# ACOUSTIC MHD GENERATOR

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**Abstract :** Acoustic MHD generator is a part in a Thermo-acoustic Radio-Isotopic Power System project SPACETRIPS for space application. The radio-isotopic source produces heat. The thermo-acoustic generator converts it into sound. The MHD generator further converts sound into electricity. For decades our Institute developed conductive MHD generators working in DC mode. Now we have to adopt our experience [1] to AC mode.

## 1. Structure of the MHD generator

For specificity we assume MHD unit made from ferromagnetic insulator (e.g. laminated steel or SOMALOY) with configuration and sizes given on Fig.1. As working fluid there serves molten sodium filled in a through-going channel. In a middle of the channel its cross-section is a narrow (high= $h$ , radius= $r$ ) annular gap surrounded by a secondary coil and a radially magnetized permanent ring magnet. Conical inlet/outlet parts of the channel are azimuthally separated into $n = 32$ insulated sub-channels. Separating walls there are ferromagnetic insulators, while short continuations in the gap are nonmagnetic insulators. The gap division size is $d = 2\pi r / n$ .

Sound from thermo-acoustic generator forces gap sodium to oscillate in axial direction with velocity $u$ in radial DC field $B$ creating closed azimuthal AC current loop. An AC magnetic field pattern adds to the permanent field and induces voltage $V = 2\pi r E_V$ in both the secondary coil and a gap sodium.

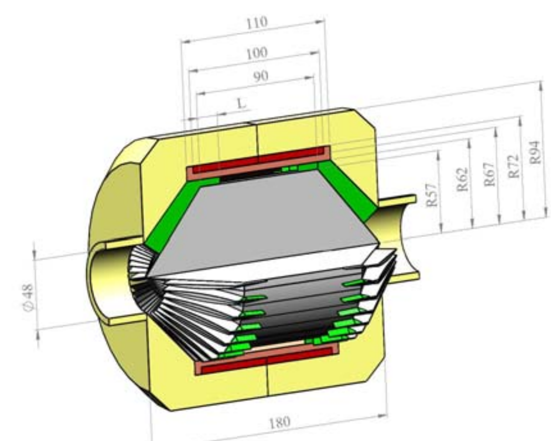



Figure1: Generator
R57—R62 sodium channel (gap)
R62—R67 x 110 secondary coil
R62—R72 x 100 permanent magnet
L — insulator

## 2. Computation

The small size of the device means low magnetic Reynolds. This allows us to combine free access simulation code FEMM4.2 with an equivalent circuit approach in a following way:

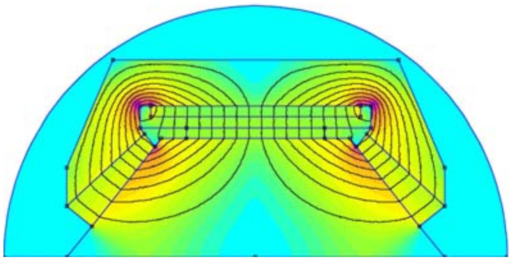


Figure 2: Magnetic field pattern.

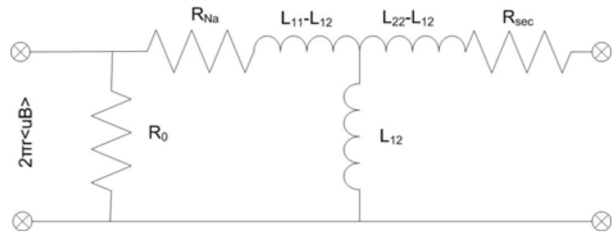



Figure 3: Equivalent circuit.

i. We define problems geometry for FEMM4.2 according to Fig.1. The inlet/outlet areas we declare as radially laminated ferromagnetic with a reduced filling factor.
ii. Operating in an axisymmetric DC mode the FEMM4.2 computes magnetic field pattern (Fig.2), as well as a single turn inductances for active part of the channel ($L_{11}$), for the secondary coil $L_{22}$ and their mutual inductance $L_{12}$.
iii. All other properties of MHD generator we get using FEMM4.2 output as an input in a simple FORTRAN code based on AC equivalent circuit approach.

