ANALYTICAL INVESTIGATION OF MHD INSTABILITY IN ANNULAR LINEAR ELECTROMAGNETIC INDUCTION PUMP

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Abstract: In this work model of spatially and temporally developing azimuthal perturbations is analyzed in an infinitely long radially averaged geometry of an annular linear induction pump (ALIP). Using linear stability analysis, it is shown that in convective type instability process these perturbations can be significantly amplified before leaving the system. Perturbation development rates and its transient nature are analyzed to be used for estimations of unhomogenity amplification in a system of finite length.

1. Introduction

It has been reported (theoretically, experimentally and numerically) that high power induction pumps operating with liquid sodium exhibit instable operating modes, which are characterized by non-axisymmetric distribution of velocity and magnetic field, vibrations, low frequency pressure fluctuations and undesirable energy and pressure losses [1, 2, 3].

A theoretical base of this phenomenon is established by A. Gailitis and O. Lielausis [1] where a fundamental instability threshold of infinite induction machine is derived – magnetic Reynolds number > 1. Since then, there are only few authors like F. Werkhoff [4] who have tried to study similar or more sophisticated models, however significant improvements over the base theory has not been reported.

2. Presentation of the problem

Let us analyze case of ALIP identical to [1]. It is simplified from real ALIP by infinite geometry, neglecting influence of channel walls and using current sheet formulation only for main harmonic of the field (fig. 1.).

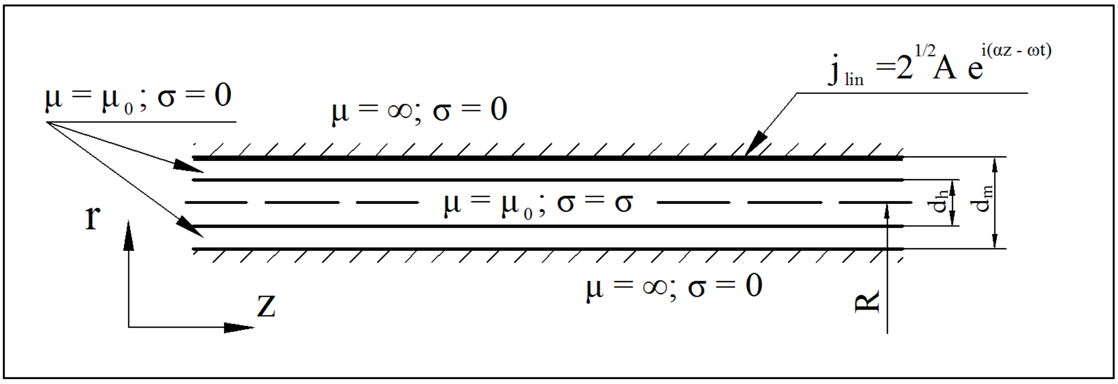

Fig. 1.A simplified model of infinite EMIP.

Moreover, consider that (where τ – pole pitch and l – length of system):

$$d_h < d_m \ll R, \tau \quad (1); \quad \tau \ll l \quad (2)$$

From condition (1) it can be rather correctly assumed that linear current density j_{lin} is evenly spread over height of non-magnetic gap d_m, significant is only radial component of magnetic field and all other effects are averaged over radius. Condition (2) declares that longitude end effects are negligible – therefore geometry can be considered as infinitely long.

Consider only r component of magnetic field and *z*, *φ* velocity components in form:

$$B = B_0(\varphi, z, t) e^{i(\alpha z - \omega t)} \boldsymbol{e_r} \quad (3); \quad \boldsymbol{v} = v_z(\varphi, z, t)\boldsymbol{e_z} + v_\varphi(\varphi, z, t)\boldsymbol{e_\varphi} \quad (4)$$

Then induction equation for *r* component of magnetic field is:

$$\Delta B - \mu_0 \sigma \left(\frac{d_h}{d_m}\right)\left[\frac{\partial B}{\partial t} + (\nabla \boldsymbol{v}) B\right] = \frac{i\mu_0 \alpha \sqrt{2} A}{d_m} e^{i(\alpha z - \omega t)} \quad (5)$$

Choosing characteristic space and time scales as:

$$L = \frac{2\tau}{2\pi} = \frac{1}{\alpha} \quad (6); \quad T = \frac{1}{2\pi f} = \frac{1}{\omega} \quad (7)$$

Defining magnetic Reynolds number *ε* and magnetic field dimensionless amplitude *b*:

$$\varepsilon = \frac{\mu_0 \sigma v_B}{\alpha}\left(\frac{d_h}{d_m}\right) \quad (8); \quad b = \frac{B_0 d_m \alpha}{\sqrt{2}\mu_0 A} \quad (9)$$

