MAGNETOGRAVITATIONAL STABILITY OF COMPRESSIBLE RESISTIVE ROTATING STREAMING FLUID MEDIUM

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Abstract: Magnetohydrodynamic stability of a gravitational medium with streams of variable velocity distribution for a general wave propagation in the present of the rotation forces has been studied. The magnetic field has strong stabilizing influence but the streaming is a destabilizing. The rotating forces have a stabilizing influence under certain restrictions. It is proved that the gravitational Jean's instability criterion is not influenced by the electromagnetic force or the rotation force or even by the combined effect of them whether the fluid medium is streaming or not and whether the rotation in one or more dimension.

Key words: Magnetogravitational, Resistive, Rotating, Streaming, Compressible

1. Introduction

The self-gravitational instability of a homogeneous fluid medium at rest has been investigated since long time ago, for its practical application in astrophysics see Jeans (1902). It is founded that the model is unstable under the restriction $k^2 C_S^2 - 4\pi G \rho_o < 0$ called after Jeans by Jeans' criterion, where k is the net wave number of the propagated wave, C_S^2 is a sound speed in the fluid, of density ρ_o, and G is the self-gravitational constant. Chandrasekhar and Fermi (1953), and later on Chandrasekhar (1981) made several extensions. The Jeans' model of self-gravitational medium has been elaborated with streams of variable velocity distribution by Sengar (1981). Recently Radwan and Elazab (1988), Radwan et al. (2001), developed the magnetogravitational stability of variable streams pervaded by the constant magnetic field (H_0, 0 0). The stability of different cylindrical models under the action of self-gravitating force in addition to other forces has been elaborated by Radwan and Hasan (2008),(2009). Hasan (2011) has investigated the stability of a oscillating streaming fluid cylinder subject to the combined effect of the capillary, self-gravitating and electrodynamic forces in all axisymmetric and non-axisymmetric perturbation modes. He (2011) has investigated the stability of oscillating streaming self-gravitating dielectric incompressible fluid cylinder surrounded by tenuous medium of negligible motion pervaded by transverse varying electric field for all the axisymmetric and non-axisymmetric perturbation modes. He (2012) has studied the instability of a full fluid cylinder surrounded by self-gravitating tenuous medium pervaded by transverse varying electric field under the combined effect of the capillary, self-gravitating, and electric forces for all the modes of perturbations. He (2012) the magnetodynamic stability of a fluid jet pervaded by transverse varying magnetic field while its surrounding tenuous medium is penetrated by uniform magnetic field.

Here in the present work we study the magnetodynamic stability of a self-gravitating rotating streaming viscous fluid medium pervaded by general magnetic field. Such studies have a correlation with the formation of sunspots. Also they have relevance in describing the condensation within astronomical bodies cf. Chandrasekhar and Fermi (1953), and also Chandrasekhar (1981).

2. Basic state

We consider an infinite self-gravitating fluid medium. The fluid is assumed to be homogeneous and viscous. The model is acting upon the following forces (*i*) the pressure gradient force, (*ii*) electromagnetic force, (*iii*) self-gravitating force, (*iv*) the forces due to rotating factors and (*v*) the forces due to resistivity. We shall utilize the Cartesian coordinates (*x*, *y*, *z*) for investigating such problem. The required equations for the present problem

$$\rho\left(\frac{\partial u}{\partial t}+(\underline{u}\cdot\nabla)\underline{u}\right)=-\nabla P+\mu(\nabla\wedge\underline{H})\wedge\underline{H}+\rho\nabla V-2\rho(\underline{u}\wedge\underline{\Omega})+\frac{1}{2}\rho(\underline{\Omega}\wedge\underline{r})^2 \qquad (1)$$

$$\frac{\partial\rho}{\partial t}+(\underline{u}\cdot\nabla)\rho=-\rho(\nabla\cdot\underline{u}) \qquad (2)$$

$$\frac{\partial\underline{H}}{\partial t}=\nabla\wedge(\underline{u}\wedge\underline{H})-\nabla(\eta\nabla\wedge\underline{H}) \qquad (3)$$

$$\nabla\cdot\underline{H}=0 \qquad (4)$$

$$\nabla^2 V=-4\pi G\rho \qquad (5)$$

$$P=K\rho^{\Gamma} \qquad (6)$$

Here ρ, $\underline{u}$, and P are the fluid density, velocity vector and kinetic pressure, μ and $\underline{H}$ are the magnetic field permeability and intensity, V and G are the self-gravitating potential and constant, η is the coefficient of resistivity, $\underline{\Omega}$ is the angular velocity of rotation, K and Γ are constants where Γ is the polytropic exponent.
We assume that the medium: (i) rotates with the general uniform angular velocity

$$\underline{\Omega}=(\Omega_x,\Omega_y,\Omega_z) \qquad (7)$$

(ii) be pervaded by the two dimensions homogeneous magnetic field

$$\underline{H}_o=(0,H_{oy},H_{oz}) \qquad (8)$$

and (iii) posses streams moving in the *x*_direction with velocity

$$\underline{u}_o=(U(z),0,0) \qquad (9)$$

varying along the z-direction of the Cartesian coordinates (*x*, *y*, *z*).