## 3. Formulation of equivalent circuit approach

An acoustic MHD generator combines two electrical machines - a conductive MHD generator converting Na flow energy into electrical current with a transformer transforming it into voltage applied to the consumer. Transformer part of effective circuit (Fig.3.) contain three inductances. Measured from left transformer shows $L_{11}$, from right $L_{22}$. If test current is applied on one side the others side voltage shows $L_{12}$.

Definition of generator part is much longer [1]. Azimuthal current in sodium

$$j = \sigma(uB(x) - E_u(x) - E_V(x))$$

contains electrical field in two terms: $E_V$ set by transformer input voltage $V$ and an insulators response to $uB$

$$E_u(x) = u\int B(x_1) g_3(x, x_1) d^3x_1 = ub(x)$$

with a formal 3D Green function $g_3(x, x_1)$. Total power generation in Na reads

$$Q = \sigma u^2 \int B(x)(B(x) - b(x)) d^3x - \sigma u \int_\Omega E_V(x) B(x) d^3x$$

In generating part of equivalent circuit (Fig.3) three numerical values $R_{Na}, V_0$ and $R_0$ should be set:

**i.** Sodium resistance for induced current. The whole our approach is valid only for large $n >> 1$ when axial size of an active volume $\Omega = 2\pi rhl_{active}$ slightly exceeds insulator-free part of the gap: $l_{active} = l_{free} + 2\times 0.22d$ {$0.22 \approx \ln(2)/\pi$, see [1]}:

$$R_{Na} = 2\pi r / \sigma hl_{active}$$

In equivalent circuit $I_{Na} = (V_0 - V)/R_{Na}$. Power generation there

$$Q_{circuit} = V_0^2 / R_0 + (V_0 - V) I_{Na} + VI_{Na} = V_0^2 (1/R_0 + 1/R_{Na}) - V_0 V / R_{Na}$$

should correspond to $Q$.

**ii.** For $V_0$ we assume $E_V(x) = V/2\pi r$ working in a volume $\Omega$ only and compare last term in $Q$ with one in $Q_{circuit}$:

$$V_0 = \sigma u R_{Na} \int_\Omega (E_V(x)/V) B(x) d^3x = 2\pi r u < B >_\Omega,$$

where $< B >_\Omega$ means $B$ average in $\Omega$.

**iii.** For $R_0$ we compare first term in $Q$ with one in $Q_{circuit}$

$$1/R_0 = \sigma \int_{whole} (B(x) - b(x)) B(x) d^3x / < B >_\Omega^2 - 1/R_{Na}$$

Integration there is over the whole channel. Azimuthal integration leads to 1D formula

$$b(z) = \int_{subchannel} B(z_1) g_1(z, z_1) dz_1$$

$$1/R_0 = (\int_{whole} (B(z) - b(z)) B(z) dz / (< B >_\Omega^2 \, l_{active}) - 1)/R_{Na}$$

For real computation we use two approximations:

$$g_1(z, z_1) \approx 2G(2|z - z_1|/d)/d$$

$$\int_{whole} (B(z) - b(z)) B(z) dz \approx \int_{free} B^2(z) dz + \int_{subchannels} (B(z) - b(z)) B(z) dz + 0.085 d (B_R + B_L)$$

