FERROFLUID BRIDGE BETWEEN TWO CONES AND A CYLINDER IN THE MAGNETIC FIELD OF A LINE CONDUCTOR

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Abstract: The behaviour of a ferrofluid bridge between two cones and a cylinder in the magnetic field of a line conductor in the presence of a pressure drop is investigated. Here we consider a particular case of right circular coaxial truncated cones with different apex angles. A line conductor is also located on their axis. The cones intersect in a circle of the conductor radius. The possibility of the fluid shape hysteresis for a cyclic increase and decrease of the current and of spasmodic changes at certain values of the current is studied.

1. Introduction

The free surface of a ferrofluid changes its shape near a line conductor while the current is slowly changing. For some values of the current, hysteresis and spasmodic phenomena may be observed. For small magnetic fields in [1], the spreading of a ferrofluid drop along a wire in case of wetting was studied theoretically and observed in the experiment. In [2] the behaviour of a ferrofluid bridge between coaxial cylinders with a line conductor on their axis for both cases of wetting and non-wetting was investigated theoretically. Taking into account the results obtained in [1], the behaviour of a ferrofluid drop on a line conductor for any values of wetting angles and magnetic fields was developed in [3]. A ferrofluid drop on a line conductor with limiting conical surfaces in case of non-wetting was studied in [4]. We take into account the results obtained in [2], [4] and state the problem of a ferrofluid bridge between two cones and a cylinder in the magnetic field of a line conductor. It should be noted that the ferrofluid bridge considered in [2] cannot sustain any pressure drop in contrast to this problem where there is a pressure drop.

2. Problem statement and its solution

We consider a heavy, incompressible, homogenous, isothermal ferrofluid (V is the ferrofluid volume) between a cylindrical surface of the radius R_c and two limiting right circular truncated conical surfaces with different apex angles α_1 and α_2. All these surfaces are coaxial, and a line conductor of the radius r_0 with the current I is located on their axis. The cones intersect in a circle of the conductor radius (fig. 1). In this geometry the ferrofluid bridge can sustain a pressure drop $\Delta p = p_1 - p_2$. The pressure p_1 is maintained above the ferrofluid and the pressure p_2 is maintained beneath the ferrofluid. The ferrofluid is immersed in a non-magnetic liquid with the same density (the case of hydroimponderability). If the ferrofluid does not wet solid boundaries then $90^\circ < \theta_1, \theta_2, \theta_3 \leq 180^\circ$, where θ_1 is the wetting angle of the upper conical surface, θ_2 – of the lower conical surface, θ_3 – of the outer cylinder. If the ferrofluid wets solid boundaries then $0^\circ \leq \theta_1, \theta_2, \theta_3 \leq 90^\circ$ (the case $\theta_i > \alpha_i$, $i = 1, 2$ is only considered). The ferrofluid has a free axially symmetric surface $z = h(r)$, $r^2 = x^2 + y^2$ (the axis z is directed along the axis of the conductor). In this geometry, the magnetic field of the conductor $|\boldsymbol{H}|$ is not deformed by the ferrofluid and $|\boldsymbol{H}| = H$, $H(r) = 2I/(cr)$, where c is the speed of light in vacuum. We consider that for a ferrofluid with small concentration of the same ferromagnetic particles its magnetization M_f can be described by the Langevin law as for paramagnetic gas: $M_f(\xi) = M_S L(\xi)$, $L(\xi) = cth\,\xi - 1/\xi$, $\xi = mH/(kT)$, $m = M_S/n$. Here M_S is the saturation magnetization of a ferrofluid, m is the magnetic moment of one ferromagnetic particle, n is the number of ferromagnetic particles per unit volume of a ferrofluid, T is the fluid temperature, k is the Boltzmann constant, ξ is the Langevin parameter which corresponds to the current in a line conductor.

