THE VISCOUS EFFECTS ON A SMALL MHD PUMP

HEE REYOUNG KIM
Ulsan National Institute of Science and Technology, 50 UNIST-gil Ulju-gun
Ulsan, KOREA
Corresponding author: kimhr@unist.ac.kr

Abstract : A one-dimensional MHD analysis has been performed for the viscous and end effects on a magnetohydrodynamic pump. The calculations show that the developed pressure difference resulted from electromagnetic and viscous forces in the liquid metal is expressed in terms of the slip, and that the viscous loss effects are negligible compared with electromagnetic driving forces except in the low-slip region where the pumps operate with very high flow velocities comparable with the synchronous velocity of the electromagnetic fields, which is not applicable to the practical MHD pumps.

1. Introduction

The linear induction MHD pumps have been employed for circulating the sodium coolant in Liquid-Metal Reactors. The MHD pumps have the advantage over the mechanical pumps due to the fact that they have no rotating parts, which results in simplicity and convenience of maintenance and repair. The basic operational experiments on pilot linear induction MHD pumps with various geometrical shapes have been carried out, and the practical applications have been made for the other areas of research like liquid metal chemistry. The linear induction MHD pump gets the driving power from Lorentz's forces generated by the time-varying magnetic fields and the current induced in the electrically conductive liquid metal. Mathematical solutions for the driving forces are obtained by solving MHD equations of incompressible viscous flow coupled with Maxwell's equations under appropriate assumptions. It is found that the mechanical pressure gradients developed in the pump duct are given as a function of the slip including other pump variables. Since the pumping pressures are developed by Lorentz's forces experiencing viscous drag forces, viscous loss effects on the electromagnetically-developed liquid metal flow need to be investigated. Generally, the Hartmann number given by the system scale length and magnetic field with viscosity and electrical conductivity is used as a measurement of viscous effect [1]. In the present work, direct comparison of the electromagnetic force with viscous force is carried out by analytical solutions obtained from related equations. The calculated results show, at the nominal conditions, that the pump performance can be analyzed by electromagnetic treatment due to negligible viscous effects. In this paper, MHD flow and electromagnetic analyses on the annular linear induction MHD pumps with flowrates of 60 L/min are carried out by solving MHD and Maxwell's equations.

2. Analysis of viscous losses by MHD laminar flow analysis

The MHD pump is driven by Lorentz's force given by products of induced current ($\mathbf{J}$) and magnetic field ($\mathbf{B}$) perpendicular to it. Electrically conductive liquid fluid also experiences viscous forces while it is developed by Lorentz's force. In this respect, an attempt has been made to obtain the expression for the pressure gradient mathematically by analyzing MHD equations coupled with Maxwell's field equation. As shown in figure 1, a typical annular linear induction MHD pump has sloted external cores in which exciting coils are inserted to generate $\boldsymbol{J} \times \boldsymbol{B}$ force in the liquid metal flowing in the annular duct [2-4].

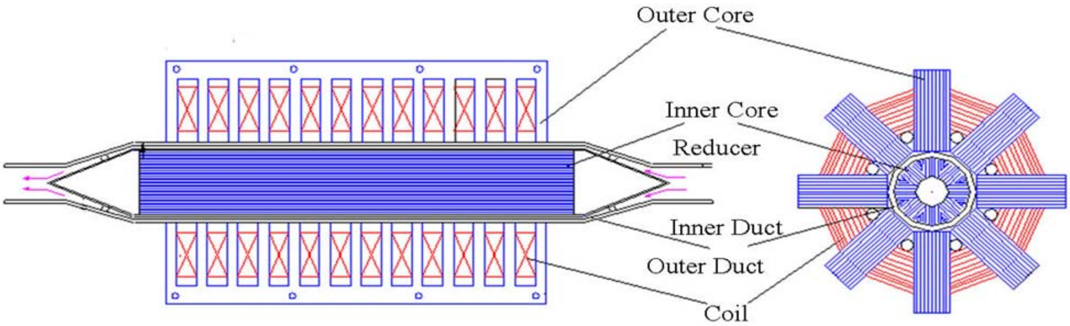

Figure 1: Cross-sectional view of an MHD pump.

But, to simplify the problem, the real pump core shape of is turned into a smooth core face replacing exciting coils by an equivalent current sheet to real coil arrangement as depicted in figure 2 [5].