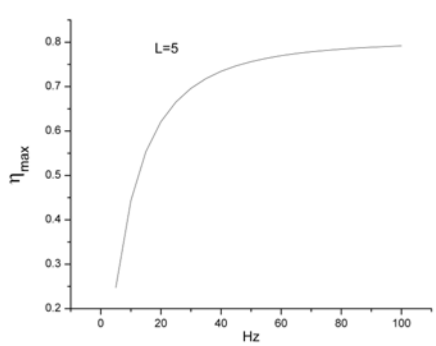
In correction term $0.085 \approx \ln(2)/\pi - 0.5 \sum_{k=1,\ldots\infty} 1/((k-0.5)\pi)^3$; $B_R, B_L$ are field values at right and left insulator tips. With such assumption in uniform field $R_0 = \infty$ as it should be.

$$G(z) = \frac{1}{\pi} \sum_{k=1}^{\infty} \exp(-(k-0.5)\pi|z|)/(k-0.5) = \frac{1}{2} \int_{-1}^{1} f(x,z) dx$$

$$f(x,z) = \sum_{k=1}^{\infty} \exp(-(k-0.5)\pi|z|) \cos((k-.5)\pi x)$$

The $G(z)$ is a Green function because $f(x,z)$ satisfies the Laplace equation. A proper normalization of $G(z)$ and a smoothness $\partial f(x,z)/\partial z|_{z=0} = 0$ both are due to the identity $\sum_{k=1,\ldots\infty} \sin(k-0.5)x/(k-0.5) = \pi/2$ for $0 < x < \pi$ [2].

## 4. Results



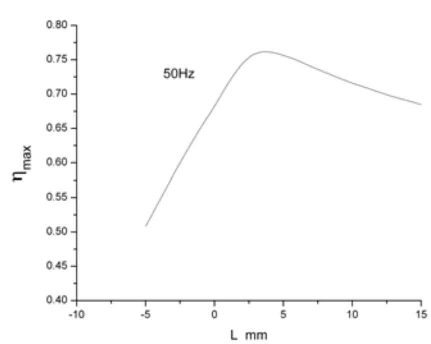




Figure 4: Efficiency dependence on the sound frequency and insulator length.

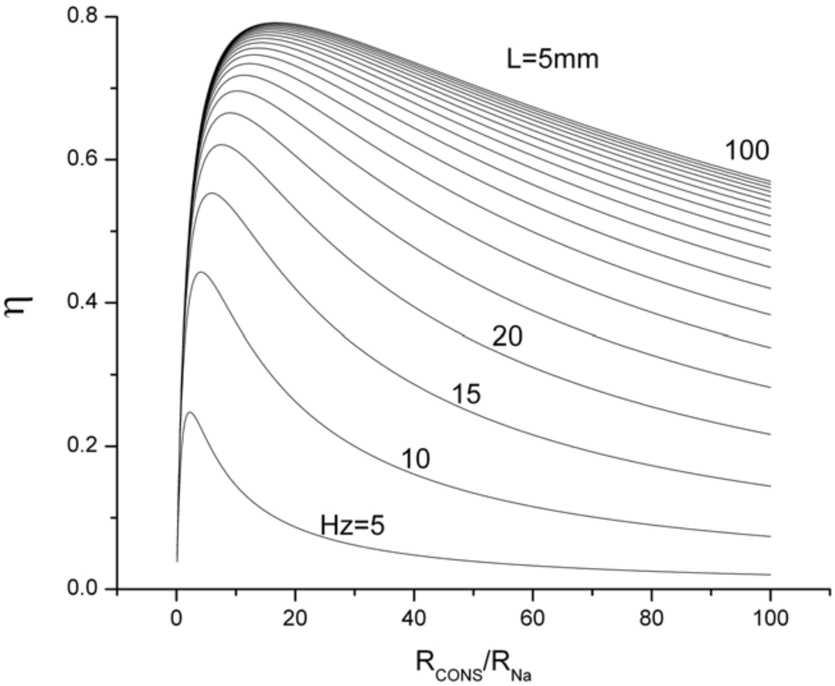



Figure 5: Efficiency vs. load and frequency.

