Self-calibrating phase-shift flowmeter for liquid metals

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Abstract: We present an improved phase-shift flowmeter that has a significantly reduced sensitivity to the variation of the electrical conductivity of the liquid metal. Two simple theoretical models of flowmeter are considered where the flow is approximated by a solid body motion of finite-thickness conducting layer. In the first model, the applied magnetic field is represented by a harmonic standing wave. In the second model, the sending coil is approximated by a couple of straight wires placed above the layer. We show that the effect of electrical conductivity can strongly be reduced by using the phase shift between the sending coil and the upstream receiving coil to rescale the measured phase shift between two receiving coils.

1 Introduction

This paper is concerned with further development of a recently invented AC induction flowmeter for liquid metal flows based on phase shift measurements [1, 2]. The flowmeter operates by measuring the phase disturbance caused by the flow of liquid metal in an applied ac magnetic field. The main advantage of the phase-shift flowmeter is the robustness to external perturbations compared to measurements of the amplitude used by conventional eddy-current flowmeters. However, the phase disturbance depends not only on the velocity of liquid but also on its electrical conductivity.

The aim of this work is to reduce the dependence of the flow-induced phase shift measurements on the electrical conductivity of liquid. The basic idea is that not only the liquid flow but also the alternation of the magnetic field itself gives rise to the phase shift in the induced magnetic field. The current version of the flowmeter employs only the former effect. The latter effect, which gives rise to phase shifts between the sending and receiving coils, is dependant on the conductivity of the liquid. Therefore it may be used to compensate for the effect of conductivity on the flow-induced phase shift. The feasibility of such an approach is investigated using two simple theoretical models of the phase-shift flowmeter, where the flow is approximated by a solid body motion of a finite-thickness conducting layer. In the first model, the applied magnetic field is represented by a harmonic standing wave. In the second model, the sending coil is approximated by a couple of straight wires placed above the layer.

2 Basic Equations

Consider a medium of electrical conductivity σ moving with the velocity $\vec{v}=\vec{e}_x V$ in an ac magnetic field with the induction $\vec{B}$ alternating harmonically with the angular frequency ω. The induced electric field follows from the Maxwell-Faraday equation as $\vec{E}=-\vec{\nabla}\Phi-\partial_t\vec{A}$, where Φ is the electric potential, $\vec{A}$ is the vector potential and $\vec{B}=\vec{\nabla}\times\vec{A}$. The density of the electric current induced in the moving medium is given by Ohm's law

$$\vec{j}=\sigma(\vec{E}+\vec{v}\times\vec{B})=\sigma(-\vec{\nabla}\Phi-\partial_t\vec{A}+\vec{v}\times\vec{\nabla}\times\vec{A}).$$

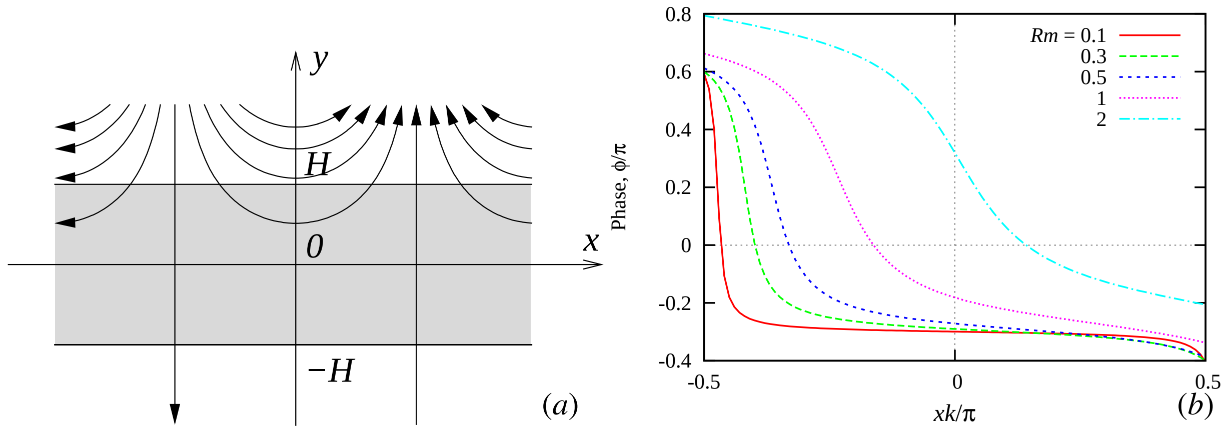

Figure 1: Model of a conducting layer of thickness $2H$ in the external magnetic field represented by a standing harmonic wave (a). Phase distribution over a half-wavelength of the applied magnetic field at various dimensionless velocities defined by Rm (b).

