MHD Simulation of Plasma Rocket Exhaust

EBERSOHN[1] Frans H., LONGMIER[1] Benjamin W., and SHEBALIN[2], John V.
[1]*Dept. of Aerospace Engineering*/University of Michigan, Ann Arbor, MI 48109, USA
ebersohn@umich.edu
[2]*Astromaterials Research Office*/NASA Johnson Space Center, Houston, TX 77058, USA

Abstract:

The magnetohydrodynamic description of magnetic nozzle physics is presented and discussed. A parametric analysis was performed of the generalized Ohm's law for relevant plasma propulsion devices. Results suggest that all terms of the generalized Ohm's law should be retained in MHD simulations to capture the relevant physics.

1. Introduction

Plasmas have a long history of use in space propulsion primarily through ion and Hall thrusters. However, a need exists for thrusters which limit the interaction of plasma with walls of the thruster which enables scaling to higher temperature, higher energy plasmas and increases the overall lifetime of the thruster.[1,2] Among the key components in the development of these thrusters is the magnetic nozzle which guides the flow of plasma through the thruster. Magnetic nozzles are strong converging-diverging magnetic fields which direct the flow of the plasma and limit interaction with the walls. These nozzles are also integral to the thrust generation process by converting non-directed magnetoplasma energy of the plasma into directed kinetic energy. Understanding the physics of magnetic nozzles is crucial to the current design of electric propulsion thrusters and also has broader applications to astrophysical jets. [3]

Magnetohydrodynamics (MHD) has been previously used to study the flow of plasma through a magnetic nozzle. A number of theoretical and computational studies have been performed with a focus primarily on ideal and resistive MHD descriptions. Herein we summarize the results of these studies and argue that more complex MHD descriptions are required to accurately capture all of the relevant physics in a magnetic nozzle. Section II of this paper further introduces the physics of magnetic nozzles and summarizes previous studies with MHD. Section III presents a parametric study of relevant magnetic nozzle devices and Section IV concludes this paper.

2. Summary of Studies and Simulations of Magnetic Nozzle with MHD

The primary function of magnetic nozzles is to control the flow of plasma and generate thrust by accelerating the plasma in the axial direction. A simple schematic of a magnetic nozzle is shown in fig. 1. The plasma is generated in the plasma source and then guided by the magnetic field which is generated by either a current coil, as shown, or by permanent magnets. A converging-diverging magnetic field is typically employed to create a nozzle contour similar to de Laval nozzles used in chemical rockets. The thrust generation process can be divided into three essential components: 1.) Energy conversion to directed kinetic energy; 2.) Transfer of momentum from the plasma to the thruster; 3.) Detachment of the plasma from the initially confining magnetic field lines. These steps of the thrust generation process are coupled with one another.[4]

In this paper we consider magnetic nozzles primarily from the MHD perspective and will limit the discussion of the relevant physics to those in the fluid, MHD regime. The physics of the magnetic nozzle can also be considered from particle and kinetic perspectives, leading to additional physical mechanisms which generate thrust.

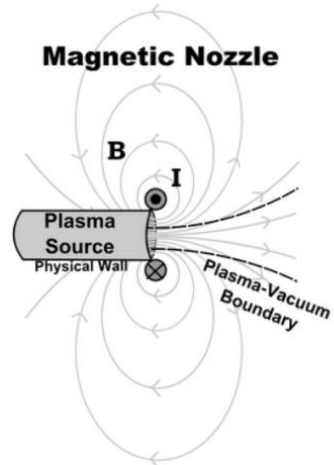

Figure 1: Schematic of magnetic nozzle.

Energy conversion mechanisms considered from an MHD perspective include generalized Hall effect and thermoelectric acceleration[5,6], directing the flow of fluid thermal energy[7], and Joule heating. Generalized Hall acceleration occurs due to the interaction of currents generated by the Hall effect with the applied field. Thermoelectric effects generate additional electric fields due to gradients in the electron density which can contribute to the Hall effect currents.[5] Hall effects can also result in a force which causes rotation of the plasma jet. [6] Fluid thermal energy is converted to axial kinetic energy due to the formation of a converging-diverging "magnetic wall". A current layer forms at the edge of the plasma which shields the inner plasma from the applied magnetic field and generates the confining force that guides the plasma flow. Simulations with resistive MHD codes (MACH2 and MACH3) have confirmed thrust generation by this process and show that the integrity of the current layer is crucial to efficient magnetic nozzle operation [8,9]. Joule heating describes the conversion of electromagnetic field energy into thermal energy in the plasma due to resistive heating.

