## FEASIBILITY ANALYSIS OF AN MHD INDUCTIVE GENERATOR COUPLED WITH A THERMO ACOUSTIC ENERGY CONVERSION SYSTEM

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**Abstract** : This paper fits in the feasibility analysis of a Magneto-Hydrodynamic (MHD) inductive generator, coupled with a Thermo-Acoustic (TA) energy conversion system. The MHD and TA processes have the great advantage to convert energy without mechanical moving components. In this work, first the design criteria are given, then the order of magnitude of the obtained parameters is used to model the system by using the finite element method (FEM) to confirm the theoretical results. The conceptual idea and the FEM model are described.

# 1. Introduction

MHD power generation systems were originally investigated starting from the fact that the interaction of a plasma with a magnetic field must take place at much higher temperatures than were possible in a mechanical turbine. This kind of machines can only be efficient if the charges concentration in the gas is at an adequate level. This is usually obtained by heating the gas to a high temperature and seeding it with ionizing elements. In conventional MHD generators a plasma passes through an intense magnetic field, so, by closing the circuit on a load, the induced electromotive force determines in the fluid an electric current [1]. The main problems of the known MHD generators are the high temperatures needed to ionize the gas and the high magnetic field (about 5T) required to have significant outlet energy. The proposed device does not require an external magnetic field to work, but it performs the energy conversion through the induction principle. The charge carriers are first created by means of an electrical discharge and then separated by an external, high voltage, electrostatic field. Once the equilibrium is reached, if the gas inside the duct gets to vibrate by the TA phenomena, the charge carriers give rise to an alternating electric current; this induces an electromotive force in a toroidal coil wrapped around the duct and connected to the load. Thermo acoustic engines can convert high temperature heat into acoustic power with high efficiencies and without moving parts. They are low mass and promise to be highly reliable. Coupling this system with an MHD generator will create a heat driven electric generator suitable for space applications [2] [3] [4].

The paper is organized as follows. In Section 2 the energy conversion process is qualitatively described. In Section 3 a first study has been done in order to propose a simplified theory about the performances of the generator and, therefore, to give the design criteria. Then a FEM analysis is performed to justify the assumptions of the design phase. In Section 4 is described the thermo-acoustic model; in Section 5 is described the electromagnetic model; some results are reported. The last section provides some conclusions.

# **2.** Energy conversion process

The TA phenomena occur when a great gradient of temperature is present in the longitudinal direction of a duct containing a gas. In order to obtain said gradient we need a heat source and

a stack inside the duct with a large surface. The TA effect allows transforming the thermal energy into vibration energy, where the gradient of temperature affects the flow rate of energy while the frequency is determined by the length of the duct. The most important features of TA are that there aren't mechanical moving parts, and the working fluid is quasi-static [5]. It represents a better solution with respect to the conventional MHD generators, which convert the energy of a flowing working fluid. The device proposed in this study (Fig. 1) allows one to perform the transformation from heat to electricity without moving parts and with a quasistatic working fluid. The thermo acoustic engine converts thermal energy into mechanical energy, then the MHD generator converts the mechanical energy into electricity. The working fluid is forced to become plasma by means of periodical electrical discharges supplied by two electrodes immersed in the gas and linked to a pulsed high voltage generator. In order to have enough charge density, the pressure of the gas plays a fundamental role. The fact that the electric charge is generated by means of an electric discharge, provided that the pulsing voltage is enough high, means that the gas can be ionized also at low temperatures and no seeding is necessary. The charge carriers of opposite sign are separated by means of two electrodes connected to a DC high voltage supply. Here, the voltage needed to maintain in equilibrium the two clouds of charges of different sign, linearly depends on the surface of the electrode, so that the shape of said electrodes have to be chosen carefully. Once the equilibrium is reached, if the gas inside the duct gets to vibrate, due to the TA effect, the unbalanced charge carriers participate to the motion of the surrounding neutrals, giving rise to an alternating current. Such current induces an electromotive force in a toroidal coil, wrapped around the duct in correspondence of the vibrating charges, which supplies the electrical load.



Figure 1: Schematic view of the MHD generator



## 3. Theoretical development and demonstrative facility sizing

A first study has been done in order to propose a simplified theory about the performances of the MHD induction ionized gas generator coupled with the TA engine. The order of magnitude of the design parameters obtained has been used to modeling and simulate the system. This study starts from the equation of Ampère and the equation of the circuit (Fig. 2):

$$2\pi R \cdot B = \mu_f \left( I - ni \right) \tag{1}$$

In this expression  $I = \overline{I} e^{j \omega t}$  is the total electric current in a cross section due to the charge oscillation,  $i = \overline{i} \cdot e^{j \omega t}$ , is the induced electric current in the toroidal coil,  $\mu_f$  is the magnetic permeability of the core of toroidal coils,  $B = \overline{B} e^{j \omega t}$ , and *R* is the mean radius the core. This expression does not take into account the displacement current. Let be *S* the cross section of the core and *n* the number of turn coils. The electrical circuit comprises also the load resistance  $R_e$  and a capacity *C* to compensate the self of the coil, in such configuration the equation controlling the electrical circuit writes:

$$j\omega nS \cdot \overline{B} = (R_e + 1/j\omega C) \cdot \overline{i} \Longrightarrow B^2 = 1/\omega^2 n^2 S^2 (R_e^2 + 1/\omega^2 C^2) \cdot i^2$$