Assuming the ac frequency to be sufficiently low to neglect the displacement current, Ampere's law $\vec{j} = \frac{1}{\mu_0}\vec{\nabla} \times \vec{B}$ leads to the following advection-diffusion equation for the vector potential

$$\partial_t \vec{A} + (\vec{v} \cdot \vec{\nabla})\vec{A} = \frac{1}{\mu_0 \sigma} \nabla^2 \vec{A}, \tag{1}$$

where the gauge invariance of $\vec{A}$ has been used to specify the scalar potential as $\Phi = \vec{v} \cdot \vec{A} - \frac{1}{\mu_0 \sigma} \vec{\nabla} \cdot \vec{A}$. We now consider an applied magnetic field varying in time harmonically as $\vec{A}_0(\vec{r},t) = \vec{A}_0(\vec{r})\cos(\omega t)$, which allows us to search for a solution in the complex form $\vec{A}(\vec{r},t) = \Re\left[\vec{A}(\vec{r})e^{i\omega t}\right]$. Equation (1) for the amplitude distribution of the vector potential takes the form

$$i\omega \vec{A} + (\vec{v} \cdot \vec{\nabla})\vec{A} = \frac{1}{\mu_0 \sigma} \nabla^2 \vec{A}. \tag{2}$$

We focus now on a simple 2D externally applied magnetic field, which is invariant along the unit vector $\vec{\varepsilon}$. Such a magnetic field can be specified by a single component of the vector potential $\vec{A} = \vec{\varepsilon} A$ which leads to the boundary conditions $[A]_S = [\partial_n A]_S = 0$, where $[f]_S$ denotes the jump of quantity f across the boundary S where$\partial_n \equiv (\vec{n} \cdot \vec{\nabla})$ is the derivative normal to the boundary.

We start with a simple model where the conducting medium is a layer of thickness $2H$ shown in Fig. 1(a), and the applied magnetic field is a harmonic standing wave with the vector potential amplitude given by

$$\vec{A}_0(\vec{r},t) = \vec{e}_z A_0(\vec{r},t) = \vec{e}_z \hat{A}_0(y)\cos(kx)\cos(\omega t),$$

where k is the wave number in the x-direction. We choose the half-thickness H as the length scale and introduce a dimensionless ac frequency $\bar{\omega} = \mu_0 \sigma \omega H^2$ and the magnetic Reynolds number $Rm = \mu_0 \sigma V H$. The latter represents a dimensionless velocity. It is important to note that this key parameter depends on the product of the physical velocity and electrical conductivity. The full derivation of a solution based on the above and its continuation to a external field generated by a couple of straight wires is to be found in [2].

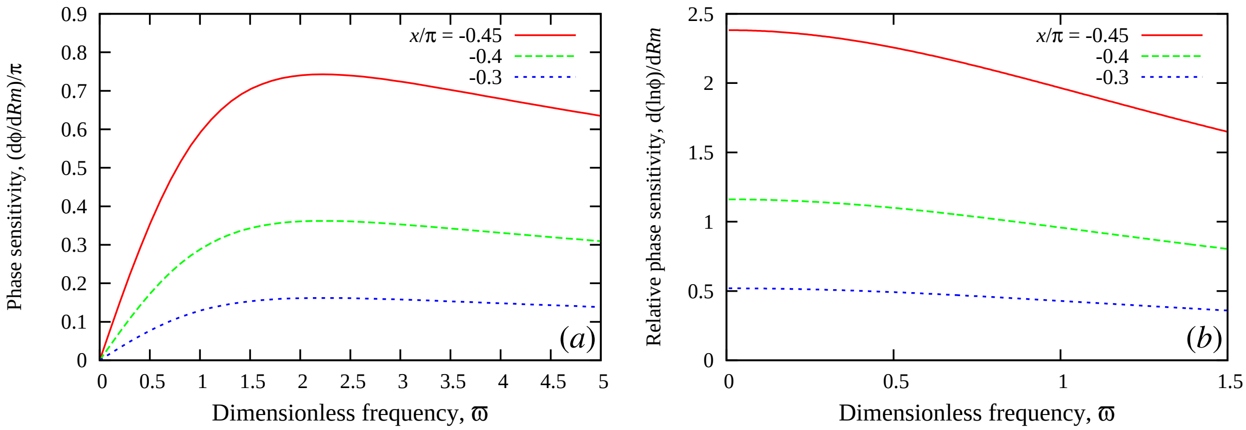

Figure 2: The phase sensitivity (a) and the relative phase sensitivity (b) versus the dimensionless frequency $\bar{\omega}$ at various horizontal observation positions below the layer for $k = 1$

3 Numerical Results

Single Harmonic of the magnetic field

Let us start with the original phase-shift flowmeter as a basis for the following development. Figure 1(b) shows the distribution of the phase between two nodes of the applied magnetic field along the bottom of the layer. For the layer at rest, the phase distribution is piecewise continuous with jumps in value at the wave nodes. These discontinuities are smoothed out by the motion of the layer and shifted further downstream as Rm is increased. Figure 1(b) also shows an important feature of the phase variation, that the strongest variation occurs at the right, immediately downstream of a node whereas the variation to the left, upstream of a node, is relatively weak, especially at lower values of Rm.

