STABILITY ANALYSIS OF A PARTICLE WITH A FINITE ENERGY OF MAGNETIC ANISOTROPY IN A ROTATEING AND PRECESSING MAGNETIC FIELD

CĪMURS[1] Jānis, CĒBERS[1] Andrejs
Affiliation: [1]*University of Latvia*, Zeļļu 8, Rīga, LV-1002, Latvia
e-mail address of corresponding author: janis.cimurs@lu.lv

Abstract : A model of a single ferromagnetic particle with a finite coupling energy of the magnetic moment with a body of particle is formulated and regimes of its motion in liquid in a rotating magnetic field are investigated. As the special case, when the field strength H are large compared to anisotropy field H_a, the stability of synchronous with the field regime of superparamagnetic particle in precessing magnetic field is studied.

1. Introduction

The dynamics of magnetic particle in AC magnetic field plays an important role in different phenomena and applications. The technique of magnetic hyperthermia uses energy dissipated by the motion of magnetic particles in an AC field for cancer therapy [1-3]. It was predicted that magnetotactic bacteria in a rotating magnetic field should follow complex trajectories [4], which were also found in experiment [5]. It was demonstrated that a suspension of the magnetic Janus particles, which possess anisotropic magnetic susceptibility, forms different structures in a precessing field depending on the field's precession frequency and angle [6]. Chain formation of superparamagnetic particles in a precessing magnetic field was studied in [7]. Precessing magnetic field are used for driving magnetic swimmers, for example, magnetic dipoles with attached flexible tail [8,9].

Here we formulate model for single ferromagnetic particle with a finite coupling energy of the magnetic moment with the body of the particle in a magnetic field. The regimes of particle dynamics in the rotating magnetic field are investigated. As special case a superparamagnetic particle in the precessing magnetic field is studied.

2. Model

We introduce single domain ferromagnetic particle with magnetic moment **m**=m**e** and easy axis of magnetization **n**. Energy in external magnetic field **H**=H**h** reads:

$$E=-mH\,\boldsymbol{e}\cdot\boldsymbol{h}-\tfrac{1}{2}KV\left(\boldsymbol{n}\cdot\boldsymbol{h}\right)^2 , \tag{1}$$

where K is a constant of magnetic anisotropy and V is volume of the particle. We assume that magnetic relaxation time is much smaller than the characteristic time of the particle motion, so the magnetic moment is in equilibrium state determined by ($\boldsymbol{K}_e=\boldsymbol{e}\times\partial/\partial\,\boldsymbol{e}$) $\mathbf{K}_e$E=0. It gives

$$\boldsymbol{e}\times\boldsymbol{h}=\frac{2\,H_a}{H}(\boldsymbol{e}\cdot\boldsymbol{n})\boldsymbol{n}\times\boldsymbol{e} , \tag{2}$$

where H_a=KV/2m. The dynamics of the easy axis is determined by the balance of viscous and mechanical torques and reads ($\boldsymbol{K}_n=\boldsymbol{n}\times\partial/\partial\,\boldsymbol{n}$)

$$-\zeta\boldsymbol{\Omega}-\boldsymbol{K}_n E=0\,;\;\; \frac{d\boldsymbol{n}}{dt}=\boldsymbol{\Omega}\times\boldsymbol{n}\,, \tag{3}$$

where ζ is the rotational drag coefficient of the particle. We assume that particle is spherical. Eq. (3) is the particular case of more general "egg-yolk" model proposed in [10,11], where it reads $\zeta\boldsymbol{\Omega}=-\boldsymbol{K}_n E-\boldsymbol{K}_e E$. In Eq. (3) internal magnetic relaxation is neglected ($\mathbf{K_e}$E=0).

