FORCES EXPERIENCED BY AN ISOLATING SPHERE MOVING CLOSE TO A WALL

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Abstract
The paper presents phase-resolving numerical simulations to compute the force acting on an insulating sphere moving in a conducting liquid along an insulating wall. The superposition of a magnetic field and a perpendicular electric field, both parallel to the wall, results in Lorentz forces that create rotating flows in the vicinity of the sphere. Complex interactions of viscous and inertial fluid forces, electric induction, and Lorentz forces take place. By carefully changing the parameters of the problem the influence of these mechanisms on the force acting on the sphere is investigated. The configuration is proposed as a suitable test case to assess numerical methods for MHD multiphase flows.

1. Introduction

The behaviour of an insulating particle exposed to electric and magnetic fields is highly relevant for electrolysis processes, electrostatic separation of particles, and metallurgy. Here, we suppose the insulating sphere to represent a gas bubble in liquid aluminium, a situation relevant when generating metal foam, for example [1,2]. Leenov and Kolin [3] derived the force F_s acting on a spherical, insulating particle exposed to a magnetic field B and a perpendicular electric current $j = \sigma E$ to be

$$F_s = \sigma E B \pi R^3 \quad . \tag{1}$$

Sellier [4-6] extended this approach, deriving force and momentum acting on spheres and arbitrarily shaped particles close to a plane wall. He also derived the resulting velocity and rotation of a sphere under the assumption of vanishing Reynolds number. Also, the induction term in the Ohm's law

$$j = \sigma(-\nabla\Phi + u \times B) \quad . \tag{2}$$

was neglected, corresponding to a high ratio of applied electric field gradient $\nabla\Phi$ and electric induction $u \times B$. Additionally, the magnetic field was supposed to be constant in space and time, which means firstly, that it is not influenced by the different material properties inside the sphere and secondly, that magnetic induction and, therefore, the magnetic Reynolds number is small.
In this work, numerical simulations of the fluid dynamics and the MHD effects are presented, to find the limits of these assumptions and to investigate the behaviour beyond.

2. CFD code

The coupled fluid dynamic and MHD problem is solved with the in-house code PRIME [7-9]. It solves the unsteady, incompressible Navier-Stokes equations on a staggered Cartesian grid. For time advancement a low-storage three-step Runge-Kutta scheme is used, combined with an implicit treatment of the viscous terms. Turbulence modelling is not applied. Instead, very fine spatial and temporal resolution is employed to resolve even the smallest turbulent structures. The particle surface is modelled with the Immersed-Boundary method [7] with the surface being represented by Lagrangian marker points. At these points additional forces are introduced into the Navier-Stokes equations to impose a no-slip boundary condition. The sum of these forces equals the action of the fluid on the particle which hence is directly available in the method.

The magnetic field is assumed to be constant in space and time, because gas bubbles and liquid aluminium both have a magnetic susceptibility close to zero. By definition, the electric conductivity σ_s of an insulating sphere is much smaller than the conductivity of the fluid $\sigma_s \ll \sigma$. Therefore, an additional Laplace equation for the correction Φ_{cor} of the electric potential has to be solved to ensure, that the electric current

$$j = \alpha\, \sigma \left(-\nabla(\Phi + \Phi_{cor}) + u \times B\right) \qquad (3)$$

is divergence-free [9]. For the different electric conductivity of sphere and fluid is accounted by the parameter α which equals one outside the sphere, zero inside and is computed with a second order cut-cell approach around the particle surface described in [7]. The applied numerical scheme can handle non-vanishing inertial effects of the fluid and electric induction in the fluid. Therefore, it can be used to extend the earlier results of Sellier [4-6].

3. Setup

In order to limit the set of variables while still including the interactions of MHD mechanisms, the setup shown in fig. 1 is proposed. An insulating sphere with radius R is moving with imposed, constant velocity u_s parallel to an insulating plane wall without rotation. The distance between particle surface and wall remains constant and is chosen to be $0.5\,R$. An electric field is oriented parallel to the direction of movement, a constant magnetic field is oriented parallel to the wall and perpendicular to the electric field.

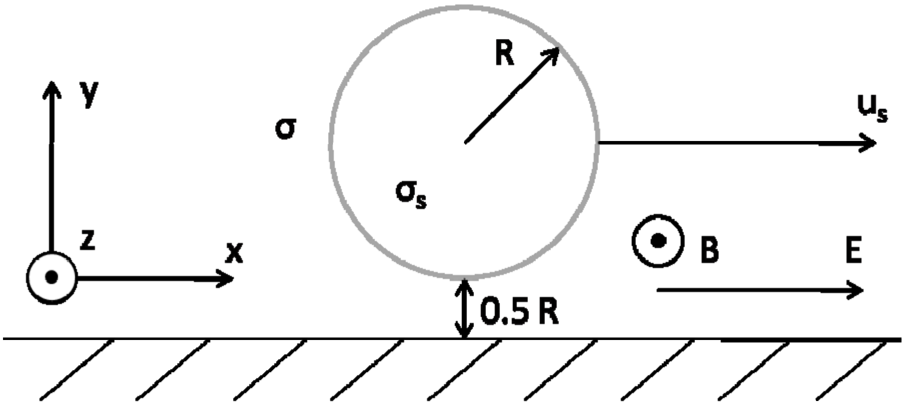

Figure 1: Geometrical setup of the simulation.

At the wall and at the surface of the particle, a no-slip condition for the fluid is applied.

