DEFORMATION OF A BODY WITH A MAGNETIZABLE POLYMER IN A UNIFORM MAGNETIC FIELD

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Abstract: The deformation of a spherical body with a magnetizable polymer in the uniform applied magnetic field is studied theoretically. Elastic properties of such composite are described by Mooney-Rivlin model. The possibility of existence of more than one form of the equilibrium of such body in the applied uniform magnetic field is obtained. The theory predicts the hysteretic behavior of the body form if the applied magnetic field increases and decreases cyclically, when many-valuedness of the body form exist.

1. Introduction

The deformation of various bodies with magnetizable polymers was explored in many works (for example, in [1]). However, in these researches usually there was one solution of a problem of static only.
In the present work, the deformation of a spherical body with a magnetizable polymer in the uniform applied magnetic field is studied theoretically. Elastic properties of such composite are described by Mooney-Rivlin model [2]. The possibility of existence of more than one form of the equilibrium of such body in the applied homogeneous magnetic field is obtained.

2. Setting problem

A spherical body (R is body radius) with a magnetizable polymer in a uniform applied magnetic fields $\boldsymbol{H}_0$. Consider that magnetization of the polymer $\boldsymbol{M}$ depend on magnetic field linearly,

$$\boldsymbol{M} = \chi \boldsymbol{H}, \qquad \chi = \text{const}, \qquad \mu = 1 + 4\pi \chi = \text{const} \tag{1}$$

Consider uniform stretch of the body. The spherical body in the magnetic field $\boldsymbol{H}_0$ becomes elongated along the applied magnetic field and becomes ellipsoid of rotation with axes a and b, $b < R < a$. Introduce some parameter λ:

$$\lambda = a/R \tag{2}$$

Condition $\lambda = 1$ means an undistorted spherical body.
Introduce the demagnetization coefficient N ($H^{(i)}$ is a uniform magnetic field inside the body):

$$N = -\frac{H^{(i)} - H_0}{4\pi M}, \qquad M = \chi H^{(i)} \tag{3}$$

In the case elongated ellipsoid of rotation the well known formula for N are [3]:

$$N = \frac{1-e^2}{e^3}(\operatorname{arctanh}(e) - e), \quad e = \sqrt{1 - b^2/a^2} \tag{4}$$

The parameter e depends on λ because the body volume is constant:

$$e=\sqrt{1-b^2/a^2}=\sqrt{1-\lambda^{-3}} \tag{5}$$

The formula for N due to (5) may be written as:

$$N(\lambda)=\frac{\operatorname{arctanh}\sqrt{1-\lambda^{-3}}-\sqrt{1-\lambda^{-3}}}{(\lambda^3-1)\sqrt{1-\lambda^{-3}}} \tag{6}$$

From (3) a dependence $H^{(i)}$ on H_0 is obtained:

$$H^{(i)}=\frac{H_0}{1+4\pi\chi N} \tag{7}$$

3. Static form of the body

Full energy of the ellipsoid in the applied magnetic field $E(H_0, \lambda)$ equals summa of elastic and magnetic energies [1, 3]:

$$E=E_m+E_0^{(e)}, \tag{8}$$

$$E_0^{(e)}=V_b\left[C_1\left(\lambda^2+\frac{2}{\lambda}-3\right)+C_2\left(\frac{1}{\lambda^2}+2\lambda-3\right)\right], \tag{9}$$

$$E_m=-\frac{V_b}{2}\chi\frac{H_0^2}{1+4\pi\chi N} \tag{10}$$

Here the coefficient C_1 has the sense of the rubber-like elasticity modulus, and the coefficient C_2 relates with initial shear modulus of the polymer η by formula $\eta = 2(C_1 + C_2)$ [1].
Using (8), (9) and (10), full energy of unit of the volume of the ellipsoid in the applied magnetic field is determined by the following formula:

$$\frac{E(H_0,\lambda)}{V_b}=\left[C_1\left(\lambda^2+\frac{2}{\lambda}-3\right)+C_2\left(\frac{1}{\lambda^2}+2\lambda-3\right)\right]-\frac{1}{2}\chi\frac{H_0^2}{1+4\pi\chi N} \tag{11}$$

Condition of the body equilibrium is

$$\left.\frac{\partial}{\partial\lambda}\left(\frac{E(H_0,\lambda)}{V_b}\right)\right|_{H_0=const}=0 \tag{12}$$

From (12) and (11) the equation of the body equilibrium may be written as:

$$2(C_1\lambda+C_2)\left(1-\frac{1}{\lambda^3}\right)+2\pi\chi^2\frac{H_0^2}{(1+4\pi\chi N)^2}\frac{\partial N}{\partial\lambda}=0 \tag{13}$$

Let us denote left part of equation (13}) as a function $\Psi(\lambda, H_0)$:

$$\Psi(\lambda,H_0)=2(C_1\lambda+C_2)\left(1-\frac{1}{\lambda^3}\right)+2\pi\chi^2\frac{H_0^2}{(1+4\pi\chi N)^2}\frac{\partial N}{\partial\lambda} \tag{14}$$

So the equation of the body equilibrium (13) is rewritten as

$$\Psi(\lambda, H_0) = 0 \tag{15}$$

Equation (15) is implicit dependency of the parameter λ on the applied magnetic field value H_0. Introduce dimensionless parameters:

$$\Psi^* = \frac{\Psi(\lambda, H_0)}{C_1}, \tag{16}$$

$$K = \frac{C_2}{C_1}, \qquad P = \frac{\chi H_0}{\sqrt{C_1}} \tag{17}$$

The formula for $\Psi^*(\lambda, P)$ is

$$\Psi^*(\lambda, P) = 2(\lambda + K)\left(1 - \frac{1}{\lambda^3}\right) + 2\pi P^2 \frac{1}{(1 + 4\pi\chi N)^2}\frac{\partial N}{\partial \lambda} \tag{18}$$

The equation of the body equilibrium may be written in dimensionless form:

$$\Psi^*(\lambda, P, K, \chi) = 0 \tag{19}$$

Equation (19) allows us to study implicit dependence of the parameter λ on the dimensionless parameter P for different parameters χ and K.