3. Perturbation analysis

For small departures from the initial state, every variable quantity Q may be expressed as

$$Q=Q_o+Q_1 \quad , \quad |Q_1| << Q_o \qquad (10)$$

where Q stands for each $\rho,\underline{H},P,\underline{u}$ and V . Based on the expansion (10), the perturbation equations could be obtained from (1)--(6).

4. Eigenvalue relation

Apply sinusoidal wave along the fluid interface. Consequently, from the viewpoint of the stability approaches given by Chandrasekhar, (1981), we assume that the space-time dependence of the wave propagation of the form

$$Q_1 \sim \exp\left[i\left(k_x x+k_y y+k_z z+\sigma t\right)\right] \qquad (11)$$

Here σ is gyration frequency of the assuming wave. k_x, k_y and k_z are (any real values) the wave numbers in the (*x*, *y*, *z*) directions. By an appeal to the time-space dependence (11), the relevant perturbation equations could be rewritten in the matrix form

$$[a_{ij}][b_j]=0 \qquad (12)$$

where the elements $[a_{ij}]$ of the matrix are given in the appendix I while the elements of the column matrix $[b_j]$ are being $u, v, w, h_x, h_y, h_z, \rho_1$ and V_1 .

For non-trivial solution of the equations (12), setting the determinant of the matrix $[a_{ij}]$ equal to zero (see Appendix I), we get the general eigenvalue relation of seven order in n in the form

$$A_7n^7 + A_6n^6 + A_5n^5 + A_4n^4 + A_3n^3 + A_2n^2 + A_1n + A_0 = 0 \qquad (13)$$

where the compound coefficients $A_i \ (i = 0,1,2,3,4,5,6,7)$ are calculated.

5. Discussions and results

Equation (13) is a general MHD eigenvalue relation of a rotating self-gravitating fluid medium pervaded by magnetic field of two dimensions. Some previously publishing results may be obtained as limiting cases here. That confirms the present analysis.

In absence of the rotating, electromagnetic forces and for inviscid fluid i.e. $\underline{\Omega} = 0$, $\underline{H}_o = 0$ and $\eta = 0$, equation (13) yields

$$k^2n^3 + k^2\left(k^2C_S^2 - 4\pi G\rho_o\right)n - k_xk_z\left(k^2C_S^2 - 4\pi G\rho_o\right)DU_o = 0 \qquad (14)$$

This relation coincides with the dispersion relation, of a pure self-gravitating fluid medium streams with variable streams ($U_0(z)$, 0,0) derived by Sengar (1981). For more details concerning stability of this case we may refer to Sengar (1981).

If $\underline{\Omega} = 0$, $\underline{H}_o = 0$, $\eta = 0$ and $U_o = 0$, equation (13) reduces to

$$n^2 = k^2C_S^2 - 4\pi G\rho_o \qquad (15)$$

This gives the same results given by Jean's (1902). For more details concerning the instability of this case, we may refer to the discussions of Jean's (1902). In absence of the magnetic field and we assume that the fluid medium is stationary i.e. $\underline{H}_o = 0, \eta = 0$ and $U_o = 0$, equation (13) gives another relation. The purpose of the present part is to determine the influence of rotation on the Jean's criterion (15) of a uniform streaming fluid. So in order to carry out and to facilitate the present situation we may choose $\Omega_x = 0$, $\underline{H}_o = 0$, $\eta = 0, k_x = 0$ and $k_y = 0$, equation (13), gives

$$n^4 + \left(4\pi G\rho_o - C_S^2k_z^2 - 4\Omega^2\right)n^2 + 4\Omega_z^2\left(C_S^2k_z^2 - 4\pi G\rho_o\right) = 0 \qquad (16)$$ with

$$\Omega^2 = \Omega_y^2 + \Omega_z^2 \qquad (17)$$

Equation (16) indicates that there must be two modes in which a wave can be propagated in the medium. If the roots of (16) are being n_1^2 and n_2^2, then we have

$$n_1^2 + n_2^2 = C_S^2k_z^2 + 4\Omega^2 - 4\pi G\rho_o \qquad (18)$$

$$n_1^2\, n_2^2 = 4\Omega_z^2\left(C_S^2k_z^2 - 4\pi G\rho_o\right) \qquad (19)$$

and so we see that both the roots $n^2{}_1$ and $n^2{}_2$ are real. The discussions of (16) indicate that if the Jean's restriction

$$C_S^2k_z^2 - 4\pi G\rho_o < 0 \qquad (20)$$

is valid, then one of the two roots n_1^2 or n_2^2 must be negative and consequently the model will be unstable. This means that under the Jean's restriction (20), the self-gravitating rotating fluid medium is unstable. This shows that the Jean's criterion for a self-gravitating medium is unaffected by the influence of the uniform rotation.