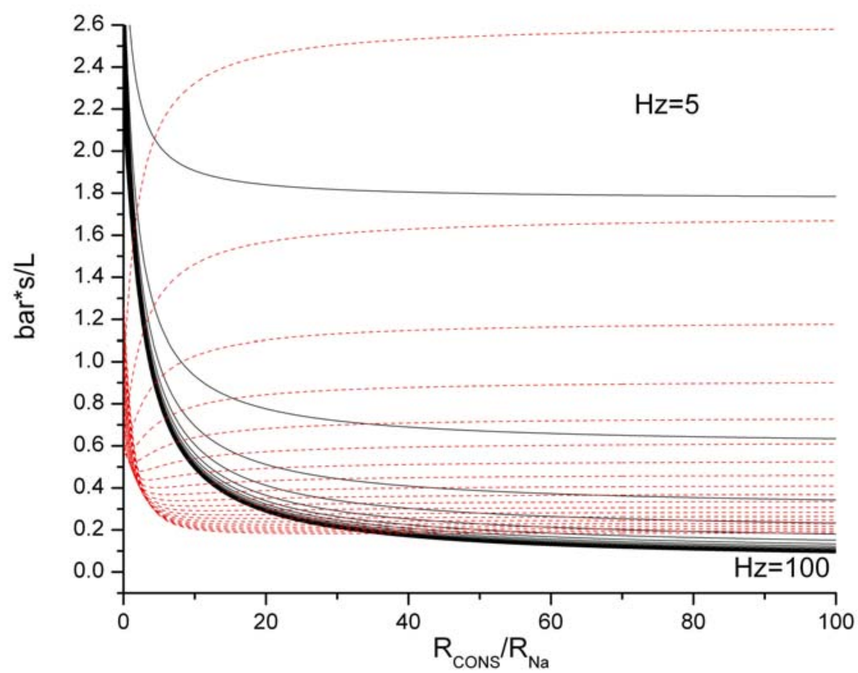
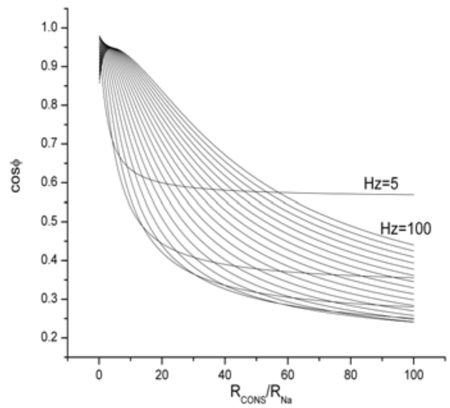



Figure 6: Acoustic impedance. Active part – solid line, reactive – dash.



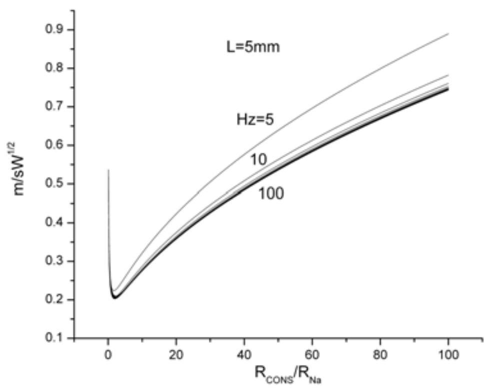




Figure 7: Acoustic cosϕ.

Figure 8: Na velocity for 1W output.

## 5. Conclusions

Effective circuit is fast and useful tool to analyze different generator aspects. In this paper only ideal generator was considered. Practical things such as sodium inertia, viscosity, loses in constructive materials etc. were omitted. No problem to include most of them into effective circuit and rerun the code.

## References

[1] Birzvalks Y.A. Basics of theory and calculation for conductive DC MHD pumps. Zinatne. Riga. 1968 (In Russ).

[2] Gradshteyn and Ryzhik's Table of Integrals, Series, and Products Alan Jeffrey (ed.) Fifth edition (January 1994) 1, 204 p. ISBN number: 0-12-294755-X.