MHD FLOW UNDER AN IMPACT OF PERMANENT MAGNETS DRIVING SYSTEM

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Abstract: Liquid metal MHD flow initiated by a system of rotating permanent magnets in rectangular cavity has been investigated. We examine the impact of influential magnetic forcing parameters on the produced hydrodynamic structures in the cavity. The spin-up modes and steady-state flow regimes have been simulated in 3D numerical model and verified experimentally on a specially designed setup using Doppler ultrasound technique.

1. Introduction

The flow regime in the process of metal melting and solidification considerably affects the dynamics of the solid-liquid interface. The possibility of flow control should ultimately lead to the possibility of controlling temperature and concentration fields within the metal volume, phase change front shape and process duration.

Electromagnetic methods of the impact of rotating (RMF) or traveling (TMF) magnetic fields on liquid metals have been known for many years [1]-[3] However, it is hard to affect purposefully the phase change front, and the impact of the fields of moving (rotating) permanent magnets on liquid metal becomes more efficient [4], [5]. Besides, using movable permanent magnets system provides some advantages in comparison with RMF impact due to their design simplicity, relatively small overall size and low power consumption.

In the papers [4], [5] analyzing such systems, in the main, integral ("pump", pressure vs flow rate) characteristics were studied. Some hydrodynamic characteristics within limited flow volumes were investigated in [6].

Here we examine a closed container of orthogonal cross-section with liquid metal, in which it is necessary to organize a flow with specified parameters that will allow us, for example, to control the shape of phase change fronts [7]. We cannot use directly the results of some known papers, e.g., those describing MHD flow in a cavity of a similar geometric form [8] under the action of RMF, since in this case a local impact on liquid metal is limited by the electromagnetic system design. In fact, in the mentioned problems with RMF-driven flows, a well-known shape function (a function of electromagnetic body force (EMBF) distribution over the coordinates) is used, the dimensionless amplitude of EMBF being characterized by the magnetic Taylor number. In particular, for cylindrical containers, the relationship between the values of the azimuthal flow velocity and the magnetic Taylor number values and aspect ratios are established, and the conditions of the appearance of flow instability with respect to various disturbances are described.

We use a 3D computer model to examine flow regimes. Numerical results are compared with experimental ones, where the components of the flow melt velocity measured by Doppler velocimeter.

2. Formulation of the problem

In the present study we use a 3D approach to solve complicated problem of metal mixing in an orthogonal container in the presence of rotating magnetic field. The configuration under study is schematically presented in Figure 1.

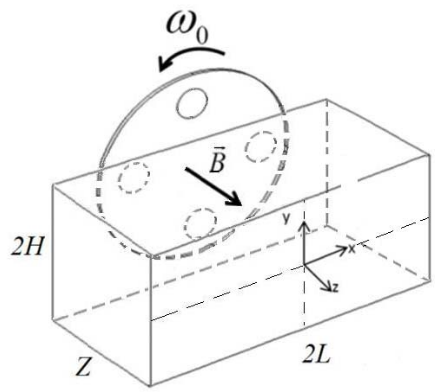

Figure 1: schematic presentation of problem statement: $2L,\ 2H,\ 2Z$ – container's dimensions; origin of coordinates is located in the center of container (cross-section in plane *x*-*y*).

The governing system of MHD equations includes a number of approximations:
Liquid metal motion under the action of electromagnetic forces are examined in the induction-free approximation (magnetic Reynolds number $\mathrm{Re}_m = \mu\sigma u_0 R_0 << 1$), which makes it possible to detach the electrodynamic part of the problem from hydrodynamic one.
Together with low-frequency approximation ($\bar{\omega} = \mu\sigma\omega_0 R_0^2 < 1$) it allows us to reduce equations of electrodynamics to the following:

$$\begin{aligned}
&\frac{\partial \vec{B}}{\partial t} = \frac{1}{\mu\sigma}\Delta\vec{B},\\
&\nabla\cdot\vec{B} = 0,\\
&\nabla\times\vec{E} = -\frac{\partial\vec{B}}{\partial t},\\
&\vec{j} = \sigma\left(\vec{E} + \vec{u}\times\vec{B}\right),\\
&\nabla\cdot\vec{j} = 0,
\end{aligned} \tag{1}$$