The variation of phase φ with Rm at low velocities can be characterized by the phase sensitivity $K = \pi^{-1}\,\partial_{Rm}\varphi|_{Rm=0}$. The dependence of this quantity on the dimensionless frequency $\bar{\omega}$ is plotted in Fig. 2(a) for several observation points. As $\bar{\omega}$ has a similar effect to Rm the reduction can be achieved by scaling the phase variation with the phase itself, which leads to the relative phase sensitivity

$$K_r = \pi K/\varphi. \tag{3}$$

As seen in Fig. 2(b) with small $\bar{\omega}$ this quantity tends to constant for given observation point. Although the relative phase sensitivity is not completely independent of $\bar{\omega}$ it can be seen that at lower values of $\bar{\omega}$ it varies much less than the unscaled phase sensitivity shown in 2(a). Following this idea the effect of conductivity can be reduced by scaling the phase shift between the voltages measured by the two receiving coils with the phase shift between the sending and receiving coils. Since, as shown above, the phase shift upstream of a node is less affected by the motion of the layer then downstream of a node, the upstream measurement shall be used to scale the phase shift between the receiving coils. Rescaling the phase shift directly with the reference phase, as done for sensitivity, is not sufficient as according to equation (3) the result still remains proportional to Rm. Since at small $\bar{\omega}$ the reference phase varies directly with $\bar{\omega}$ which, similar to Rm, is proportional to

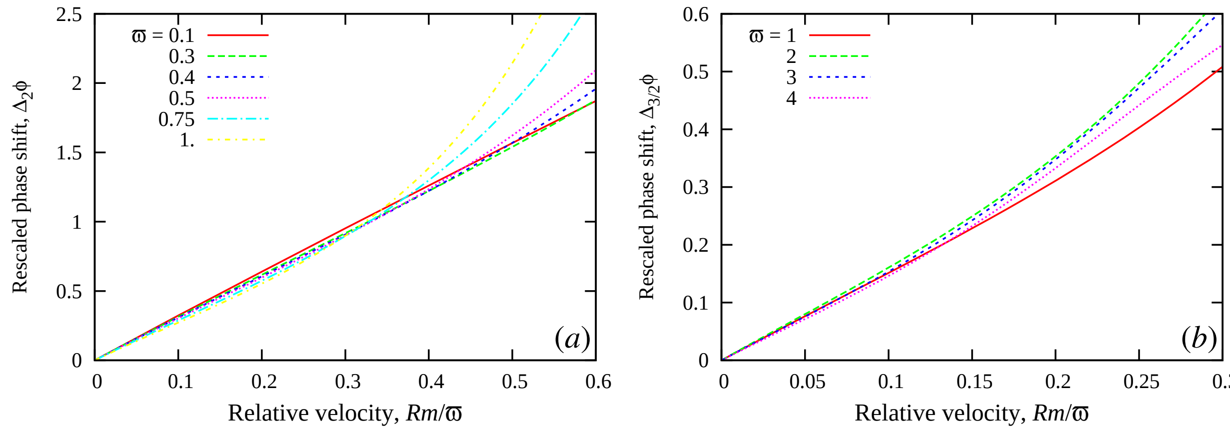

Figure 3: Rescaled phase shift $\Delta_2\varphi$(a) and $\Delta_{3/2}\varphi$ (b) between two observation points placed below the layer at $\pm x = 0.3$ versus the relative velocity $Rm/\bar{\omega}$ for $k = 1$ and various dimensionless frequencies $\bar{\omega}$.

conductivity. The following scaling by the square of the reference phase will eliminate conductivity

$$\Delta_2\varphi = \frac{\Delta_0\varphi}{\varphi^2}, \tag{4}$$

where $\Delta_0\varphi = \varphi_+ - \varphi_-$is the difference between the downstream and upstream phases which are denoted by φ_+ and φ_- respectively.

For the rescaled phase shift to be insensitive to σ it cannot be dependent directly on $\bar{\omega}$ or Rm, but must be a function of these control parameters such that σ is eliminated. We choose this to be $Rm/\bar{\omega} = V/(\omega H)$. Henceforth this ratio shall be referred to as the relative velocity.