From Eq. (2) it can be seen that **e** is in the plane defined by the vectors **n** and **h** and reads:

$$\boldsymbol{e}=\frac{H/H_a\boldsymbol{h}+2(\boldsymbol{e}\cdot\boldsymbol{n})\boldsymbol{n}}{H/H_a(\boldsymbol{e}\cdot\boldsymbol{h})+2(\boldsymbol{e}\cdot\boldsymbol{n})^2}\,, \tag{4}$$

where $(\boldsymbol{e}\cdot\boldsymbol{n})^2=1-(2H_a/H)^2[1-(\boldsymbol{e}\cdot\boldsymbol{n})^2](\boldsymbol{e}\cdot\boldsymbol{n})^2$. From [12] we know that in the range $H/H_a<1$ magnetic moment **e** has one stable states, in the range $H/H_a>2$ **e** has two stable states, but in the range $1<H/H_a<2$ number of stable states (one or two) are determined by angle between **n** and **h**. In this range of magnetic field strength irreversible jumps of the magnetic moment can take place and should be taken into account, when the motion of the particle is considered. The orientation of the magnetic moment of the particle is found minimizing dimensionless equation of energy (1):

$$\hat{E}=-(H/H_a)\boldsymbol{e}\cdot\boldsymbol{h}-(\boldsymbol{n}\cdot\boldsymbol{h})^2\,, \tag{5}$$

with has one or two minimums for **e** for fixed **n·h**.

We introduce dimensionless time $\tilde{t}=\omega_H t$, where ω_H is angular frequency of rotating magnetic field. Using Eq. (4) the equation of motion of particle (2), can now be written in the form (tilde is omitted henceforth):

$$\frac{d\boldsymbol{n}}{dt}=\frac{\omega_a}{\omega_H}C(\boldsymbol{n}\cdot\boldsymbol{h}\,,H/H_a,\xi)[\boldsymbol{h}-\boldsymbol{n}(\boldsymbol{n}\cdot\boldsymbol{h})]\,, \tag{6}$$

where $\omega_a=KV/2\zeta$. The function

$$C(\boldsymbol{n}\cdot\boldsymbol{h}\,,H/H_a,\xi)=\frac{H/H_a(\boldsymbol{e}\cdot\boldsymbol{n})_\xi^2}{H/H_a(\boldsymbol{n}\cdot\boldsymbol{h})+2(\boldsymbol{e}\cdot\boldsymbol{n})_\xi}\,, \tag{7}$$

in general depends on history due to hysteresis of vector **e**. Here history dependence is introduced by variable ξ, which has two values (e.g. 1 and 2) and changes its value in jumps of **e**.

3. Rotating field

In a rotating magnetic field $\boldsymbol{h}=(\cos t\,,\sin t\,,0)$ synchronous with field regimes, when particle rotates with angular velocity $\boldsymbol{\omega}=(0{,}0{,}1)$, $d\boldsymbol{n}/dt=\boldsymbol{\omega}\times\boldsymbol{n}$, can be calculated from Eq. (6):

$$\frac{H/H_a(\boldsymbol{e}\cdot\boldsymbol{n})^2(\boldsymbol{e}_z\cdot\boldsymbol{n})(\boldsymbol{n}\cdot\boldsymbol{h})}{H/H_a(\boldsymbol{n}\cdot\boldsymbol{h})+2(\boldsymbol{e}\cdot\boldsymbol{n})}=0\,, \tag{8}$$

We see that three types of synchronous regimes are possible:

- Planar regime, where $\mathbf{e_z}\cdot\mathbf{n}$=0
- Precession regime, where **n·h**=0
- Unstable stationary regime, where **e·n**=0

The existence intervals and stability analysis of these regimes can be found in [13]. It can be found that besides synchronous regimes there exists asynchronous planar regime. The analysis of this regime can also be found in [13]. In Table 1-2 and Fig. 1 the results of [13] is reviewed.