where $\vec{B},\ \vec{E},\ \vec{j}$ are the magnetic induction, electric field intensity and current density, $\vec{u}$ is the flow velocity, $\rho,\ \nu,\ \sigma,\ \mu$ – liquid metal density, kinematic viscosity, conductivity and magnetic permeability, respectively.
Magnetic system consists of cylindrical permanent magnets with diameter *d* and height *h* are arranged at a distance R_0 from the rotation axis in parallel to the two side walls of the container and rotate with the angular velocity ω_0. The magnetic field on the end-face of each rotating permanent magnet is $\pm B_0 \vec{e}_z$.

In our problem, the magnetic field is specified on d-wide ring surfaces on the disks as $B_z\big|_{z=\pm(Z+\delta)} = B_0 \cos(\omega_0 t - p\ arctg\,\frac{y}{x})$, where p – the number of pole pairs, which depends on magnets polarity alternation, δ – the distance between the plane of driving magnets end-faces and inner surfaces of corresponding container's lateral walls.

The cause of the conducting liquid flow in a rotating field is an EMBF, whose density is determined by

$$\vec{f}_{em} = \sigma(\vec{E} + \vec{u} \times \vec{B}) \times \vec{B}. \qquad (2)$$

Besides, in the approximation of a small magnetic interaction parameter $St = \frac{Ha^2}{\mathrm{Re}} < 1$ (where $Ha = B_0 R_0 \sqrt{\frac{\sigma}{\rho\nu}}$, $\mathrm{Re} = \frac{u_0 R_0}{\nu}$ are Hartmann and Reynolds numbers) we do not take into account the variable electromagnetic force component and examine its constant part only (here $T = 2\pi/\omega_0$ – period of disk revolution):

$$\langle f_{em} \rangle = \frac{1}{T}\int_0^T \left|\vec{f}_{em}\right| dt. \qquad (3)$$

We should note another simplification assumed in the problem, which is connected with the smallness of the magnetic Reynolds number – the effect of the term $\left(\vec{u} \times \vec{B}\right)$ of Eq. (2) on flow computation results. It was estimated before [7] for several sets of parameters and established that this term can be also neglected.

In this case, the flow is described by the following equations

$$\frac{\partial \vec{u}}{\partial t} + (\vec{u} \cdot \nabla)\vec{u} = -\frac{1}{\rho}\nabla P + \nu \cdot \Delta \vec{u} + \frac{1}{\rho}\vec{f}_{em}, \qquad (4)$$

$$\nabla \cdot \vec{u} = 0,$$

where flow velocity at the initial time: $\vec{u}\big|_{t=0} = 0$, and on the container inner face $\vec{u}\big|_S = 0$ (here P is a pressure).

3. Results and discussion

Calculated values of the magnetic field and the flow velocity in the cavity of the container were compared with the experimental ones obtained by Doppler ultrasound velocimeter (description of the experiment is given in [7]). Figure 2 shows the distribution of longitudinal mean velocity component, and Figures 3 and 4 illustrate some typical hydrodynamic structures generated in the container.