Induction equation can be rewritten in dimensionless form:

$$\left[\frac{\partial^2}{\tilde{R}^2 \partial \varphi^2} - 1 + i\varepsilon(1 - \widetilde{v_z})\right] b + \left[\frac{\partial^2}{\partial \tilde{z}^2} + 2i\frac{\partial}{\partial \tilde{z}} - \varepsilon\left(\frac{\partial}{\partial \tilde{t}} + \widetilde{v_z}\frac{\partial}{\partial \tilde{z}} + \widetilde{v_\varphi}\frac{\partial}{\tilde{R}\partial \varphi}\right)\right] b = i \quad (10)$$

In order to analyze the balance of momentum in flow it is necessary to calculate distribution of electromagnetic force density:

$$\boldsymbol{f_{EM}} = \left(\frac{d_m}{d_h}\right)\left[\frac{\nabla \times \boldsymbol{B}}{\mu_0} - \boldsymbol{j}\right] \times \boldsymbol{B} \quad (11)$$

From which dimensionless form of quasi – stationary electromagnetic force:

$$\tilde{\boldsymbol{f}}_{\boldsymbol{EM}} = \sigma v_B \left(\frac{\mu_0 A}{d_m \alpha}\right)^2 \varepsilon^{-1}\left[Re(b)\boldsymbol{e_z} - \tilde{\nabla}\left(\frac{bb^*}{2}\right)\right] \quad (12)$$

In order to simplify solution, it is more convenient to solve vorticity equation, therefore gradients become zeros and single equation for radial vorticity component is:

$$\beta\left(\frac{\partial}{\partial \tilde{t}} + \widetilde{v_z}\frac{\partial}{\partial \tilde{z}} + \widetilde{v_\varphi}\frac{\partial}{\tilde{R}\partial \varphi}\right)\widetilde{\omega} = -\left(\tilde{\nabla} \times (\tilde{\boldsymbol{v}}|\tilde{\boldsymbol{v}}|)\right)_r + j^2 \varepsilon^{-1} Re\left(\frac{\partial b}{\tilde{R}\partial \varphi}\right) \quad (13)$$

j^2 is modified interaction parameter (ratio of electromagnetic and friction forces) and *β* is modified Reynolds number (ratio of inertial and friction forces), k_m' - geometric parameter:

$$j^2 = \frac{2\sigma d_h}{\lambda \rho v_B}\left(\frac{\mu_0 A}{d_m \alpha}\right)^2 \quad (14); \quad \beta = \frac{2 d_h \alpha}{\lambda} \quad (15); \quad \kappa'_m = 1 + \left(\frac{1}{\alpha R}\right)^2 \quad (16)$$

(10), (13) and continuity equation (17) closes the system of described problem:

$$\frac{1}{\tilde{R}}\frac{\partial \tilde{v}_\varphi}{\partial \varphi} = -\frac{\partial \tilde{v}_z}{\partial \tilde{z}} \quad (17)$$

We look for solution with perturbations that have spatial and temporal dependency:

$$\tilde{v}_z = q + \delta\tilde{v}_z\left[e^{(n\tilde{z} - \gamma\tilde{t})} \cos(m\varphi)\right] \quad (18); \quad b = b_c + \delta b_m\left[e^{(n\tilde{z} - \gamma\tilde{t})} \cos(m\varphi)\right] \quad (19)$$

n and *γ* are spatial and temporal development rates of perturbation:

$$n = \frac{k}{\alpha} \quad (20); \quad \gamma = \frac{\Omega}{\omega} \quad (21)$$

q and b_c are unperturbed solutions of system as described in [1].

Considering only real *n* and *γ*, after linearization system becomes:

$$\left(2in + \varepsilon_\gamma - \kappa'_m + i\varepsilon_q\right)\delta b_m = i\varepsilon b_c \delta\tilde{v}_z \quad (22); \quad \left(2|q| - \beta\varepsilon_\gamma\right)\delta\tilde{v}_z = j^2\varepsilon^{-1} Re(\delta b_m) \quad (23)$$

$$\varepsilon_q = \varepsilon(1-q) \quad (24); \quad \varepsilon_\gamma = \varepsilon(\gamma - q) \quad (25)$$

The determinant of system (22, 23) considering only linear (with respect to *n*, *γ*) terms:

$$2|q| - \frac{\beta}{\varepsilon}\varepsilon_\gamma = \frac{j^2}{\left(1+\varepsilon_q^2\right)} \cdot \frac{\varepsilon_\gamma - \kappa'_m + \varepsilon_q^2 + 2n\varepsilon_q}{{\kappa'_m}^2 - 2\kappa'_m\varepsilon_\gamma + \varepsilon_q^2 + 4n\varepsilon_q} \quad (26)$$

