FEM Solution of MHD Flow Equations Coupled on a Pipe Wall in a Conducting Medium

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Abstract

The Galerkin FEM with triangular linear elements is applied for solving MHD pipe flow equations coupled through boundary conditions on the pipe wall to the induced current Laplace equation of the exterior electrically conducting medium. The well known MHD characteristics as flattening velocity and boundary layer formation are observed inside the pipe for increasing values of Reynolds and magnetic Reynolds numbers. The continuation of induced currents of the fluid and external medium are maintained on the pipe wall accordingly with the corresponding magnetic Reynolds numbers values.

1 Introduction

The electrically conducting, viscous and incompressible fluid is driven down a straight pipe of sufficient length and of circular cross-section by a constant pressure gradient under an external magnetic field applied perpendicular to the axis of the pipe. MHD pipe flow finds some engineering and biomedical applications as MHD generators, pumps and instruments for measuring blood pressure.

Then, the MHD equations inside the pipe Ω_f located in an electrically conducting medium Ω_{ex} are given in nondimensional form as [1]

$$\begin{aligned} &\left.\begin{aligned} \nabla^2 V + Re \cdot Rh \cdot \frac{\partial B^f}{\partial y} &= -1 \\ \nabla^2 B^f + R_{m_f} \cdot \frac{\partial V}{\partial y} &= 0 \end{aligned}\right\} \quad \text{in } \Omega_f \\ &\nabla^2 B^{ex} = 0 \qquad \text{in } \Omega_{ex} \end{aligned} \tag{1}$$

with the boundary conditions on the pipe wall Γ

$$\left.\begin{aligned} V(x,y) &= 0 \\ B^f(x,y) &= B^{ex}(x,y) \\ \frac{1}{R_{m_f}}\frac{\partial B^f}{\partial n} &= \frac{1}{R_{m_{ex}}}\frac{\partial B^{ex}}{\partial n'} \end{aligned}\right\} \quad \text{on } \Gamma \tag{2}$$

and on the far away fictitious external boundary Γ_{ex} either $B^{ex}(x,y)=0$ (insulated) or $\dfrac{\partial B^{ex}(x,y)}{\partial n}=0$ (conducting) conditions are taken (see Figure 1 for the domain of the problem). n and n' are unit outward normals on Γ for Ω_f and Ω_{ex} regions, respectively. Equations (1) are derived from the interaction of Navier-Stokes equations for conducting fluids with the Maxwell equations for electromagnetic field through Ohm's law.

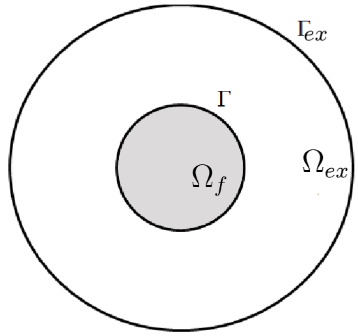


Figure 1: Domain of the problem

V, B^f and B^{ex} are the fluid velocity, induced magnetic fields of the fluid and external medium, respectively. Re and Rh are the Reynolds number and magnetic pressure of the fluid. Rm_f and Rm_{ex} are the magnetic Reynolds numbers of the fluid and external medium, respectively.

This coupled MHD problem (1)-(2) is solved by using BEM [2] for square and circular pipes and DRBEM [3] for circular pipe with the assumption that $B^{ex}\to 0$ as $x^2+y^2\to\infty$.

The Galerkin FEM application to the equations (1) with coupled boundary conditions (2), and one of the fictitious external boundary conditions is carried with linear approximations for V, B^f and B^{ex} as

$$V=\sum_{j=1}^{3} N_j V_j, \qquad B^f=\sum_{j=1}^{3} N_j B_j^f, \qquad B^{ex}=\sum_{j=1}^{3} N_j B_j^{ex} \tag{3}$$

where N_j's are linear triangular shape functions, V_j, B_j^f and B_j^{ex} are nodal values of V, B^f and B^{ex}.

The system of equations for one 3-nodal element 'e' is given as

$$\begin{bmatrix} K & -ReRhC & 0 \\ -Rm_fC & K & -\dfrac{Rm_f}{Rm_{ex}}D \\ 0 & 0 & K-D \end{bmatrix} \begin{Bmatrix} V_i \\ B_i^f \\ B_i^{ex} \end{Bmatrix} = \begin{Bmatrix} T \\ 0 \\ 0 \end{Bmatrix} \tag{4}$$