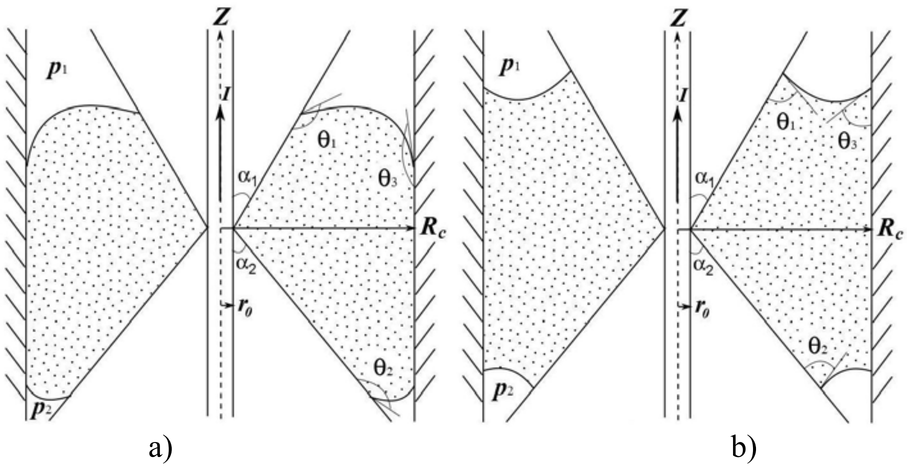


Figure 1: Ferrofluid bridge between coaxial conical and cylindrical surfaces in the magnetic field of a line conductor under a pressure drop in case of a) non-wetting and b) wetting

We use the hydrostatic equation:

$$-\nabla p_i + M_i(H)\nabla H + \rho_i \boldsymbol{g} = \boldsymbol{0}, \quad i = f, l \quad , \tag{1}$$

where the indexes f and l designate the ferrofluid and the non-magnetic liquid surrounding the ferrofluid (the magnetization $M_l = 0$), p is the fluid pressure, ρ is the fluid density, $\boldsymbol{g}$ is the gravitational acceleration. We also use the boundary condition on the free surface $h(r)$:

$$p_l - p_f = \pm 2\sigma K, \quad 2K = (h'' + h'^3/r + h'/r)/(1 + h'^2)^{3/2}, \tag{2}$$

where σ is the coefficient of surface tension and K is the mean curvature of the surface. The sign "+"("−") should be chosen when the non-magnetic liquid is situated above (beneath) the ferrofluid.

From (1) and (2) we get the general, inhomogeneous, non-linear, second-order differential equation. In case of hydroimponderability, when $\rho_f = \rho_l$, we may reduce the order of this equation and get the general analytical solution for any axially symmetric shape of the ferrofluid free surface $h(r)$ in any axisymmetric magnetic field in the dimensionless form [2]. Here we need to describe the upper contact surface of fluids $h_1^*(r^*)$ and the lower contact surface of fluids $h_2^*(r^*)$ separately in the dimensionless form:

$$\begin{aligned} h_1^*(r^*) &= -\int_{r^*}^{R_c^*} G_1/(1-G_1^2)^{1/2}\, dr^* + D_1, \quad G_1(r^*) = C_1/r^* + B_1 r^* - P_1/r^* \int_{r_1^*}^{r^*} r^* P(r^*, \xi_0)\, dr^*, \\ h_2^*(r^*) &= -\int_{r^*}^{R_c^*} G_2/(1-G_2^2)^{1/2}\, dr^* + D_2, \quad G_2(r^*) = C_2/r^* + B_2 r^* + P_1/r^* \int_{r_2^*}^{r^*} r^* P(r^*, \xi_0)\, dr^*. \end{aligned} \tag{3}$$

The following dimensionless parameters are introduced: $r^* = r/r_0$, $R_c^* = R_c/r_0$, $h_i^* = h_i/r_0$, $r_i^* = r_i/r_0$, $i = 1, 2$, $H^* = H/H_0 = 1/r^*$, $H_0 = 2I/(cr_0)$, $\xi_0 = mH_0/(kT)$, $P_1 = nkTr_0/\sigma$,

$P(r^*, \xi_0) = ln\ [sh\ (\xi_0 H^*)/(\xi_0 H^*)]$. Later, the signs "*" are omitted and parameters are considered as non-dimensional, unless otherwise specifically agreed.