By expanding last term of (26) in Taylor series we transform identity that *n* and *γ* can be easily expressed:

$$\frac{2|q|\left({\kappa'_m}^2 + \varepsilon_q^2\right)(1-q)}{\varepsilon_q^2 - \kappa'_m} + An - B\gamma = \frac{j^2(1-q)}{\left(1+\varepsilon_q^2\right)} \quad (27)$$

$$A = \left[2|q|\left(\frac{4\varepsilon_q + 2\varepsilon q\kappa'_m}{{\kappa'_m}^2 + \varepsilon_q^2} - \frac{2\varepsilon_q - \varepsilon q}{\varepsilon_q^2 - \kappa'_m}\right) + \beta q\right]\frac{\left({\kappa'_m}^2 + \varepsilon_q^2\right)(1-q)}{\varepsilon_q^2 - \kappa'_m} \quad (28)$$

$$B = \left[2|q|\left(\frac{2\varepsilon\kappa'_m}{{\kappa'_m}^2 + \varepsilon_q^2} + \frac{\varepsilon}{\varepsilon_q^2 - \kappa'_m}\right) + \beta\right]\frac{\left({\kappa'_m}^2 + \varepsilon_q^2\right)(1-q)}{\varepsilon_q^2 - \kappa'_m} \quad (29)$$

Now using equation for dimensionless developed pressure (30) [1]:

$$p = \frac{j^2(1-q)}{\left(1+\varepsilon_q^2\right)} - q|q| \quad (30)$$

And inserting (27) into (30) we have:

$$p = An - B\gamma + \frac{2|q|\left({\kappa'_m}^2 + \varepsilon_q^2\right)(1-q)}{\varepsilon_q^2 - \kappa'_m} - q|q| \quad (31)$$

Last two terms of (31) are solution for stability threshold derived in [1] (32). Development rates n and γ are function of pressure difference from stability threshold (33):

$$p_0 = \frac{2|q|\left({\kappa'_m}^2 + \varepsilon_q^2\right)(1-q)}{\varepsilon_q^2 - \kappa'_m} - q|q| \quad (32); \quad \Delta p = p - p_0 = An - B\gamma \quad (33)$$

By expressing γ from (33) and inserting it into exponent of perturbation development:

$$\delta v(\tilde{z},\tilde{t}) = \delta v \cdot e^{\frac{\Delta p}{B}\tilde{t}} \cdot e^{n(\tilde{z}-\tilde{v}_g\tilde{t})} \quad (34); \quad \tilde{v}_g = \frac{A}{B} = q + \Delta_g \quad (35)$$

$$\Delta_g = \frac{4|q|\varepsilon_q\left(\varepsilon_q^2 - 2\kappa'_m - {\kappa'_m}^2\right)}{\left(4|q|\varepsilon\kappa'_m + \beta\left({\kappa'_m}^2 + \varepsilon_q^2\right)\right)\left(\varepsilon_q^2 - \kappa'_m\right) + 2|q|\varepsilon\left({\kappa'_m}^2 + \varepsilon_q^2\right)} \quad (36)$$

Group velocity of perturbation (35) is not exactly equal to mean velocity of flow, but has correction (36). For $\beta >> 1$, $k_m' \approx 1$, $q > 0$ (case of real ALIP) Δ_g will be positive in stable regime ($0 < \varepsilon_q < 1$), near ($\varepsilon_q \approx 1$) it will have singularity and negative sign ($1 < \varepsilon_q < 2^{0.5}$), after ($\varepsilon_q = 2^{0.5}$) Δ_g is nonnegative. As singular behavior is not common in nature, pump can experience uncontrolled transient from stable to unstable regime, which is experimentally observed in [3]. Moreover, (34) is solution of first order partial differential (transport) equations' initial value problem (IVP) similar as discussed in [5]. Perturbation in the initial moment (t = 0) can be expressed in Fourier's series (37), then solution of this IVP is (38):

$$\delta v(\tilde{z},0) = \sum_{h=-\infty}^{\infty} a_h\, e^{ih\tilde{z}} \quad (37); \quad \delta v(\tilde{z},\tilde{t}) = e^{\frac{\Delta p}{B}\tilde{t}} \cdot \sum_{h=-\infty}^{\infty} a_h\, e^{ih(\tilde{z}-\tilde{v}_g\tilde{t})} \quad (38)$$

Expression (38) captures the nature of perturbation development in a convective instability process. In (38) second term describes movement of perturbation with (35) while sustaining its shape, however, first term states that it will exponentially develop in time. Such behavior described above is illustrated in (fig. 2). Consider idealized pump with length L, and some randomly shaped perturbation in initial time moment t_0 in the inlet. If pump is stable, perturbation will move towards outlet while decreasing (t_4) and after leaving the system. Similarly, if pump is unstable (fig. 3) perturbation will also move towards outlet, but being amplified. Reaching some maximum in the outlet (t_4) it eventually leaves the system.