Regime	Existence interval
Synchronous planar regime (**e**$_z$·**n**=0)	$\frac{\omega_H}{\omega_a}<1 \wedge \frac{\omega_H}{\omega_a}<\frac{H}{H_a}$
Precession regime (**n**·**h**=0)	$\left(\frac{\omega_H}{\omega_a}\right)^2>\left(\frac{H}{H_a}\right)^2-\left(\frac{H}{2H_a}\right)^4 \wedge \frac{H}{H_a}<2$
Asynchronous planar regime (**e**$_z$·**n**=0)	$\frac{\omega_H}{\omega_a}>1 \vee \frac{\omega_H}{\omega_a}>\frac{H}{H_a}$

Table 1: Existence intervals of regimes of single domain ferromagnetic particle dynamics in rotating magnetic field

Regime	Stability interval
Synchronous planar regime (**e**$_z$·**n**=0)	$\left(\frac{\omega_H}{\omega_a}\right)^2>\left(\frac{H}{H_a}\right)^2-\left(\frac{H}{2H_a}\right)^4 \vee \frac{H}{H_a}<\sqrt{2}$
Precession regime (**n**·**h**=0)	$\frac{H}{H_a}<\sqrt{2}$
Asynchronous planar regime (**e**$_z$·**n**=0)	$\int_0^{2\pi}\frac{C(\cos\beta, H/H_a, \xi(\beta))}{\omega_H/\omega_a-C(\cos\beta, H/H_a, \xi(\beta))}d\beta>0$

Table 2: Stability intervals of regimes of single domain ferromagnetic particle dynamics in rotating magnetic field

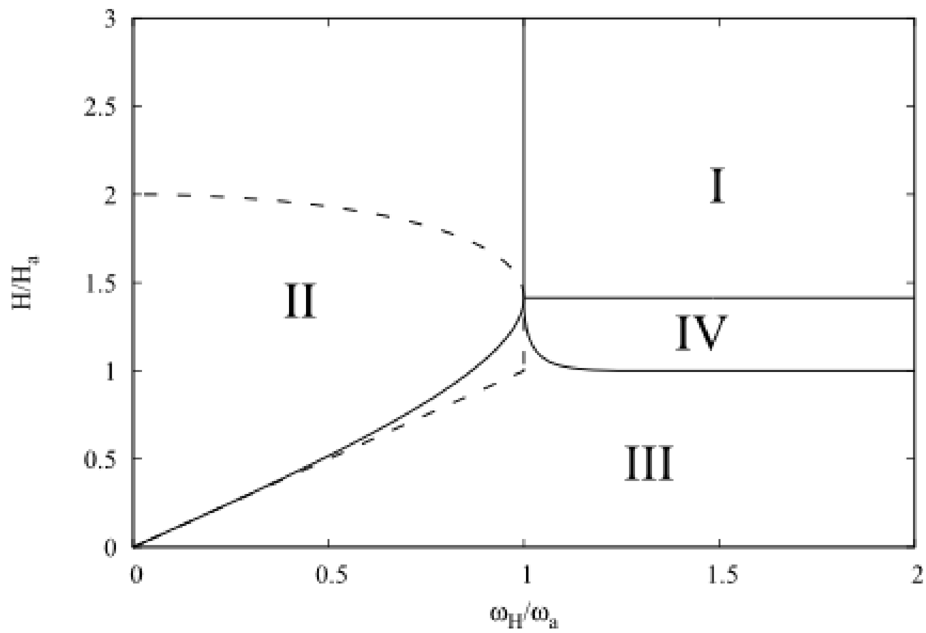

Figure 1: Phase diagram. The solid line indicates boundary between regions where stability of regime changes. In region I only asynchronous planar regime is stable. In region II only synchronous planar regime is stable and dashed line in this region is boundary of existence of

unstable precession regime. In region III only precession regime is stable and dashed line in this region is boundary of existence of unstable synchronous planar regime. In region IV precession and asynchronous planar regimes are stable.