where

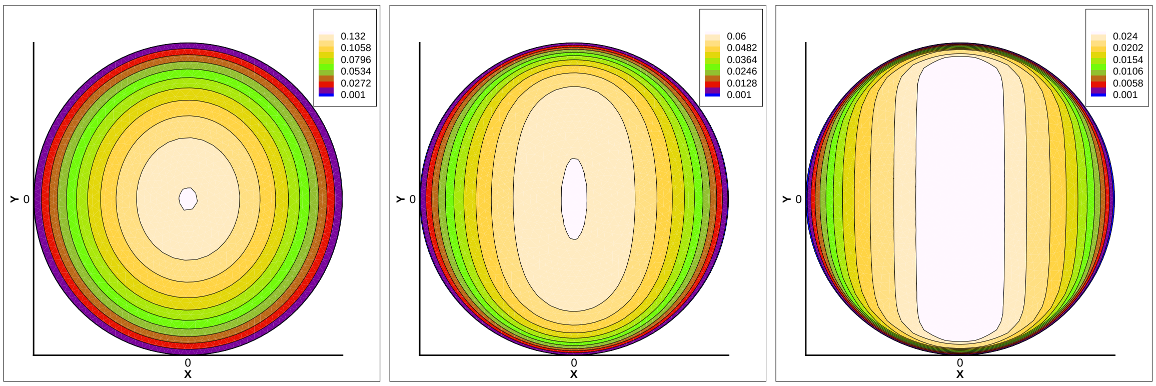
$$\begin{aligned} K_{ij} &= \int_{\Omega^e}\left(\frac{\partial N_i}{\partial x}\frac{\partial N_j}{\partial x}+\frac{\partial N_i}{\partial y}\frac{\partial N_j}{\partial y}\right)d\Omega \quad , \quad C_{ij}=\int_{\Omega^e}\frac{\partial N_i}{\partial y}N_j d\Omega \ , \\ & \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad i,j=1,2,3 \\ D_{ij} &= \int_{\Gamma^e}\frac{\partial N_i}{\partial n'}N_j d\Gamma \quad , \quad T_i=\int_{\Omega^e} N_i d\Omega \end{aligned} \tag{5}$$

and K and D matrices refering to B^{ex} are evaluted in the region Ω_{ex}.

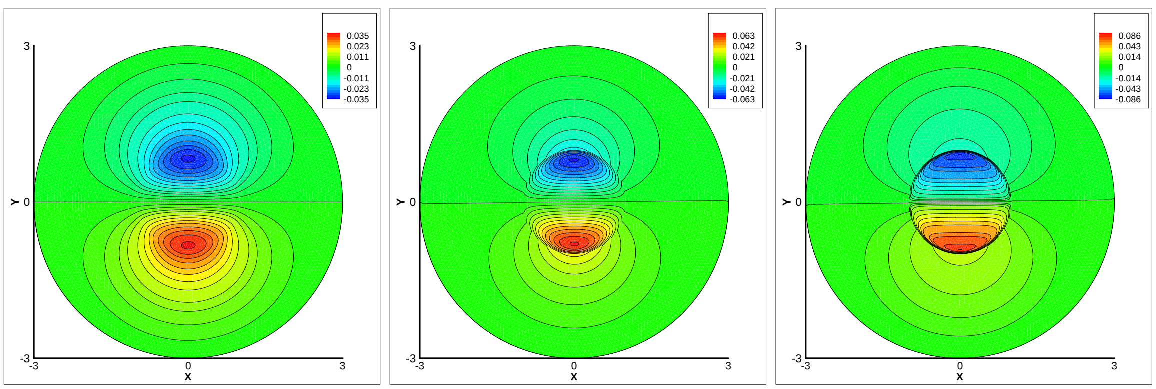
2 Numerical Results

Numerical results are obtained from the solution of final assembled global system of equations for V^f, B^f and B^{ex} nodal values in Ω_f and Ω_{ex}, respectively.

For the insulating external boundary case fluid velocity, and fluid and external medium induced current contours are presented in Fig 2 and Fig 3 for increasing values of Rm_f and for increasing values of Rh, respectively.



(a) Velocity



(b) Induced currents

Figure 2: Solution contours for $Rm_f = 1$ (left), $Rm_f = 10$(center) and $Rm_f = 100$(right) for $Re = 1, Rh = 10, Rm_{ex} = 1$

As fluid Rm_f increases flattening velocity is observed. Fluid becomes stagnant at the pipe center. Boundary layers are developed. Increase in Rm_f causes increase in B^f magnitude and fluid induced current tries to close itself inside the pipe. For equal Rm_f and Rm_{ex}, B^f and B^{ex} continue smoothly through the pipe wall.

As magnetic pressure Rh of the fluid increases both V and B^f, B^{ex} magnitudes drop but the continuation of B^f and B^{ex} through the pipe wall Γ is very well observed in Fig 3.

When the fluid $Rm_f = 10, 50, 100$ is much grater than external medium $Rm_{ex} = 0.01$ magnitude of B^f and correspondingly interaction through the conducting pipe wall is separated. Fluid induced current B^f behaves as if the induced current in a conducting wall pipe. Magnitude of B^{ex} is decreased and small compared to B^f (Fig 4).

In Fig 5, induced magnetic field lines are presented for the case of conducting external fictitious boundary Γ_{ex}. Fluid induced current B^f again closes itself inside the pipe as Rm_f increases, external induced current B^{ex} obeys the exit condition $\dfrac{\partial B^{ex}}{\partial n} = 0$ far away the pipe, they both continue on the pipe wall Γ.