Taking into account that  $R_e = P_0/i^2$ :

$$B^{2} = (1/\omega^{2}n^{2}S^{2})[(P_{0}^{2}/i^{2}) + (i^{2}/\omega^{2}C^{2})] \Rightarrow dB^{2}/di^{2} = (1/\omega^{2}n^{2}S^{2})[(-P_{0}^{2}/i^{4}) + (1/\omega^{2}C^{2})] = 0$$

Therefore, we can learn the value of the current which corresponds to the minimum of the magnetic induction:  $i^2 = \omega CP_0$  and reminding again that  $R_e = P_0/i^2 \Rightarrow R_e = 1/\omega C$ The previous position allows us to strongly reduce the complexity of the formulation of the problem and it tells us that the minimum value of the magnetic induction occurs when the impedance of both capacitor and resistor have the same modulus.

$$-\omega nS \cdot \overline{B} = (1/\omega C)(1+j) \cdot \overline{i} \Longrightarrow \overline{i} = -(\omega^2 nSC/2)(1+j) \cdot \overline{B}$$

By considering the expression of the power:  $P_0 = R_e i^2 = i^2 / \omega C \Rightarrow C = 2P_0 / \omega^3 n^2 S^2 B^2$ Such value of capacitance can be substituted in the Ampère equation:

$$\left[2\pi R/\mu_f - P_0/\omega SB^2 \left(1+j\right)\right] \cdot \overline{B} = \left[2\pi R \cdot B/\mu_f - \left(P_0/\omega SB\right)\left(1+j\right)\right] e^{j\varphi} = I = \pi R_D^2 \rho v_0$$
(2)

where  $\varphi$  is the phase of *B*. This equation allows us to calculate the electric current in the gas which allows to obtain the desired power and magnetic induction. First of all, taking into account that the *I* is the reference for the phases, we can calculate the phase  $\varphi$ . In fact, the phase of the term within brackets must be opposite to  $\varphi$ . We obtain:

$$\tan \varphi = \left[ 2\pi R \omega SB^2 / \mu_f P_0 - 1 \right]^{-1}$$
(3)

Secondly, we can calculate the modulus of B which corresponds to the minimum value of the gas current I. To do that we have to minimize the modulus of the term within brackets, obtaining:

$$B^2 = \mu_f P / 2\pi R \omega S \tag{4}$$

This result has an important consequence. In fact, by (2) and (3) derives that we have the minimum of both B and I when the magnetic induction is in quadrature with respect to the gas current. By substituting (4) in the (2) we obtain:

$$\pi R_D^2 \rho v_0 = \sqrt{2\pi RP / \mu_f \, \omega S} \tag{5}$$

The equations (4) and (5) allow us to perform the device sizing. The (4) establishes a direct relationship, for a given material of the torus core, between the required power, the size of the core and the frequency of the current. On the other hand, the (5) gives indications on the size and the operative conditions of the duct. As in the case of the magnetic induction, frequency and cross section of the core contribute to limit this parameter, while the permeability and the radius of the torus have an opposite effect. Therefore, if we have heavy constraints on both magnetic induction and electric current in the gas, it is preferable to act on frequency and cross section of the torus. Finally, in order to obtain the desired current I in the gas, we can observe that the radius of the duct has a stronger effect with respect to both density of charge and velocity amplitude. Taking into account that charge density and velocity amplitude have in general strict limits, we can foresee that the size of the device will be the key parameter in order to fulfill the requirements.

By means of a few substitutions we can obtain the current circulating in the coil:

$$\bar{i} = -\sqrt{\left(2\pi R P_0 / \mu_f n^2 \omega S\right)\left(1+j\right)} \tag{6}$$

and the voltage drops in the gas:

$$\overline{U} = -\omega S \sqrt{\mu_f P_0 / 2\pi R \omega S}$$
<sup>(7)</sup>

Finally, we can calculate the voltage to apply to the electrode in order to maintain the charges in equilibrium into the gas.

$$C_{D} = \varepsilon \left( 2\pi R_{D} L\beta / \delta \right) = \pi R_{D}^{2} L\rho / \Delta \varphi \Longrightarrow \Delta \varphi = \delta R_{D} \rho / 2\varepsilon \beta = \delta I / 2\varepsilon \beta \pi R_{D} \nu_{0}$$
(8)

where  $C_D$  is the capacitance of the electrode-gas system,  $\varepsilon$  is the vacuum dielectric constant,  $A_D$  is the gas-electrode interface surface, L is the length of the electrode,  $\beta$  is the ratio between the surface  $A_D$  and the surface of the internal wall of the duct corresponding to the electrode. The design parameters and the results for a demonstrative facility are reported in Table 1.