4. Precessing field

As a special case, when magnetic field strength H is large compared to anisotropy field H_a, the particle can be modelled as superparamagnetic ($H \gg H_a$). In this case **e**=**h** and $C(\boldsymbol{n}\cdot\boldsymbol{h}, H/H_a, \xi)=\boldsymbol{n}\cdot\boldsymbol{h}$.

It is found in [14] that in a precessing magnetic field $\boldsymbol{h}=(\sin\vartheta\cos t, \sin\vartheta\sin t, \cos\vartheta)$ synchronous and asynchronous with the field regimes are possible for superparamagnetic particle. If we choose magnetically oblate particles (particles with negative ω_a) than analysis is the same as for prolate particle, but with negative time. That makes stable states unstable and vice versa. The results of [14] is reviewed in Fig. 2

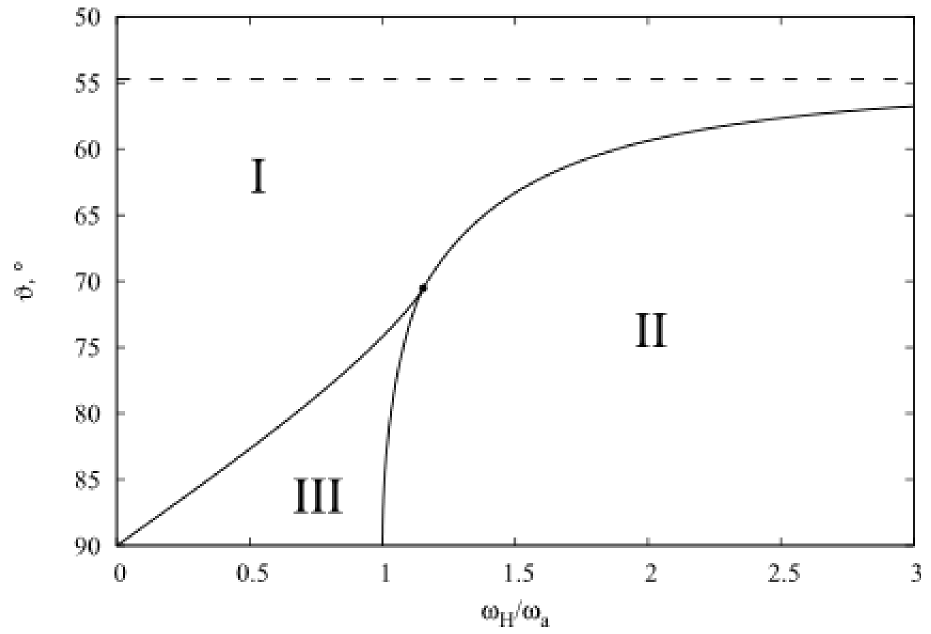

Figure 2: Phase diagram. The prolate superparamagnetic particle has a stable synchronous regime in region I∪III, and the oblate superparamagnetic particle in region II∪III. The point shows a codimension-2 bifurcation point with coordinates (2/√3,arccos(1/3)). The dashed line is asymptote $\vartheta=\arccos(1/\sqrt{3})$ of the solid line.

5. Conclusion

It shows that synchronous and asynchronous regimes are possible for a particle with a finite energy of the magnetic anisotropy in a rotating magnetic field. In the synchronous regime the easy axis of the particle is in the plane of a rotating field at low frequencies (a planar regime) and on the cone at high frequencies (precession regime). The stability of both regimes is investigated and it is shown that precession regime is stable for the magnetic field strength below the critical value. Taking into account irreversible jumps of the magnetic moment it is shown that the planar asynchronous regime is unstable for the field strength below the critical value. In addition the bifurcation diagram for the prolate and oblate superparamagnetic particles in precessing magnetic field has been shown. It generalizes results for the case of rotating field. In spite differences in behaivior of prolate and oblate particles in the precessing field, their bifurcation diagrams are identical expect for interchanged stable and unstable fixed points.

6. References

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