Momentum transfer occurs due to the pressure of the plasma on the source walls and the Lorentz force resulting from the interaction of the induced currents and applied magnetic field. Diamagnetic induced currents are necessary to generate thrust in the plasma while paramagnetic currents can cause a drag force which reduces efficiency. [10]

MHD detachment scenarios considered include induced self-magnetic field effects[10], super-Alfvénic plasma flows[11], resistive diffusion[7,12], and magnetic reconnection. Self-field detachment can be achieved if strong diamagnetic currents are present which effectively cancel out the applied magnetic field.[10] Super-Alfvénic detachment can occur when the plasma flow velocity exceeds the Alfvén velocity. In this scenario the plasma drags the magnetic field lines along with it and significant field line stretching should occur.[11] Detachment through resistive diffusion occurs due to the plasma diffusing across the magnetic field lines due to collisions. This mechanism can result in drag losses on the plasma and significant anomalous diffusive transport can occur. [7,12] Finally, magnetic reconnection can facilitate detachment due to field line tearing and the formation of magnetic islands.

3. MHD Description and Parametric Analysis

The primary assumptions made in the derivation of the MHD governing equations are that the fluid behaves as a continuum, the ion mass is much greater than the electron mass ($m_{ion} \gg m_e$), and that the characteristic electron time scales are much faster than the bulk fluid time scales

($\omega_e \gg \omega_{fluid}$). The latter assumption implies that the electrons respond instantaneously to changes in the flow which leads to a simplification of the electron momentum equation into a force balance. This force balance can then be used to derive the generalized Ohm's law which provides closure to the MHD equations and allows for calculation of self-consistent magnetic fields.[13] The generalized Ohm's law is

$$\boldsymbol{E} = -\mathbf{U} \times \mathbf{B} + \frac{1}{n_e e}\boldsymbol{J} \times \boldsymbol{B} - \frac{1}{n_e e}\nabla(n_e k T_e) + \eta \boldsymbol{J} \tag{1}$$

In this equation the terms on the right hand side are referred to as the convective, Hall, electron pressure, and resistive terms respectively. The type of MHD model (ideal, resistive, Hall, etc.) and the physics captured by this model is defined by the terms which are kept in the Ohm's law. The relative importance of the terms in Ohm's law is found by an analysis comparing the terms.

The first relation of Ohm's law terms is shown in (2) and is found by comparing the convective to the resistive effects. The resulting non-dimensional number is known as the magnetic Reynolds number which gives a ratio of the convective to the resistive effects. High Re_m implies that the resistive effects are small which implies that the resistive term in Ohm's law can be ignored. For high Re_m the induced magnetic field can be large, which is important for the physical mechanisms such as formation of the current layer, induced field detachment, super-Alfvénic detachment, and magnetic reconnection.

$$Re_m = UL\sigma\mu_0 \tag{2}$$

The non-dimensional number comparing the Hall to the convective term and is shown in (3). The resulting non-dimensional number relates to the magnetization of the ions and compares a characteristic fluid frequency to the ion cyclotron frequency. Large values of this ratio imply that Hall effects are important compared to the convective contributions. Physically this relation implies that the ions are effectively demagnetized and a non-linear description is needed to describe their motion. Ion demagnetization is an important consideration in the detachment process as well as in the formation of Hall currents.

$$\Omega_{m,i} = \frac{\omega_{fluid}}{\omega_{c,i}} = \frac{U/L}{\omega_{c,i}} \tag{3}$$

Relation (4) compares the Hall term to the resistive term in Ohm's law. This ratio, known as the electron Hall parameter, compares the electron cyclotron frequency to the electron-ion collision frequency. The Hall parameter gives insight into the effective magnetization of the particles. Large Hall parameters suggest that the particles can complete cyclotron orbits many times before experiencing a collision, suggesting that the magnetic field significantly affects that species. This parameter is important in the thrust generation process because it can result in the formation of Hall currents which can contribute to the energy conversion process. For Hall currents to exist the ions must be effectively demagnetized so that a net current is produced.

$$\Omega_{H,e} = \frac{\omega_{ce}}{\nu_{ei}} \tag{4}$$

The final ratio is found by comparing the Hall to the electron pressure term and is shown in (5). The resulting non-dimensional number is known as the thermal β which gives a ratio of the plasma to the magnetic pressure. When this ratio is small the magnetic pressure is greater than the electron pressure and the electron pressure can be ignored in comparison to the Hall effects.

$$\beta = \frac{nkT_e}{B^2/2\mu_0} \tag{5}$$

These non-dimensional parameters are used to compare the terms in the generalized Ohm's law and identify the type of MHD model necessary to capture the important physical mechanisms in magnetic nozzles. We calculate these non-dimensional parameters for theoretical and experimental designs which incorporate magnetic nozzles shown in Table 1. The devices considered include the VAriable Specific Impulse Magnetoplasma Rocket (VASIMR) [14,15], fusion-based propulsion systems [8], the magnetoplasmadynamic arc jet (MPDA) [16], and the Helicon Double Layer Thruster (HDLT) [2,17]. The electron magnetization, $\Omega_{m,e}$, is also included as a measure of the validity of the MHD assumption that electron time scales are much faster than the bulk fluid timescales. Conditions change significantly as the plasma flows through the nozzle and the values selected represent only a single point.