On contact lines of three media, for $r = r_1$ and $r = R_c$, the Jung condition should be satisfied and it gives the following boundary conditions:

$$G_1(r = r_1) = -\cos(\theta_1 - \alpha_1), \quad G_1(r = R_c) = \cos\theta_3. \tag{4}$$

From (4) the constants B_1 and C_1 may be determined as functions of r_1. On contact lines of three media, for $r = r_2$ and $r = R_c$, other boundary conditions hold true:

$$G_2(r = r_2) = \cos(\theta_2 - \alpha_2), \quad G_2(r = R_c) = -\cos\theta_3. \tag{5}$$

From (5) the constants B_2 and C_2 may be determined as functions of r_2. The constants $D_1 = h_1\ (R_c)$ and $D_2 = h_2\ (R_c)$ are determined from the following conditions:

$$h_1(r = r_1) = (r_1 - 1)\ ctg\alpha_1, \quad h_2(r = r_2) = -(r_2 - 1)\ ctg\alpha_2. \tag{6}$$

The relation between the constants B_1 and B_2 follows from the condition of fluid equilibrium:

$$B_1 + B_2 = r_0 \Delta p/(2\sigma). \tag{7}$$

In turn, the variables r_1 and r_2 have to satisfy equation (7) and the conservation law of ferrofluid volume.

It should be noted that for $p_1 > p_2$ in case of non-wetting, the ferrofluid bridge can take two different positions: to contact simultaneously the upper and the lower conical surfaces (fig. 1a) or to contact only the lower conical surface. In case of wetting, the ferrofluid bridge can take all three different positions: to contact simultaneously the upper and the lower conical surfaces (fig. 1b), to contact only the upper conical surface or to contact only the lower conical surface. If the ferrofluid contacts only the lower conical surface, then instead of (4) the following boundary conditions hold true:

$$G_1(r = r_1) = -\cos(\theta_2 + \alpha_2), \quad G_1(r = R_c) = \cos\theta_3. \tag{8}$$

If the ferrofluid contacts only the upper conical surface, then instead of (5) the following boundary conditions hold true:

$$G_2(r = r_2) = \cos(\theta_1 + \alpha_1), \quad G_2(r = R_c) = -\cos\theta_3. \tag{9}$$