4. Results

In Fig. 1 for $\chi = 3$ and $K = 15, 54, 82, 130$ dependences λ on P are shown. It is clear that for enough large magnetic susceptibility χ for some value of the parameter P three solutions for form of the body exist. With increasing of the parameter K, values of the parameter P, for which three equilibrium forms of the body exist, increase.

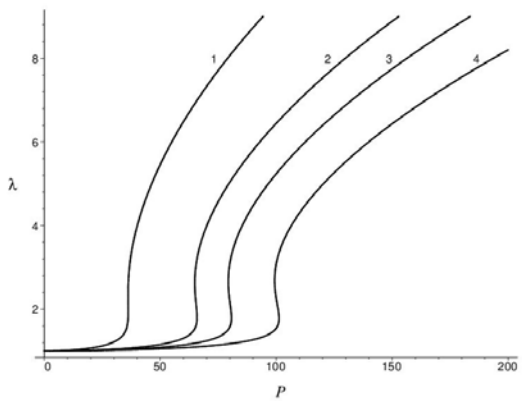

Figure 1: Dependencies λ on P for $\chi = 3$, line 1 – $K = 15$, 2 – $K = 54$, 3 – $K = 82$, 4 – $K = 130$.

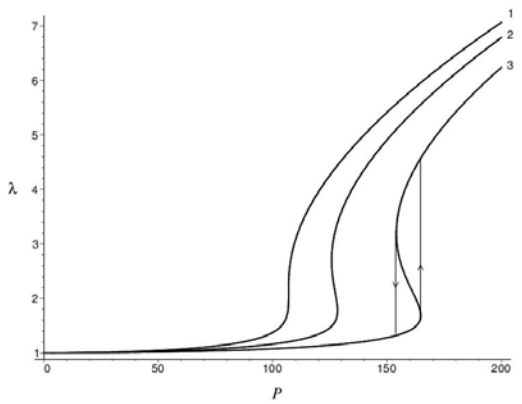

Figure 2: Dependencies λ on P for $K = 210$, line 1 – $\chi = 2.4$, 2 – $\chi = 2.5$, 3 – $\chi = 3$.

In Fig. 2 for $K = 210$ and $\chi = 2.4, 2.5, 3$ dependencies λ on P are shown. From Fig. 2 we can see that for enough small magnetic susceptibility χ many-valuedness of the solution does not exist. If solution many-valuedness exist (for example, for $K = 210$ and $\chi = 3$) the hysteretic behavior of the body form may be observed if the applied magnetic field increases and decreases cyclically, see Fig. 2.

It is shown that for $\chi < 10^{-1}$ for all value of the parameter K many-valuedness of solution does not exist (Fig. 3).

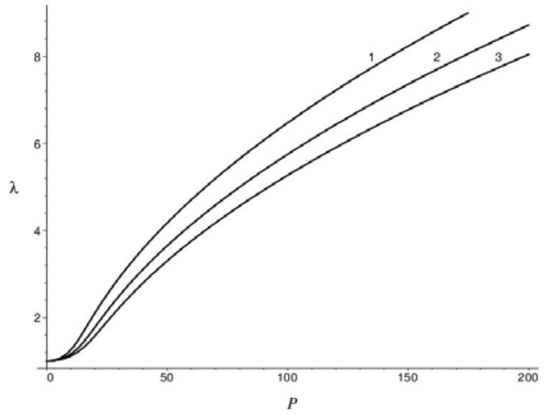

Figure 3: Dependences λ on P for $\chi = 0.1$, line 1 – K=100, 2 – K=150, 3 – K = 200.

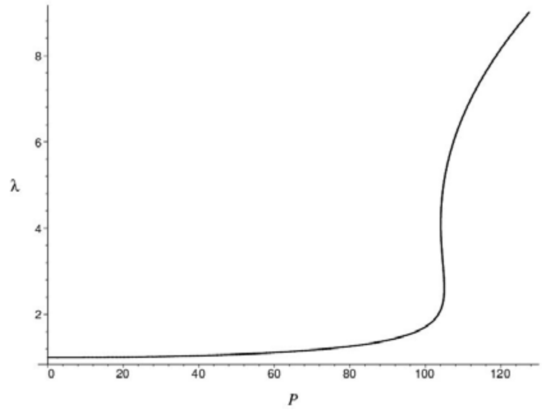

Figure 4: Dependence λ on P for $K = 0$ and $\chi = 30$.

At last, the dependence λ on P for $K = 0$ (the empirical rule for swollen rubbers [2] and $\chi = 30$ is shown on Fig. 4. When $K = 0$ it needs very large value of the magnetic susceptibility χ for existence many-valuedness of the body form.

5. Conclusion

The deformation of a spherical body with a magnetizable polymer in a uniform applied magnetic field is studied theoretically. Elastic properties of such polymer are described by Mooney-Rivlin model. The possibility of existence of more than one form of the equilibrium of such bodies in the applied uniform magnetic field is obtained. It is shown that for any values of the elasticity moduli for enough large magnetic susceptibility three equilibrium forms of the body exist for some value of the applied magnetic field. It is needed to note that for enough small magnetic susceptibility many-valuedness of the solution does not exist for any values of the elasticity moduli. The presented theory predicts the hysteretic behavior of the body form if the applied magnetic field increases and decreases cyclically.

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7. References

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