Numerical study of magnetic field driven micro-convection in the Hele-Shaw cell

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The interfacial instability of miscible magnetic fluids in a horizontal Hele-Shaw cell under the action of a vertical magnetic field is studied numerically. The fingering at the interface between the magnetic and non-magnetic fluids is induced by a uniform external magnetic field greater than a critical value. The numerical results of the simulation are compared with the experimental results.

I. INTRODUCTION

The studies on interfacial phenomenon between miscible magnetic fluids have been started more than thirty years ago [1] extending the previous work on magnetostatic instabilities of magnetic liquids in the Hele-Shaw cell for miscible fluids. Due to the dipolar interactions between the nanoparticles, a demagnetizing field appears in the volume of the magnetic fluid. As a result, the magnetic field is larger outside than inside. The ponderomotive force on the magnetizable fluid is proportional to the concentration of magnetic particles and the local gradient of the magnetic field strength. A gradient of magnetic field appears at the frontier: this gradient is the origin of the destabilizing magnetic force. Hence, the theoretical model of the magnetic micro-convection considers the Hele-Shaw flow in the Darcy approximation [2] under the action of the ponderomotive forces due to the self-magnetic field of the fluid, the equations for the magnetostatic field, and the diffusion equation for the concentration of the magnetic nanoparticles.

$$-\nabla p - \frac{12\eta}{h^2}\mathbf{u} - \frac{2M(c)}{h}\nabla\psi_{\mathrm{m}} = 0,\ \nabla\cdot\mathbf{u} = 0\ , \tag{1}$$

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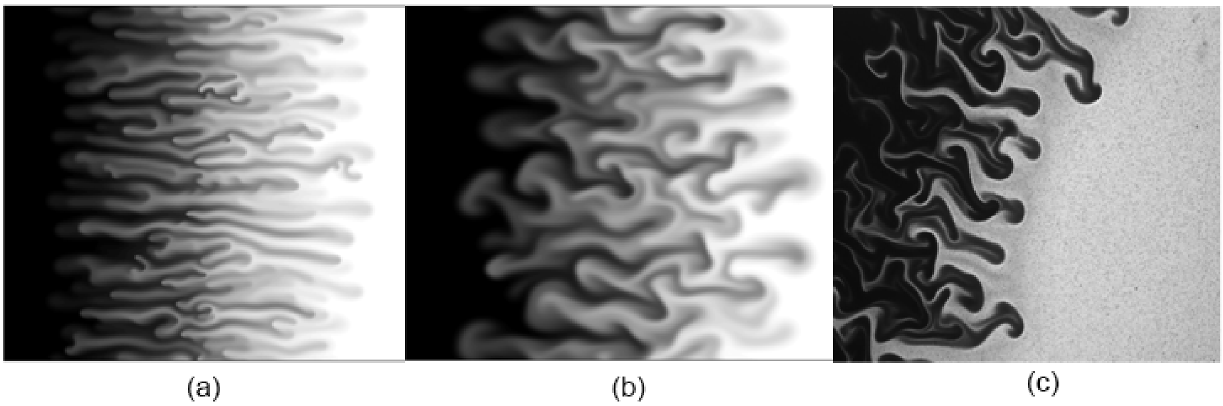

FIG. 1: Concentration snapshots of miscible magnetic and non-magnetic fluids in a Hele-Shaw cell (a) Darcy model for Rayleigh number $Ra_m = 1250$, (b) Darcy model with viscous part for Rayleigh number $Ra_m = 1250$, (c) experimental data for magnetic field strength $B = 28$ Oe, which corresponds to $Ra_m = 168$

$$\frac{\partial c}{\partial t} + (\mathbf{u} \cdot \nabla) c = D \Delta_2 c \ . \tag{2}$$

The magnetostatic potential ψ_{m} is given by [3, 4]

$$\psi_{\mathrm{m}}(\mathbf{r}, t) = M_0 \int c(\mathbf{r}', t) K(\mathbf{r} - \mathbf{r}', h) \mathrm{d}S' \ , \tag{3}$$

where $K(\mathbf{r}, h) = 1/ \mid \mathbf{r} \mid -1/\sqrt{\mid \mathbf{r} \mid^2 + h^2}$ and the magnetization $M(c)$ is proportional to the concentration of the magnetic fluid c ($M = M_0 c$)