		VASIMR [15]	**Fusion [8]**	**MPDA [16]**	**HDLT [17]**
Inputs	**Gas**	Argon	He	He	Argon
	n (#/m^3)	$1.00\cdot10^{19}$	$7.53\cdot10^{21}$	$5.00\cdot10^{20}$	$5.00\cdot10^{16}$
	T_{ion} (eV)	$1.00\cdot10^{2}$	$1.00\cdot10^{2}$	$1.50\cdot10^{1}$	$1.00\cdot10^{-1}$
	T_e (eV)	$6.00\cdot10^{0}$	$1.00\cdot10^{2}$	$5.00\cdot10^{0}$	$5.50\cdot10^{0}$
	U (m/s)	$4.00\cdot10^{4}$	$1.70\cdot10^{5}$	$2.00\cdot10^{4}$	$8.70\cdot10^{3}$
	L (m)	$5.00\cdot10^{-1}$	$1.80\cdot10^{-1}$	$3.00\cdot10^{-2}$	$1.50\cdot10^{-1}$
	B (Gauss)	$5.50\cdot10^{2}$	$9.44\cdot10^{3}$	$1.00\cdot10^{3}$	$1.38\cdot10^{2}$
Non-Dimensional Parameters	$\boldsymbol{\Omega_{m,e}}$	$8.27\cdot10^{-6}$	$5.69\cdot10^{-6}$	$3.79\cdot10^{-5}$	$3.03\cdot10^{-5}$
	$\boldsymbol{Re_m}$	$6.54\cdot10^{2}$	$2.83\cdot10^{5}$	$1.88\cdot10^{1}$	$3.03\cdot10^{1}$
	$\boldsymbol{\Omega_{m,i}}$	$6.04\cdot10^{-1}$	$4.16\cdot10^{-2}$	$2.77\cdot10^{-1}$	$1.75\cdot10^{0}$
	$\boldsymbol{\Omega_{H,e}}$	$4.56\cdot10^{2}$	$7.34\cdot10^{2}$	$1.59\cdot10^{1}$	$1.63\cdot10^{4}$
	$\boldsymbol{\beta}$	$7.98\cdot10^{-2}$	$3.41\cdot10^{-1}$	$1.01\cdot10^{-1}$	$5.81\cdot10^{-4}$

Table 1: Parametric Analysis of Magnetic Nozzle Systems

Small values of $\Omega_{m,e}$ found for all devices suggest that the basic MHD assumption of fast electron time scales is satisfied. The Re_m suggests that the resistive effects can not be ignored in most cases, with the exception being the fusion propulsion system. These calculations only consider classical diffusion, which has been shown to underestimate the resistive effects in plasmas. Bohm diffusion and anomalous diffusion processes may decrease the magnetic Reynolds number, increasing the importance of resistive effects. [12] The values of $\Omega_{m,i}$ are generally found to be near one, suggesting that the ions are nearly demagnetized and Hall effects may be on the order of the convective effects. Furthermore, as the plasma expands to weaker magnetic fields this term will become larger, increasing the effects of the Hall term. The ratios found for the electron Hall parameter also suggest that the Hall effects should be included when compared with the resistive effects. The electron pressure is found to be marginally important in all cases with the magnetic field pressure being stronger than the electron pressure as expected. Although this term is small in most, it is shown to be non-negligible in some.

Preliminary simulations have been performed using the Magneto Gas Kinetic Method (MGKM) which is generalized Ohm's law MHD code. The major result of these simulations shows rotation of the plasma jet and the formation of helical structure both in the velocity and currents within the jet. Challenges to future simulation are identified, with incorporation of the Hall term imposing restrictive time steps due to the unbounded Whistler wave characteristic.[18]

5. Conclusions

The physics of magnetic nozzles from an MHD perspective is discussed and previous studies are summarized and discussed. A parametric study of the generalized Ohm's law is performed. It is found from the parametric analysis and from a physics perspective that all terms of the generalized Ohm's law should be retained to capture all aspects of the thrust generation process. Future simulations and studies should thus include all terms in the generalized Ohm's law. Further considerations need to be taken concerning the continuum assumptions made in the MHD fluid model which are not considered here. [4]

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