# NUMERICAL CALCULATIONS OF 3D PbLi MHD FLOW IN A SQUARE DUCT WITH DIFFERENT WALL ELECTRICAL CONDUCTIVITIES

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**Abstract:** Numerical results of 3D steady MHD flow in a meter-long square duct with applied strong magnetic field are presented. They could be of use in developing liquid-metal (LM) blanket, in which PbLi is assumed as a breeding material, also for estimation of influence of MHD effects on liquid-metal corrosion processes of structural materials. Results show that change of wall conductance ratio from zero to a finite value in streamwise direction leads to creation of flow transition region, which is basically proportional to the Reynolds number and inversely proportional to some power of Hartmann number as shown in results.

## 1. Introduction

This study is more of academic interest, because of its simplicity and physical certainty. The problem is similar to that described by Sterl [1]. In the first half of the channel all walls are electrically insulating, but in the second half Hartmann walls are electrically conducting. In this kind of configuration in the vicinity of cross section at x = 0 there is a region where the flow changes from bulk type flow in the insulating part of the channel to strongly irregular flow in the conducting part of the channel with characteristic M-shaped 2D velocity profile.

Numerical results are based on velocity field obtainment and interpretation, varying inlet velocity (mass inflow) and also external magnetic field in high Hartmann number region, where turbulence can be neglected, and laminar flow assumption is used. Typical flow velocities in this kind of reactor blankets are few centimeters per second in order of magnitude, so flows with small velocities are considered. The transition region length dependence on Re number as well as Ha parameter is also determined.

ANSYS FLUENT 12.0 code [2] benchmarking is done by comparing velocity distribution in channel's outlet with Hunt's analytical solution [3].

In general, material properties are chosen from real-life experimental setup: applied external magnetic field B = 1..10 T, LM density $\rho = 9400$ kg/m$^3$, electrical conductivity $\sigma = 0.77$ MS/m, dynamic viscosity $\rho\nu = 0.00187$ Pa·s; channel width a = 25 mm, wall thickness (thin-wall boundary condition is used) h = 3 mm.

## 2. Governing equations

The following Navier-Stokes equation of steady, incompressible MHD flow with continuity equation are solved numerically

$$\rho(\mathbf{v}\,\nabla)\mathbf{v} = -\nabla p + \rho\nu\nabla^2\mathbf{v} + (\mathbf{j}\times\mathbf{B}),$$
$$\nabla\,\mathbf{v} = 0.$$

Here $\mathbf{v}, p, \rho, \nu, \mathbf{j}$ and $\mathbf{B}$ are flow velocity, pressure, LM density, kinematic viscosity, induced currents and magnetic field respectively. From the following system of equations

$$\begin{cases} \nabla\times\mathbf{b} = \mu_0\mathbf{j} \\ \mathbf{j} = \sigma(\mathbf{E} + (\mathbf{v}\times\mathbf{B})) \\ \nabla\,\mathbf{b} = 0 \end{cases}$$

steady magnetic induction equation is derived and solved

$$\nabla^2 \mathbf{b} = \mu_0 \sigma[(\mathbf{v}\cdot\nabla)\mathbf{B} - (\mathbf{B}\cdot\nabla)\mathbf{v}],$$

where $\mathbf{B} = \mathbf{B}_0 + \mathbf{b}$ – the total magnetic field consisting of external and induced fields.

## 3. Boundary conditions

Mass flow inlet is chosen to reduce the entrance length, and Dirichlet boundary condition for pressure is used at the outlet. For an electrically insulating walls normal component of current density $j_n = 0$ , which is equivalent to induced magnetic field tangential components $b_{\tau 1} = b_{\tau 2} = 0$ from Ampere's relation. For Hartmann walls in conducting part of the channel thin-wall condition is applied to better match used Hunt's analytical model.