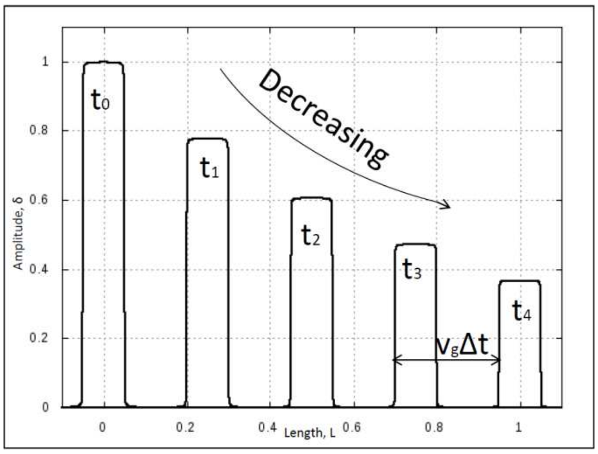

Fig. 2. Principal schematic of perturbation development in stable regime.

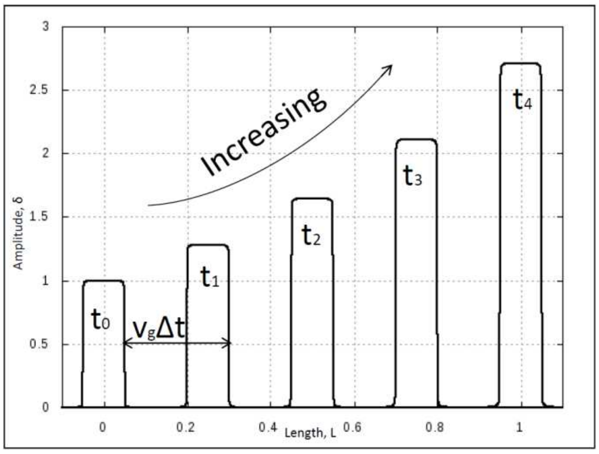

Fig. 3. Principal schematic of perturbation development in unstable regime.

Now consider (fig. 4) that pump is unstable and some perturbation always exist in the inlet (point B). In a static case ($\gamma = 0$), it will be amplified up to point E (Bold line B – E) by a exponential factor (39).

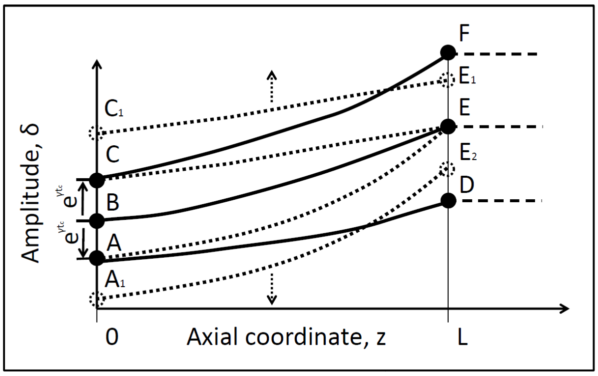

Fig. 4. Principal schematic of perturbation development in unstable regime.

Suppose that for some particular reason perturbation in the inlet starts to increase from B with $\gamma < 0$ for characteristic time interval - same as necessary for perturbation to travel from 0 to L - and stops at point C. Apparently, it results in lower spatial development rate *n* and it is described by dotted line C-E and (40).

However, as $\gamma = 0$ again, after characteristic time interval it is described by bold line C – F and spatial development rate is (39). If it hadn't been stopped spatial development would remain as in (40), but amplitude would continue to increase in time, dotted line C_1 – E_1, and so forth. Similarly, if perturbation in inlet decreases from B with $\gamma > 0$ and stops at point A, after characteristic time interval it can be described by dotted line A – E corresponding to (40) and afterwards with bold line A – D (39). Also if it hadn't been stopped amplitude would continue to decrease increase in time, dotted line A_1 – E_2, and so forth.

3. Conclusion

Preformed linear stability investigation reveals nature of convection type instability in ALIP. It is shown that some random perturbation will travel with group velocity (35) while sustaining its shape and develop (fig. 2 and 3) until it leaves the system. Development rates of perturbation can be calculated using (27 – 29).

If some mechanism exists that generates small static perturbation in the inlet of ALIP (e.g. geometrical imprecision) its amplification can lead to inhomogeneous flow in the outlet. Moreover, if transient behavior exists in the inlet (e.g. turbulent flow) perturbation will have different rates of amplification (fig. 4) which might lead to significant fluctuations of developed pressure.

4. References

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