conductivity.

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6. References

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