## 4. Solution methodology

Numerical calculations are carried out in ANSYS FLUENT 12.0. By default, steady, incompressible laminar flow is solved using finite volume method and pressure-based method. In advance MHD module is activated, and solution is obtained using magnetic induction method, which solves induction equation [4]. Pressure-velocity coupling is achieved by using SIMPLE algorithm. Least Square Cell Based, Standard and First Oder Upwind methods are used in spatial discretization for gradient, pressure and momentum, as well as magnetic field, respectively.

Gradually increasing applied external magnetic field, solution is obtained for 1..10 T field after checking whether there is no more significant change in monitored residuals.

## 5. Mesh

3D structured mesh with dimensions of 1000 mm × 25 mm × 25 mm is used (fig 1). It consists of 1.4 million cells with special refinement at the walls. First cell size at the Hartmann wall is 4 µm, and every successive cell is 1.5 times larger, first cell size at the sidewall is 15µm, with size ratio of 1.3 for every successive cell in the direction of the center.

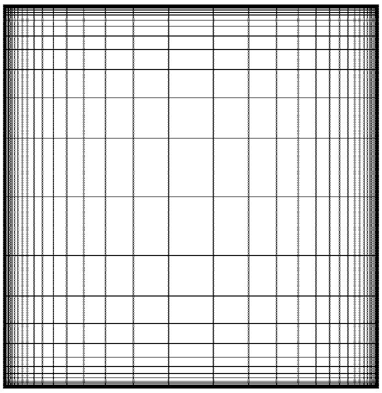

Figure 1: Mesh structure in flow cross section.

## 6. Results

<u>MHD flow transition region</u>. To show the transition region length dependence on the Reynolds number, mass flow rate at the inlet was varied, while other quantities were left unchanged. This was done for Re values ranging from 0.1 up to 7000 (fig 2).

Two regions can be distinguishable – the constant one with no dependence on Re, and the other one, which shows linear correlation between transition region length and Re number. Thus, there is some critical velocity value beyond which transition region length is independent of the flow velocity.

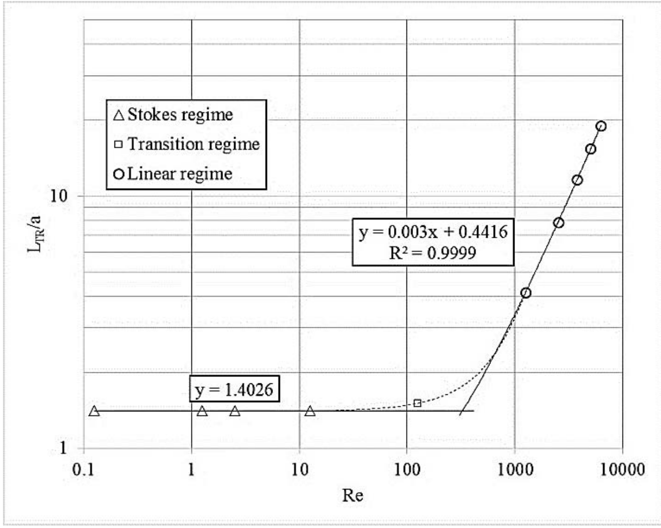


Figure 2: Non-dimensional transition region length dependence on Reynolds number at fixed magnetic field (Ha = 1015).

At fixed flow velocities external magnetic field value was varied. Data points were obtained at various magnetic field values ranging from 1 to 10 T. Taking the logarithm from non-dimensional transition region length and Ha number, plotted data points can be approximated with a linear function $Y = kX + C$ (fig 3).