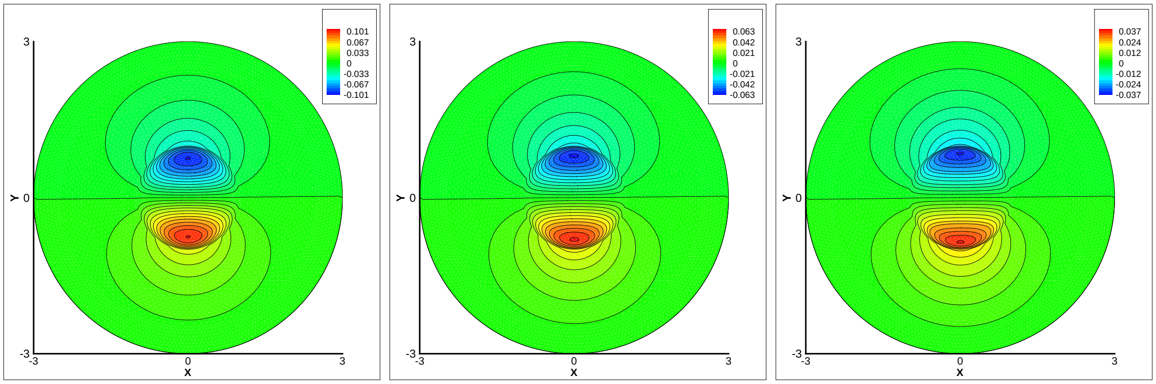


Figure 3: Induced current solution contours for $Rh = 5$ (left), $Rh = 10$(center) and $Rh = 20$(right) for $Re = 1, Rm_f = 10, Rm_{ex} = 1$

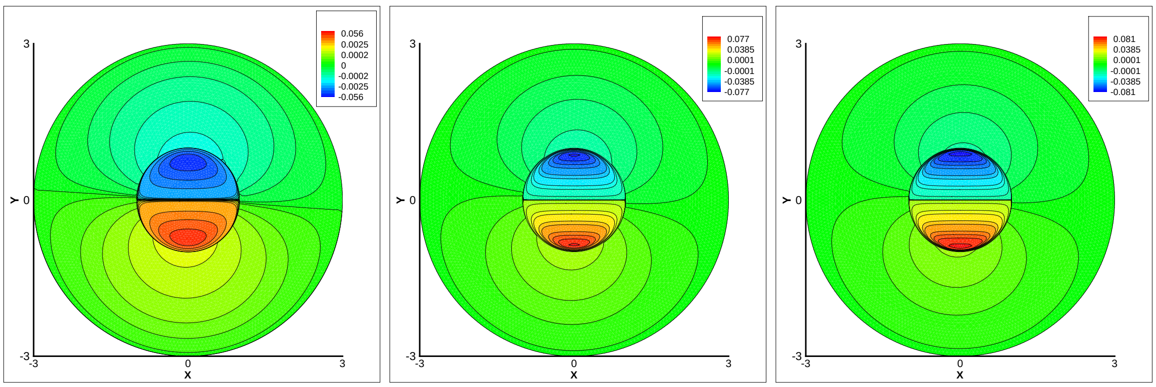


Figure 4: Induced current solution contours for $Rm_f = 10$ (left), $Rm_f = 50$(center) and $Rm_f = 100$(right) for $Rm_{ex} = 0.01, Re = 1, Rh = 10$

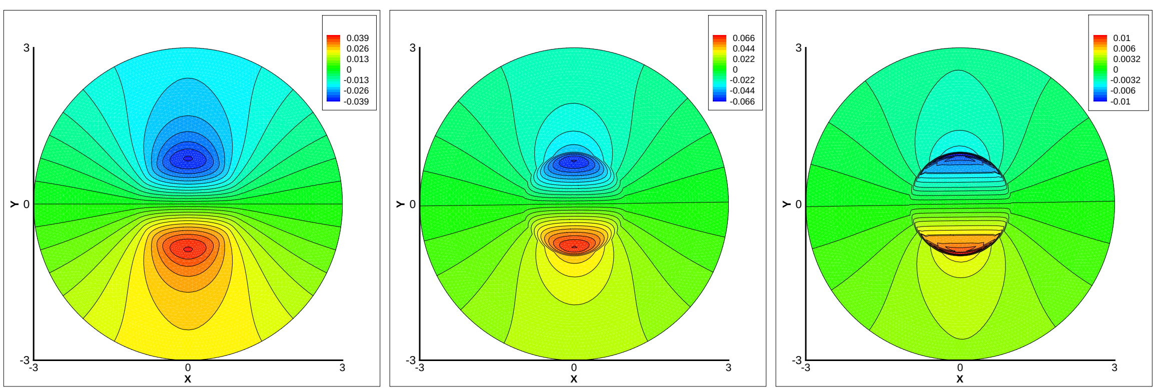


Figure 5: Induced current solution contours for $Rm_f = 1$ (left), $Rm_f = 10$(center) and $Rm_f = 100$(right) for $Re = 10, Rh = 10, Rm_{ex} = 1$

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