Table 1 Design 1 arameters and Results				
Design Parameters			Results	
$P_0 = 200 \text{ W}$	$V_0 = 30 \text{ m/s}$	$\mu_f = \mu_0 \cdot 5 \cdot 10^4 \text{ H/m}$	$i_0 = 1.6 \text{ A}$	$R_e = 200 \ \Omega$
R = 12  cm	n = 10  tr	$\delta = 0.5 \text{ mm}$	$V_{coil} = 177 \text{ V}$	$C = 0.8 \ \mu F$
$R_D = 7 \text{ cm}$	$S = 3 \ 10^{-3} \ \mathrm{m}^2$	$\beta = 1000$	$I_0 = 11.28 \text{ A}$	$\Delta \varphi = 37.9 \text{ kV}$
$\rho = 15 \text{ C/m}^3$	$\omega = 2\pi \cdot 10^3 \text{ rad/s}$	,	$U_0 = 17.72 \text{ V}$	$\dot{B} = 0.94 \text{ T}$

**Table 1 Design Parameters and Results** 

#### 4. Thermo-acoustic analysis

The thermo-acoustic analysis has been done in order to study the velocity profiles. The device to be modeled is a glass tube containing Helium (He) in which the propagation of the vibration occurs at a temperature equal to 273°K and a pressure of 50 [bar]. As can be noted



Figure 3: Velocity distribution in axial direction

from the previous study (Section 3), in order to optimize the performance of the device, the dimensionless parameter  $\beta$  has to be maximized. This can be obtained using both a thicker cross section and materials with high surface roughness. Different than in normal (isentropic/lossless) acoustics, the thermoacoustic formulation takes the dissipative effects of viscous shear and heat conduction into account. These effects cannot be neglected in acoustic wave propagation through narrow geometries. In fact, near walls viscosity and thermal conduction become important because it create a viscous and a thermal boundary layer where losses are significant [6]. The model is able to solve the equations

simultaneously for the acoustic pressure, p, the particle velocity vector,  $\mathbf{u}$ , and the acoustic temperature T. The length scale at which the thermo-acoustic description is necessary is given by the thickness of the viscous boundary layer, and the thickness of the thermal boundary layer. The thickness of both boundary layers depends on the frequency. The properties of the gas are taken from [7]. In order to simulate the thermo-acoustic effect, a vibration with known amplitude and frequency was applied to the gas. In literature, several numerical studies have highlighted the phenomenon of "Dark space" [8]. The charge density starts to decrease rapidly near the boundary and becomes almost zero at the sleeve. These considerations appear to be very useful in the analysis of the device under study. In fact the charges will not adhere exactly to the wall, but will thicken in a cloud that will remain at a certain distance from the electrode; this allow one to avoid that the thickening of charging it occurs in correspondence

of the boundary layers. The thermo-acoustic simulation solves a linearized, small parameter expansion of the Navier-Stokes equation, the continuity equation, and the energy equation. Between the results of the thermo-acoustic analysis, particularly it obtained the velocity distribution as function of the frequency and of the duct radius. As for a given gas the density and the fluid viscosity are known, the frequency, the radius of the duct, and the pressure are the main parameters that affect the velocity distribution. As the pressure is fixed, the only parameters that can be modified are the dimension of the tube and the working frequency. Finally, the velocity distribution at working frequency (1kHz) is reported in Fig. 3, with the detail near duct wall. The velocity profile has a flat shape in the center, with small peaks close to the wall.

### 5. Electromagnetic analysis

The 2D axisymmetric geometry model consists in a glass tube closed at the ends containing ionized helium gas. Two copper sleeve electrodes have been positioned at an equal distance

from the duct extremities, and have been connected to a HVDC power supply. The electric potential V represents the dependent variable of the problem. A space charge density  $\rho$  (see Table 1) has been inserted near the sleeves. Between a parametric analysis, by varying the external source HVDC, an optimal value of voltage has been found. Applying this value to the electrodes, as can be noted from Fig. 4, the electrical potential profile is null and flat in the central zone between the two electrodes where the electrical field is thus equal to zero. This allows to achieve an equilibrium condition for the charge distributions that the electric field will cannot alter.



Figure 4: Optimal distribution of the electrical potential along the duct

## 6. Conclusions

The charge distribution thickened near to the electrode can be obtained by applying a suitable external potential difference. As can be noted from the electrostatic study the electrical potential profile is null and flat in the central zone between the electrodes where the electrical field is thus equal to zero. This allows to achieve an equilibrium condition for the charge distributions that the electric field will not be able to alter. Considering the "Dark space" phenomenon (see Section 4), from the previous thermo acoustic study is apparent that by acting properly on frequency and radius, one can get the best velocity profile for ensure an enough intense vibration of the particles thickened near the electrodes.

### 7. References

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