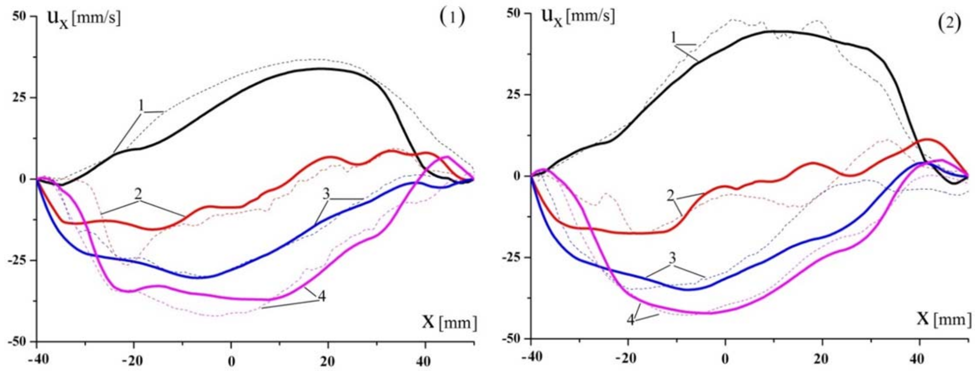

Figure 2: comparison of *x*-component of the flow at *x*-*y* section (*z* = 20mm), for different rotation velocities - 1) $\omega_0 = 10.74\, rad/\mathrm{s}\ (100 rpm)$; 2) $\omega_0 = 15.7\, rad/\mathrm{s}\ (150 rpm)$. Numbers on the curves correspond to *y*-coordinate: 1 – *y* = -23mm; 2 – *y* = 0; 3 – *y* = 11.5mm; 4 – *y* = 23mm; solid lines – computation, dashed lines – experiment (ultrasonic Doppler velocimeter DOP 2000).

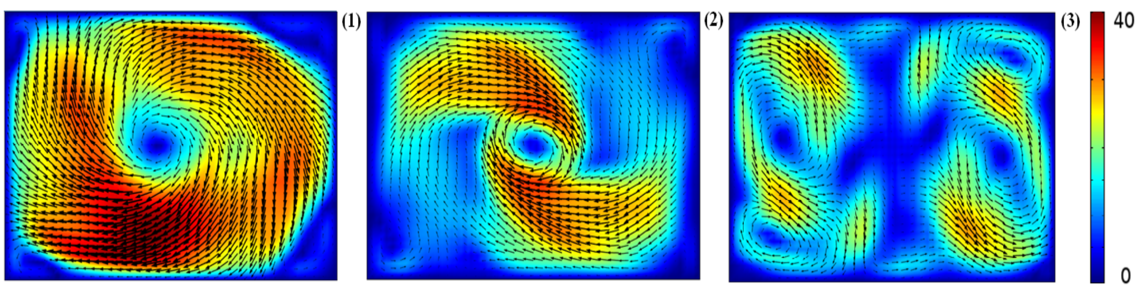

Figure 3: comparison of steady-state flow regime for different cross-sections:
1) *x*-*y* plane (*z* = 20mm); 2) center of *x*-*y* plane (*z* = 0); 3) center of *x*-*z* plane (*y* = 0)

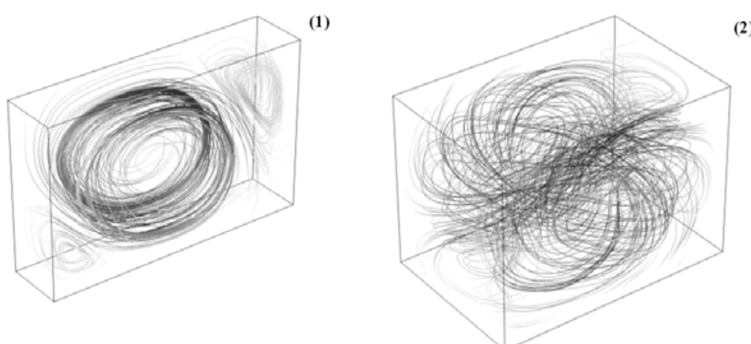

Figure 4: 3D flow patterns at $\omega_0 = 15.7 rad / s$ in containers with different width $(Z_{02} = 2Z_{01})$.

Depending on the parameters of the control of the magnetic system and the geometric relationships between the dimensions of the container we have calculated several scenarios of development and steady-state flow with characteristics, which agree satisfactorily well with experimental data. The analyzed variants indicate wide opportunities of systems with permanent magnets using, for example, for melts homogenization or for control the shape of the phase change front in the processes of solidification and melting.

4. Conclusion

3D computer simulation of MHD flow activated by rotating permanent magnets in a container of orthogonal cross-section was carried out using a laminar model, and the results were validated by experimental data obtained using Doppler ultrasound velocimetry. That allows to examine the features of the various hydrodynamic structures and even to realize the required flow regime by setting the magnetic driving system parameters.

The obtained results complement the existing findings and expand the applicability region and the potential of the used method of controlling the processes of liquid metals stirring. In the framework of this research we intend to optimize the parameters of the magnetic driving system conformably both to laminar and turbulent flow regimes.

5. References

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