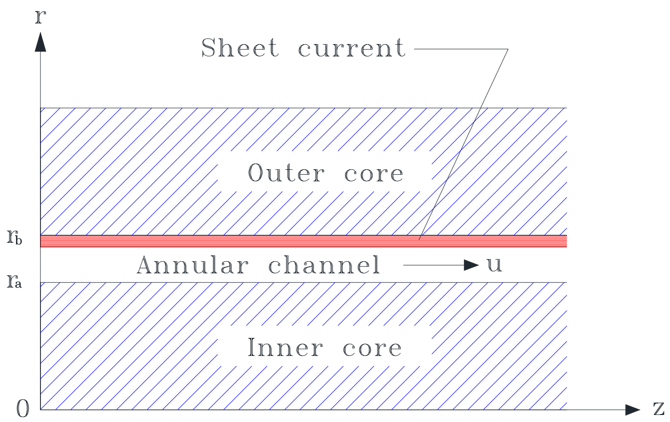

Figure 2: Simplified laminar model with equivalent current sheet .

A few more assumptions are introduced as follows [5].

(1) The pump has an infinitely-long annular channel with an equivalent current sheet replaced by discrete primary windings slotted in the outer core.

(2) All fields in the pump are axisymmetric ($\frac{\partial}{\partial\theta}=0$) in view of cylindrical arrangement of the pump system.

(3) The equivalent current sheet representing the three-phase currents (I) of continuous primary windings of N turns having pole pairs of p and pole pitch of τ is given by $J_a(r_2,z,t)=J_m e^{i(\omega t-kz)}\vec{\theta}$ where $J_m=3\sqrt{2}k_w NI/p\tau$.

(4) The sheet current produces traveling sinusoidal fields in the same form of J_a with angular frequency of ω and wave number of k for ***B*, *E*** and ***J***.

(5) Radial magnetic field (B_r) is uniform due to negligible skin effect in a narrow liquid metal gap.

(6) The liquid metal flow is incompressible.

To analyze the system, first of all, MHD equations are needed for expressing conservations of fluid and momentum. The pressure gradient (∇P) arouses from the combined action of electromagnetic driving and hydrodynamic drag forces in the conducting fluid of density ρ, viscosity η, and fluid velocity $\boldsymbol{u}$. Electromagnetic term by Lorentz's force is related with Maxwell's equations. Besides, induced current density (***J***) in the Lorentz's force is represented from Ohm's law given by electrical conductivity (σ) and fields ($\boldsymbol{E}+\boldsymbol{u}\times\boldsymbol{B}$) induced both by traveling sinusoidal magnetic fields and conductive flow movement across them. Then governing equations are given as :

- MHD equations

$$\nabla \cdot \boldsymbol{u} = 0 \quad (1)$$

$$\rho\left\{\frac{\partial u}{\partial t} + (\boldsymbol{u} \cdot \nabla)\boldsymbol{u}\right\} = -\nabla P + \eta\nabla^2\boldsymbol{u} + \boldsymbol{J} \times \boldsymbol{B} \quad (2)$$

In the MHD equations, in practical sense, when velocity and flowrate are indicated, they generally mean the averaged values over time. Therefore, we will try to analyze the system in the time-averaged point of view. The time-averaged MHD equations are resulted in two reduced equations for flow velocity and magnetic field.

$$\frac{d^2\bar{u}}{dr^2} + \frac{1}{r}\frac{d\bar{u}}{dr} - \alpha^2\bar{u} = \frac{1}{\eta}\frac{dP}{dz} - \alpha^2 U_s \quad (3)$$

$$\frac{dB_z}{dr} = \{\mu_0\sigma(-U_s + u) + f\}B_r \quad (4)$$

where $\alpha^2 = \frac{\sigma B_r^2}{2\eta}$, $U_s = \frac{\omega}{k}$ (synchronous speed).

B_r can be treated as a constant since radial magnetic field does rarely change at a narrow inter-core gap in induction machines having negligible skin effects. In general, magnetic core materials have very large permeabilities compared with those of liquid metals, and the differences of tangential magnetic fields between core ($\boldsymbol{H_1}$) and liquid metal ($\boldsymbol{H_2}$) regions are given by sheet current density ($\boldsymbol{K}$) as $\boldsymbol{n} \times (\boldsymbol{H_1} - \boldsymbol{H_2}) = \boldsymbol{K}$. Thus for the present pump model, magnetic fields at inner and outer radius give $B_z(r_2) = \mu_0 J_m$, $B_z(r_1) = 0$. Applying no-slip boundary conditions for velocity ($\bar{u}$) at the cylindrical walls (r_1, r_2), i.e., $\bar{u}(r_1) = \bar{u}(r_2) = 0$, The solutions were expressed for the time-averaged velocity expressed in terms of the modified Bessel functions, I_0 and K_0, of zeroth order as