If $G = 0, \underline{\Omega} = 0, \underline{H}_o = 0$ and $\eta \neq 0,$ equation (13) degenerates to a somewhat complicated relation. The purpose of the present part is to determine the effect of the viscosity of fluid. So in order to carry out and to facilitate the present situation we may choose $k_x = 0$ and $k_y = 0$ so equation (13), at once, yields

$$\left(\sigma + k^2\eta\right)^2\left(\sigma^5k^2 + k^4\sigma C_S^2\right) = 0 \qquad (21)$$

Equation (21) indicates that resistivity has s destabilizing influence under certain restrictions.

6. Conclusion

The gravitational Jeans instability criterion is not influenced by the electromagnetic force or the rotation forces or even by the combined effect of them whether the fluid medium is streaming or not and whether the rotation in one dimension or more. The resistivity has s destabilizing influence under certain restrictions.

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Appendix I

The elements $a_{ij}\ (i=1,2,......,8 \text{ and } j=1,2,....,8)$ of the matrix $[a_{ij}]$ in equation (41) of the linear algebraic equations (30)-(38) are being

$a_{11}=(n\rho_o)$, $a_{12}=(2\rho_o\Omega_z)$, $a_{13}=(\rho_o DU_o-2\rho_o\Omega_y)$

$a_{14}=i\,\mu(k_y H_{oy}+k_z H_{oz})$, $a_{15}=ik_x\,\mu H_{oy}$, $a_{16}=ik_x\,\mu H_{oz}$, $a_{17}=ik_x C^2$, $a_{18}=-i\,\rho_o k_x$

$a_{21}=(-2\rho_o\Omega_z)$, $a_{22}=(n\rho_o)$, $a_{23}=(2\rho_o\Omega_x)$, $a_{24}=0$

$a_{25}=i\,\mu(2k_y H_{oy}+k_z H_{oz})$, $a_{26}=ik_y\,\mu H_{oz}$, $a_{27}=ik_y C^2$, $a_{28}=-i\,\rho_o k_y$

$a_{31}=(-2\rho_o\Omega_y)$, $a_{32}=(2\rho_o\Omega_x)$, $a_{33}=(n\rho_o)$, $a_{34}=0$

$a_{35}=ik_z\,\mu H_{oy}$, $a_{36}=i\,\mu(k_y H_{oy}+2k_z H_{oz})$, $a_{37}=ik_z C^2$, $a_{38}=-i\,\rho_o k_z$

$a_{41}=i\,(k_y H_{oy}+k_z H_{oz})$, $a_{42}=0$, $a_{43}=0$, $a_{44}=-(n+i(k_x^2+k_y^2))$, $a_{45}=ik_xk_y$,

$a_{46}=DU_o+ik_xk_z$, $a_{47}=0$, $a_{48}=0$

$a_{51}=-ik_x H_{oy}$, $a_{52}=ik_z H_{oz}$, $a_{53}=-ik_z H_{oy}$, $a_{54}=ik_xk_y$, $a_{55}=-(n+i(k_x^2+k_z^2))$,

$a_{56}=ik_yk_z$, $a_{57}=0$,$a_{58}=0$

$a_{61}=-ik_x H_{oz}$, $a_{62}=-ik_y H_{oz}$, $a_{63}=-ik_y H_{oy}$, $a_{64}=ik_xk_z$, $a_{65}=ik_yk_z$,

$a_{66}=-(n+i(k_x^2+k_y^2))$, $a_{67}=0$, : $a_{68}=0$; $a_{71}=i\,\rho_o k_x$, $a_{72}=i\,\rho_o k_y$, $a_{73}=i\,\rho_o k_z$, $a_{74}=0$,

$a_{75}=0$, $a_{76}=0$, $a_{77}=n$, $a_{78}=0$; $a_{81}=0$, $a_{82}=0$, $a_{83}=0$, $a_{84}=0$, $a_{85}=0$, $a_{86}=0$,

$a_{87}=-4\pi G$, $a_{88}=k^2$