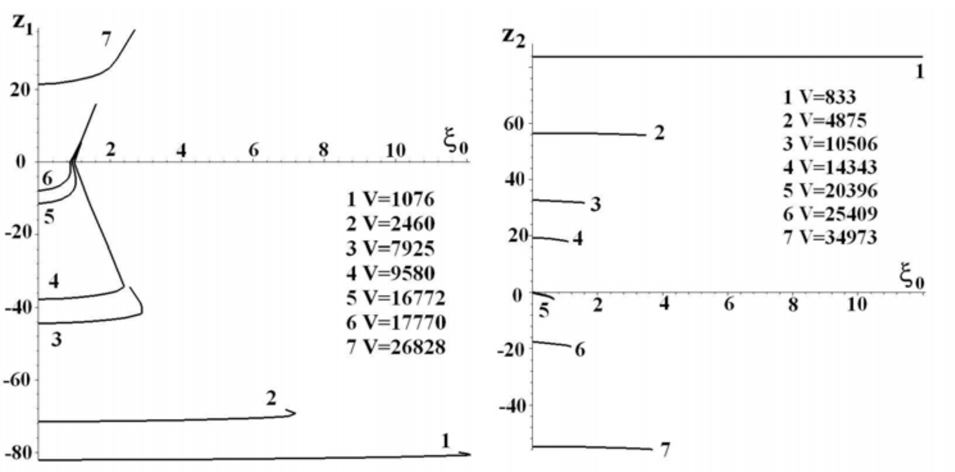
3. Numerical simulation

To simulate numerically the static shapes of the ferrofluid free surface, we fix the following values of the problem parameters: $r_0 = 5\cdot10^{-4}$ m, $R_c = 50\cdot10^{-4}$ m, $T = 300^{\circ}$K, $\Delta p = 101.325$ Pa, $n = 0.19\cdot10^{24}$ m^{-3} ($M_S = 56.6\cdot10^{-4}$ T), $\sigma = 20\cdot10^{-3}$ N/m, $a_1 = a_2 = 5°$. In case of non-wetting $\theta_1 = \theta_2 = \theta_3 = 175°$, and in case of wetting $\theta_1 = \theta_2 = \theta_3 = 30°$.

By varying the parameter r_1, for each value of the current ξ_0 it is possible to calculate the ferrofluid shapes with the fixed volume V before we reach the value of current $\xi_0 = \xi_{break}$. At this value of current the surface $h_1(r)$ contacts the surface $h_2(r)$, the ferrofluid volume becomes minimal to bridge the gap between conical and cylindrical surfaces and the ferrofluid bridge breaks up (at the same time, in case of non-wetting the constants $D_1 = D_2$). However, at some critical value of current $\xi_0 = \xi_{cr}$ solution (3), which describes the static shape of the ferrofluid free surface, may stop existing earlier than the surface $h_1(r)$ contacts the surface $h_2(r)$. In this case, for some value of the radius r the absolute values $|G_1|$ and $|G_2|$ are equal to 1 and the ferrofluid bridge breaks up unpredictably.

4. Hysteresis and spasmodic phenomena

We consider the dependence of the value $z_1 = h_1(r_1)$ on the current ξ_0 for different values of the ferrofluid volume V in case of non-wetting (fig. 2a). For $0 < V < 2460$ (for example, line 1 for $V = 1076$ in fig. 2a) the dependence $z_1 = z_1(\xi_0)$ at first monotonically increases, and later it has a range of values with no physical sense, for which the ferrofluid tends to the conductor while the current is decreasing. For $V = 2460$ the line $z_1 = z_1(\xi_0)$ stops being simply connected: the lower part has its former state, but a new second branch of solutions appears as a dot (line 2 in fig. 2a). For $2460 < V < 9580$ (for example, line 3 in fig. 2a for $V = 7925$) the dependence $z_1 = z_1(\xi_0)$ is biconnected and multivalued, one value of the current ξ_0 may be associated with one, two or three values of z_1. For $V = 9580$ the dependence $z_1 = z_1(\xi_0)$ becomes again simply connected, the lower and the upper solutions grow together (line 4 in fig. 2a). For $9580 < V < 17770$ (for example, line 5 in fig. 2a for $V = 16772$) the dependence $z_1 = z_1(\xi_0)$ continues to be simply connected and multivalued. For $V = 17770$ the dependence $z_1 = z_1(\xi_0)$ has an inflection point and it becomes single-valued (line 6 in fig. 2a). For $17770 < V < 40720$ (for example, line 7 in fig. 2a for $V = 26828$) the dependence $z_1 = z_1(\xi_0)$ continues to be simply connected and single-valued. The volume $V = 40720$ ($5 \cdot 10^{-6}$ m^3) corresponds to the maximal ferrofluid volume, which can be placed in the gap between conical and cylindrical surfaces. The lines in fig. 2a come abruptly to an end when the ferrofluid bridge breaks up either predictably at the current $\xi_0 = \xi_{break}$, or unpredictably at the current $\xi_0 = \xi_{cr}$.



a) b)

Figure 2: Dependences a) $z_1 = z_1(\xi_0)$ in case of non-wetting and b) $z_2 = z_2(\xi_0)$ in case of wetting for different values of the volume V.

In case of wetting the dependence of the value $z_2 = h_2(r_2)$ on the current ξ_0 for different values of the ferrofluid volume V is shown in fig. 2b. We can see that for all range of the volumes $0 < V < 40720$ the dependence $z_2 = z_2(\xi_0)$ decreases monotonically. The lines in fig. 2b come abruptly to an end at the critical values of current $\xi_0 = \xi_{cr}$. Hence, in case of wetting for these parameters of the problem hysteresis and spasmodic phenomena are not observed.

