ACCELERATION OF CONDUCTION LIQUID IN THE CYLINDER OF FINAL LENGTH UNDER THE INFLUENCE OF A ROTATING MAGNETIC FIELD

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Abstract: The non-stationary three-dimensional flow of a conducting liquid driven by a rotating magnetic field in the cylindrical vessel of the limited length is investigated

This work is considering non-stationary axisymmetric three-dimensional flow of a viscous incompressible electro-conducting liquid in the cylinder of final length. This flow arises under the influence of coaxially rotating magnetic field of any rotary symmetry. The problem is described by the system of two dimensionless equations with respect to azimuthal velocity v_φ and φ-component of vector potential of velocity ψ_φ:

$$\frac{\partial v_\varphi}{\partial t} - Lv_\varphi + \frac{\mathrm{Re}_\omega}{h}\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\psi_\varphi\right)\frac{\partial v_\varphi}{\partial z} - \frac{\partial \psi_\varphi}{\partial z}\frac{1}{r}\frac{\partial}{\partial r}\left(rv_\varphi\right)\right] = Ha_{ac}^2 r^{2p-1}\left(1 - v_\varphi/r\right),$$

$$\frac{\mathrm{Re}_\omega}{h}\left[\frac{\partial\left(L\psi_\varphi\right)}{\partial z}\frac{1}{r}\frac{\partial\left(r\psi_\varphi\right)}{\partial r} - r\frac{\partial}{\partial r}\left(\frac{L\psi_\varphi}{r}\right)\frac{\partial \psi_\varphi}{\partial z} + 2\frac{v_\varphi}{r}\frac{\partial v_\varphi}{\partial z}\right] = \tag{1}$$

$$= L^2\psi_\varphi - Ha_{ac}^2 r^{2p-2}\left[2\Delta\psi_\varphi + \frac{\partial^2\psi_\varphi}{\partial z^2} + 4\left(p-1\right)\frac{1}{r^2}\frac{\partial\left(r\psi_\varphi\right)}{\partial r}\right]$$

with boundary conditions

$$v_\varphi\left(0,z,t\right) < \infty,\ v_\varphi\left(r,z,t\right)\big|_\Gamma = 0,\ \frac{\partial v_\varphi\left(r,0,t\right)}{\partial z} = 0,\ v_\varphi\left(r,z,0\right) = 0,$$

$$\psi_\varphi\left(0,z\right) < \infty,\ \psi_\varphi\left(r,z\right)\big|_\Gamma = \left.\frac{\partial \psi_\varphi\left(r,z\right)}{\partial n}\right|_\Gamma = 0, \tag{2}$$

where $T_0 = 1/\omega$ is the time scale, $v_0 = \omega R_0/p$ is the characteristic value of velocity, $Ha_{ac} = Ha/\sqrt{2}$ is the Hartmann number based on the active value of induction, $\mathrm{Re}_\omega = \omega R_0^2/p\nu$ is the Reynolds number defined by the relative velocity of the area boundary in the magnetic field, ψ_φ is the φ-component of the velocity vector potential **v** = rot **ψ**, $h = Z_0/R_0$, p is the number of pole pairs (the order of rotary symmetry of a rotating magnetic field), Γ is the internal surface of the cylinder,

$$\Delta = \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{1}{h^2}\frac{\partial^2}{\partial z^2},\ L = \Delta - \frac{1}{r^2}.$$

The analysis of the second equation of system (1) shows, that the reason of occurrence of a three-dimensional flow is the spatial change of the azimuthal velocity. In this connection change at time of the azimuthal velocity is defined from the first equation of system (1), and evolution in time arising meridional vortices structures is defined through change of the azimuthal velocity.

The solution of the system of the equations (1) with boundary conditions (2) is obtained by an iterative method with applying the Galerkin's method at each step of iteration. On the i-step of iteration from the first equation of the system (1) on known value ψ_{i-1} is defined a value of azimuthal velocity v_i. Then known values ψ_{i-1} and v_i are substituted in the second equation of the system from which a value ψ_i is find. Iteration begins with a value $\psi_0 = 0$. Used computing scheme of the method looks like:

$$\frac{\partial v_i}{\partial t} - Lv_i + Ha_{ac}^2 r^{2p-2} v_i + \frac{\mathrm{Re}_\omega}{h}\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\psi_{i-1}\right)\frac{\partial v_i}{\partial z} - \frac{\partial \psi_{i-1}}{\partial z}\frac{1}{r}\frac{\partial}{\partial r}\left(rv_i\right)\right] = Ha_{ac}^2 r^{2p-1},$$

$$L^2\psi_i - Ha_{ac}^2 r^{2p-2}\left[2\Delta\psi_i + \frac{\partial^2\psi_i}{\partial z^2} + 4\left(p-1\right)\frac{1}{r^2}\frac{\partial\left(r\psi_i\right)}{\partial r}\right] - \tag{3}$$

$$-\frac{\mathrm{Re}_\omega}{h}\left[\frac{\partial\left(L\psi_{i-1}\right)}{\partial z}\frac{1}{r}\frac{\partial\left(r\psi_i\right)}{\partial r} - r\frac{\partial}{\partial r}\left(\frac{L\psi_i}{r}\right)\frac{\partial\psi_{i-1}}{\partial z}\right] = 2\frac{v_i}{r}\frac{\partial v_i}{\partial z}.$$

Other schemes of iterations are possible, however direct numerical experiment also has shown, that the scheme of iteration used above possesses the best convergence on parameters. Using procedure of the Fourier method, values v_φ and ψ_φ we define by decomposition in the series, satisfying boundary conditions (2):

$$v_\varphi\left(r,z\right) = \sum_{m,n} C_{mn} J_1\left(\beta_m r\right)\cos\gamma_n z\, e^{-\left(\beta_m^2+\gamma_n^2\right)t}, \tag{4}$$

$$\psi_\varphi\left(r,z\right) = \sum_{m,n} D_{mn}\left[J_1\left(\lambda_m r\right) + A_m I_1\left(\lambda_m r\right)\right]\cdot\left(\sin\alpha_n z + B_n sh\alpha_n z\right), \tag{5}$$

where β_m is the roots of equation $J_1\left(\beta_m\right) = 0$, γ_n is the roots of equation $\cos\gamma_n = 0$, i.e. $\gamma_n = \left(n - 1/2\right)\pi$, λ_m is the roots of equation $J_1\left(\lambda_m\right)I_1'\left(\lambda_m\right) - J_1'\left(\lambda_m\right)I_1\left(\lambda_m\right) = 0$, α_m is the roots of equation $\sin\alpha_n ch\alpha_n - -\cos\alpha_n sh\alpha_n = 0$, $A_m = -J_1\left(\lambda_m\right)/I_1\left(\lambda_m\right)$, $B_n = -\sin\alpha_n/sh\alpha_n$.
Numerical experiment made it possible to study the driven spatial flow patterns and to track their evolution in time. The solution is obtained both for small and for big Hartmann numbers at which iterative process still converges. In asymptotics by time the form of hydrodynamic structures comes close to the hydrodynamic structures obtained by solving corresponding stationary problem [1-3].
The completed research promotes the best understanding of the hydrodynamic processes occurring in a cylindrical vessel under the influence of the rotating magnetic field, operating limited time (for example, at metal processing in installations of continuous action).

References

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