II. NUMERICAL SIMULATIONS

The equations Eq.(1)-(3) are put in dimensionless form by introducing the following scales: length h, time h^2/D, velocity D/h and magnetostatic potential $M_0 h$ are solved numerically in the vorticity-stream function formulation. The stream function ψ is defined as $u_x = \partial\psi/\partial y$ and $u_y = -\partial\psi/\partial x$ and the vorticity ω as $\omega = -\nabla^2\psi$. The growth increment of perturbation of quiescent state depends on the magnetic Rayleigh number $Ra_{\mathrm{m}} = M_0^2 h^2/12\eta D$ and the smearing of the interface between the magnetic and non-magnetic liquids. A Fourier spectral method is used as the basic scheme for numerical

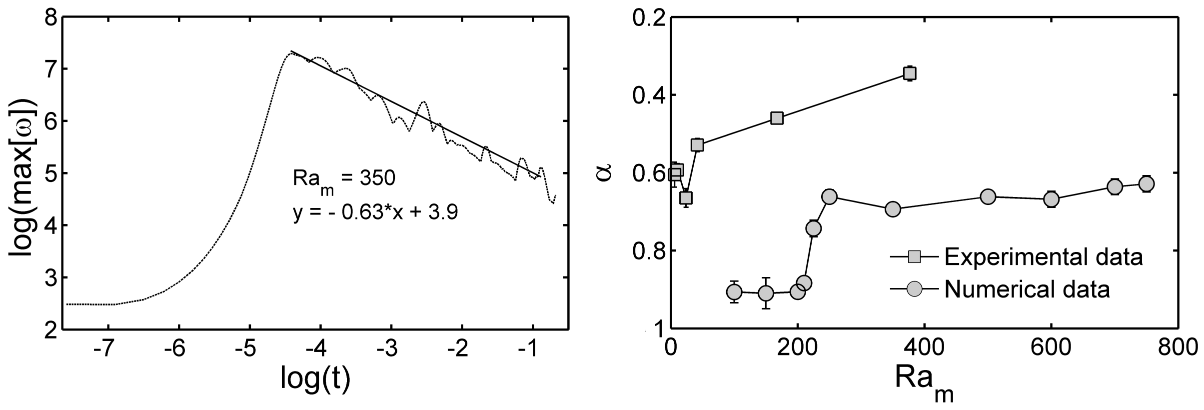

FIG. 2: (a) Maximal value of vorticity as a function of time for a magnetic Rayleigh number $Ra_m = 350$ on a logarithmic scale. (b) The exponent of the power law α for the experimental data given by (square) and numerical data (circle) for the magnetic Rayleigh numbers Ra_m.

simulations, the problem can be reduced to two algebraic equations for flow quantities and a first-order ordinary differential equation in time for the concentration and is solved by applying the linear propagator method and three-step Adams-Bashforth method.

In Fig.1 the qualitative comparison of characteristic concentration images between Darcy (a), Darcy with viscous part (b) and experiment bright field intensity image (c) are shown. A much better agreement of Darcy model and experimental results is reached when viscous stress, tangential to the boundary velocity gradients, is taken into account.

The time dependence of the maximal vorticity for several values of the magnetic Rayleigh number Ra_m is calculated to make a comparison with the experimental data. This dependence allows us to obtain the evolution of the vorticity field during the development of the magnetic micro-convection and its decay due to the diffusion of particles. It corresponds to a rapid increase of the vorticity followed by its slow decay Fig.2(a). Experimental and numerical data for the vorticity decay may be fitted with a power law $\omega_{max} \sim t^{-\alpha}$. The obtained values of the exponent α are shown in Fig.2(b). The experimental and the numerically obtained exponent α values correlate. Although, having a noticeable displacement, both have small values for Ra_m close to the critical field and become clearly higher for greater Ra_m values. The magnetic Rayleigh number $Ra_{\mathrm{m}} = M_0^2 h^2/12\eta D$ of experimental data is calculated with a diffusion coefficient $D = 2.1 \cdot 10^{-5}$ cm^{2}s^{-1}, which is estimated from the critical field ($H_c = 5.3$ Oe [2]), and is close to a diffusion coefficient $D = 2.8 \cdot 10^{-5}$ cm^{2}s^{-1} calculated from a magnetic fluid and water diffusion experiment measurements. This imprecision together with the resolution limitations of the experimental vorticity measurements play the main role for the differences of the comparison.

III. CONCLUSIONS

The proposed model of the magnetic micro-convection qualitatively describes the experimental data on the development and decay of the magnetic micro-convection. The theoretical analysis for the Darcy model shows that a non-potential magnetic force at magnetic Rayleigh numbers Ra_m greater than a critical value causes fingering at the interface between the miscible magnetic and nonmagnetic fluids. Fingering with its subsequent decay due to diffusion of particles significantly increases the mixing at the interface. The vorticity decay rate dependence on Ra_m show a similar behavior for simulations and experiments. To verify the differences, a more precise experimental study on vorticity formation must be carried out.

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