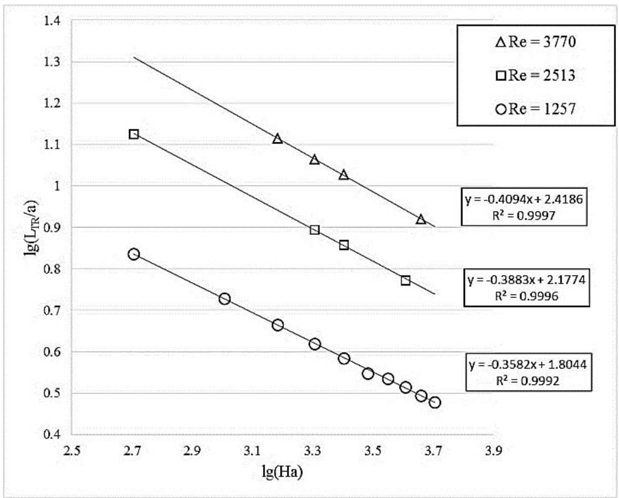


Figure 3: Non-dimensional transition region length dependence on Hartmann number at fixed inflow velocities.

Expression can be rewritten in form $\lg(y) = k \cdot \lg(x) + \lg(c)$. Taking the exponent from the latter expression, the true relation can be derived: $y = cx^k$. Displayed correlation coefficient squares confirm this dependence. From the slope values of the plotted lines can be concluded that the transition region length is inversely proportional to Ha number to the power of ≈0.4.

Invention of flow parameter. Combining the both previous relations, transition region length can be expressed as a function of Re and Ha numbers. Expression takes form $L_{TR} \sim Re \cdot Ha^{-0.4}$. One can guess that coefficient, which would make an equation from the expression, could consist of wall conductance ratio $c$ or its power, because it is one of the main parameters of the MHD flow.

Transition region of quasi-static flow. If flow velocity tends to zero, then interaction parameter tends to infinity, and nonlinear elements of Navier-Stokes equation can be neglected, achieving the so-called Stokes regime, when viscous forces are dominant in fluid. From the plotted MHD flow core velocity (fig 4) can be seen that decreasing flow velocity by two orders no changes are visible. To confirm this observation flow direction in the channel was changed to opposite. Results showed perfect agreement. Thus, decreasing integral velocity $v_0$, at some critical value $v_{cr}$ the transition to so called Stokes flow occurs, where the flow transition region and velocity distribution in this region no more depends on the magnitude of $v_0$ (Reynolds number), but depends only on Hartmann number and the magnitude of wall parameter. At flow velocities larger than the critical value ($v_0>v_{cr}$), the increase of the velocity $v_0$ leads to the increase of inertial forces, and, consequently, to the increase in length of flow transition region in the channel.

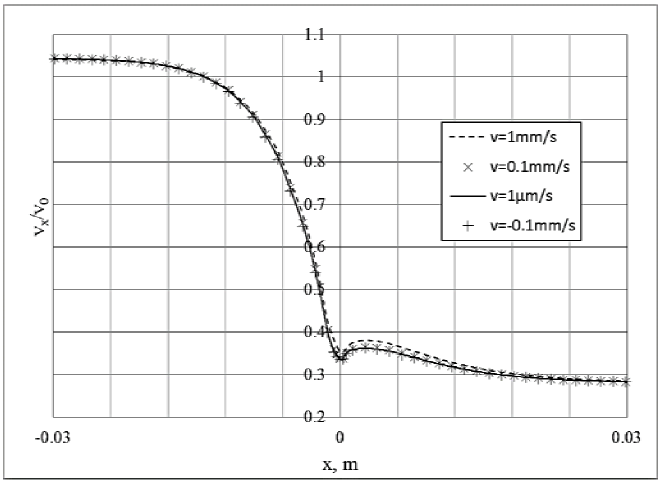


Figure 4: Non-dimensional flow velocity in transition region (Stokes regime)

## 7. Conclusions

When flow velocity is larger than critical velocity ($v>v_{cr}$), transition region length is proportional to Reynolds number. It is inversely proportional to the Hartmann number to a power of ≈ 0.4.

When flow velocity is smaller than critical velocity ($v<v_{cr}$) and tends to zero, transition region length tends to constant. No matter how small was the flow velocity, transition region length stays the same ≈ 1.4 a.

## References


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