$$u(r) = \{AI_0(\alpha r) + BK_0(\alpha r) - 1\}\left\{\frac{1}{\alpha^2\eta}\frac{dP}{dz} - U_s\right\} \quad (5)$$

where

$$A = \frac{K_0(\alpha a) - K_0(\alpha b)}{K_0(\alpha a)I_0(\alpha b) - K_0(\alpha b)I_0(\alpha a)}$$

$$B = \frac{I_0(\alpha b) - KI_0(\alpha a)}{K_0(\alpha a)I_0(\alpha b) - K_0(\alpha b)I_0(\alpha a)}$$

If an average slip (s) over the channel gap is defined by $s = \frac{1}{r_2 - r_1}\int_{r_1}^{r_2}\left(1 - \frac{\bar{u}}{U_s}\right)dr$, the radial magnetic field (B_r) and axial pressure gradient ($\frac{dP}{dz}$) are obtained as function of s together with other pump parameters.

$$B_r = \frac{\mu_0 J_m}{(-\mu_0\sigma s U_s + fk)(r_2 - r_1)} \quad (6)$$

$$\frac{dP}{dz} = \frac{1}{2}\sigma U_s B_r^2 \left\{ s + (1-s)\left(1 + \frac{1}{\frac{1}{r_2 - r_1}\int_{r_1}^{r_2}(AI_0(\alpha r) + BK_0(\alpha r) - 1)\,dr}\right)\right\} \tag{7}$$

The axial pressure gradient has been developed by two components, Lorentz's force and viscous term. The first term of right-hand side, $\frac{1}{2}\sigma U_s B_r^2$ is originated from the electromagnetic force density which is simply obtained by direct calculation of $\boldsymbol{J} \times \boldsymbol{B}$. The other terms of right-hand side correspond to viscous force density. To compare the viscous force density with electromagnetic force density, as an example, numerical values of pressure differences versus slip (or flowrate) are represented graphically in figure 3 for a pump system with flowrate of 40,000 L/min under a pressure difference of 15 atm. Figure 3 indicates that viscous force density is negligible compared with electromagnetic force density through all flowrate values except for near synchronous speed (s = 0). Practically induction pumps are not operated at near the synchronous speed with very low slip. Due to real hydraulic load like valve or piping system, such low slip value needs to be avoided so that the system can generate quite realistic developing force. Since MHD pumps are generally operated at sufficiently high slip region to generate a considerable developing force overcoming heavy hydraulic pressure load (more than a few atms), the pump system analyses can be treated by electromagnetic analysis alone neglecting viscous effects. After all, it is thought that mechanical pressure gradient in the system can be replaced by electromagnetic force density ($\boldsymbol{J} \times \boldsymbol{B}$) alone.

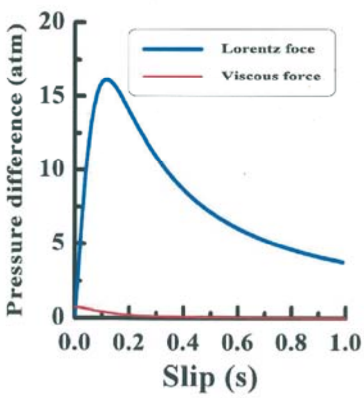

Figure 3: Comparison of viscous force effects with Lorentz's ones on producing the pressure difference between the inlet and the outlet.

Conclusion

Calculated results of the MHD flow analysis shows that the viscous loss effects on producing pressure differences are negligible compared with electromagnetic driving forces except in the low slip region where the pumps operate with very high flow velocities comparable with the field synchronous speed.

References

[1] Branover, H.: Magnetohydrodynamic flow in ducts, A John Wiley & Sons, New York (1978).
[2] Nasar, S.: Linear motion electromagnetic systems, A John Wiley & Sons, New York (1985).
[3] Nasar, S.: Linear motion electric machines, A John Wiley & Sons, New York (1976).
[4] Hee Reyoung, K and Yong Bum, L: Design and Characteristic Experiment of the Small Annular Linear Induction Electromagnetic Pump, Annals of Nuclear Energy, 38(2011) 1046-1052.
[5] Lawson, P.: Hydromagnetic channel flows: A John Wiley & Sons, New York (1960).