In fig. 3 we consider in detail the dependence $z_1 = z_1(\xi_0)$ in case of non-wetting, namely line 5 for $V = 16772$ from fig. 2a. While the current is increasing quasistatically from $\xi_0 = 0$ to $\xi_{02} = 1.042$ (36 A), the value z_1 increases monotonically from $z_1 = -11.4$ ($-57 \cdot 10^{-4}$ m) to $z_1 = -5.7$ ($-28.5 \cdot 10^{-4}$ m), in other words the ferrofluid moves to the region of bigger magnetic fields. At the current ξ_{02} the ferrofluid jumps from the point $z_1 = -5.7$ on the lower conical surface to the point $z_1 = 2.2$ ($11 \cdot 10^{-4}$ m) on the upper conical surface. Later, while the current is increasing quasistatically from ξ_{02} to $\xi_{break} = 1.537$ (53 A), the value z_1 increases monotonically from $z_1 = 2.2$ to $z_1 = 14.3$ ($71.5 \cdot 10^{-4}$ m). At the current ξ_{break} the ferrofluid bridge breaks up and all ferrofluid volume turns into a drop on conical surfaces. However, if the current does not reach the value ξ_{break} and the ferrofluid bridge does not break up, then while the current is decreasing from some value $\xi_{02} < \xi_0 < \xi_{break}$ to the value $\xi_{01} = 0.94$ (32.4 A), then the value z_1 decreases monotonically to $z_1 = 0.1$ ($0.5 \cdot 10^{-4}$ m). At the current ξ_{01} the ferrofluid jumps from the point $z_1 = 0.1$ on the upper conical surface to the point $z_1 = -8.4$ ($-42 \cdot 10^{-4}$ m) on the lower conical surface. Later, while the current is decreasing quasistatically from the value ξ_{01} to $\xi_0 = 0$, the value z_1 decreases monotonically from $z_1 = -8.4$ to $z_1 = -11.4$. Hence, in case of non-wetting the ferrofluid free surface can change spasmodically and the shape hysteresis may be observed, that is the change of the ferrofluid shape, while the current is increasing, does not coincide with the change of the ferrofluid shape, while the current is decreasing.

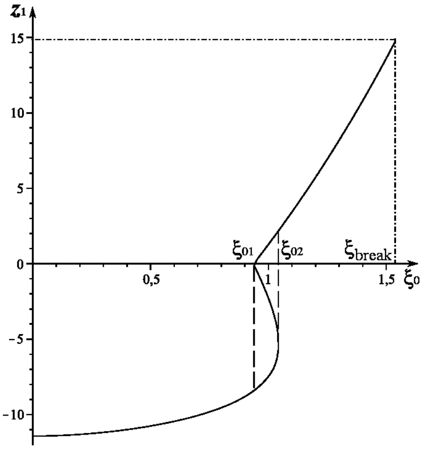


Figure 3: The dependence $z_1 = z_1(\xi_0)$ for $V = 16772$ in case of non-wetting.

5. Conclusion

It is shown that the presence of conical surfaces allows the ferrofluid bridge in the magnetic field of a line conductor to sustain a pressure drop. In case of non-wetting, spasmodic and hysteresis phenomena may be presented for some ferrofluid volumes and currents in a line conductor. In case of wetting, such phenomena are not found. The ferrofluid bridge breaks up either at the critical value of current, for which the static shape of the ferrofluid free surface stops existing, or at the value of current for which the ferrofluid volume is minimal to bridge the gap between conical and cylindrical surfaces. Presence or absence of hysteresis and spasmodic behaviour of a ferrofluid shape should be taken into account for the construction of different devices with controlled ferrofluid volumes, in which the magnetic field is changed periodically, such as seals, interrupters, valves, batchers.

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6. References

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