Figure 3(a) shows that, when plotted against the relative velocity, the rescaled phase shift given by Eq. (4) has a weak dependence on $\bar{\omega}$ as long as $\bar{\omega}$ is low. For sufficiently low relative velocities, the variation of the rescaled phase shift with $\bar{\omega}$ is weak up to $\bar{\omega} \approx 1$. This range of low relative velocities depends on the locations of the observation points. The closer the observation point to the nodes ($x = \pm 0.5$) the shorter the range of relative velocities for which the rescaled phase difference remains invariant with $\bar{\omega}$. Figure 3(a) shows that far enough from the nodes the relationship between rescaled phase difference and relative velocity is invariant for a range of dimensionless frequency from 0.1 to 1, which corresponds to a change of an order of magnitude to the conductivity.

The scaling given in Eq. (4) only holds for low $\bar{\omega}$ where shielding effect causes the reference phase to vary non-linearly as $\sim \bar{\omega}^{1/2}$. This non-linearity is compensated for by taking the reference phase to the power of 3/2 instead of the square. It results in a second rescaled phase difference $\Delta_{3/2}\varphi = \Delta_0\varphi/\varphi^{3/2}$ which is plotted against the relative velocity in Fig. 3(b). It can been seen that the proportionality of the relative velocity and this rescaled phase difference is invariant for a greater range of $\bar{\omega}$.

Sending coil modelled by two straight wires

In this section we consider the case of an external magnetic field generated by a couple of straight wires. Figure 4(a) shows that the range of $\bar{\omega}$ where the phase sensitivity varies linearly is rather

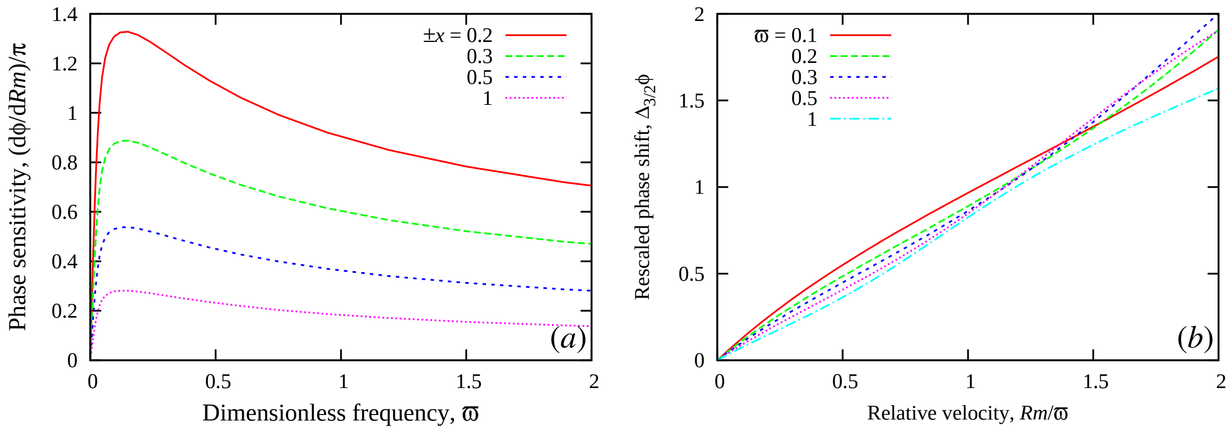

Figure 4: The phase sensitivity versus the dimensionless frequency $\bar{\omega}$ at various horizontal observation positions at below the layer (a) and rescaled phase shift $\Delta_{3/2}\varphi$ between two observation points placed below the layer at $\pm x = 1$ versus the relative velocity $Rm/\bar{\omega}$ for various dimensionless frequencies $\bar{\omega}$(b) for a sending coil modelled by two straight wires.

short As above this non-linearity is compensated by the rescaled phase shift $\Delta_{3/2}\varphi$ which is plotted in Fig. 4(b) with observation points placed symmetrically at $x = \pm 1$ As seen the rescaled phase shift depends essentially on the relative velocity whilst the variation with $\bar{\omega}$ is relatively weak.

4 Conclusions

We have demonstrated that the dependence of the flow induced phase shift measurements on the electrical conductivity of the liquid can be reduced by including the phase shift between the sending and receiving coils into the measurement scheme. It was shown that a strong reduction in the effect of conductivity is attainable by rescaling the measured phase difference by the reference phase shift between the sending coil and the upstream receiving coil. This rescaling is dependent on the nature of the variation of the phase shift with the frequency. At low frequencies, where this variation is linear, the reduction is achieved by rescaling with the square of the reference phase. At higher frequencies, the shielding effect results in non-linear variation of the reference phase with the frequency. This non-linearity can be compensated by a rescaling with the reference phase shift to the power of $3/2$ instead of the square.

References

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[2] Priede, J.; Buchenau, D.; Gerbeth, G.: Contactless Electromagnetic Induction Flowmeter For Liquid Metals. Meas. Sci. Technol. 22 (2011) 055402.