The force on the bubble is computed in the way described above. The force in x-direction, F_x , results primarily from the fluid drag and is therefore normalized by the Stokesian drag to yield the non-dimensional force

$$f_x = \frac{F_x}{\square} 6\pi\mu R u_s \, . \qquad (4)$$

The force in y-direction results primarily from the Lorentz force, with (1) describing the homogeneous case, and hence is normalized by the corresponding value [3]

$$f_y = \frac{F_y}{\square} \sigma\, E\, B 5\pi\, R^3 \, . \qquad (5)$$

The problem has three independent parameters: the velocity of the sphere, resulting in the Reynolds number $Re_s = 2\, u_s R \frac{\square}{\nu}$, the magnetic field strength B , and the product of the electric and the magnetic field strength $\sigma\, E\, B$, in the following referred to as electromagnetic field strength. The reason for discussing the magnetic field strength separately here is that it acts by electric induction on the moving conducting liquid, independently of the applied electric field.

Table I provides the considered material properties which are those of a small gas bubble in liquid aluminium, while Table II gives an overview of the different simulations carried out.

Table I: Physical parameters of the setup.

Bubble diameter	$D\,[m]$	$2x10^{-3}$		Fluid density	$\rho\,[kg\,m^{-3}]$	2400
Fluid viscosity	$\nu\,[m^2 s^{-1}]$	$5x10^{-7}$		Electric conductivity	$\sigma\,[S\,m^{-1}]$	$5x10^6$

Table II: Combination of parameters defining Cases A..E. Bold numbers indicate the parameters which are modified to investigate their influence.

Case		A	B	C	D	E
Reynolds number	Re_s	0	0	$\mathbf{10..10^3}$	3	$\mathbf{3..30}$
Electromagnetic field strength	$0.75\dfrac{\sigma EB}{\rho}$ [m	$\mathbf{\pm 10^{-3}..10}$	$\mathbf{\pm 10}$	0	0	-10^{-1}
Magnetic field strength	$B\,[T]$	10^{-3}	$\mathbf{10^{-3}..10}$	0	$\mathbf{10^{-3}..10^{-1}}$	10^{-3}

4. Results for an immobile sphere

In a first step, forces on a fixed sphere are addressed. Figure 2a shows the force f_y for small magnetic field strength as a function of the electromagnetic field strength (Case A).

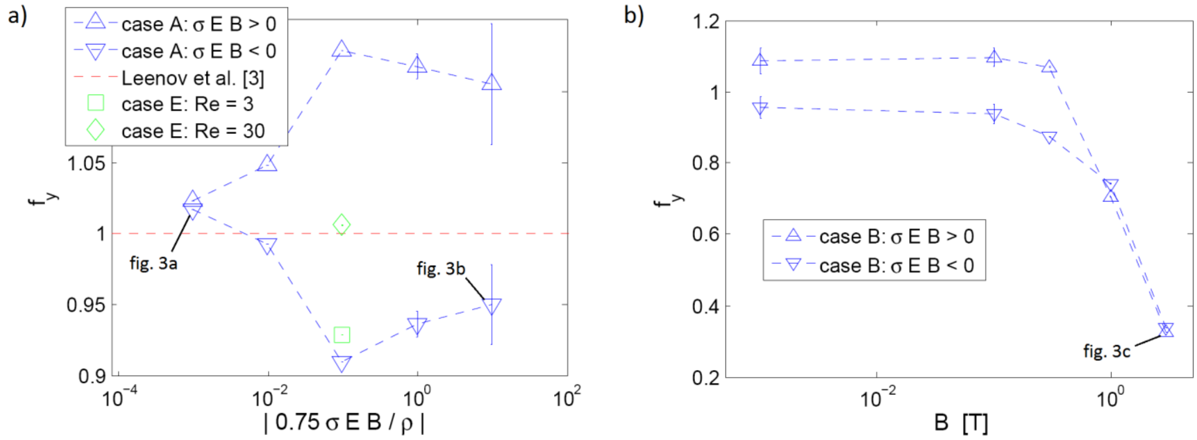

Figure 2: Wall-normal force on a sphere close to a wall. (a) Force as a function of electromagnetic field strength at constant, small magnetic field strength for a fixed sphere (Case A) and slowly moving sphere (Case E). (b) Force as a function of magnetic field strength at a constant, high electromagnetic field strength (Case B). Labeled simulations are visualized in fig. 3.

For $0.75\dfrac{\sigma EB}{\rho} = 10^{-3}$, the velocity of the fluid is small and its inertia negligible. Thus, f_y is independent of the sign of σEB . The difference to Leenov and Kolin [3] (fig. 2a) stems from

the interaction with the wall which is not present in their work. The electric current is concentrated in the gap between sphere and wall (fig. 3a), resulting in higher Lorentz forces in this region and hence, in a larger force on the sphere than in the absence of a wall. With increasing electromagnetic field strength, nonlinear inertia terms in the Navier-Stokes equation come into play with the sign of the force modification depending on the sign of the electromagnetic field (fig. 2a). At even higher field strengths, the flow becomes turbulent, resulting in fluctuating forces on the sphere.
In the second step, the magnetic field strength is varied (Case B). At constant, high electromagnetic field strength the magnetic field is increased while the electric field is decreased correspondingly. As shown in fig. 3, the stronger magnetic field damps the fluid motion, so that the inertia effects become less prominent. Fluctuations and dependency on the sign of the electromagnetic field disappear (fig. 2b). At the same time, the total force on the sphere is significantly reduced, even though the product of applied electric and magnetic field strength remains constant in this investigation.

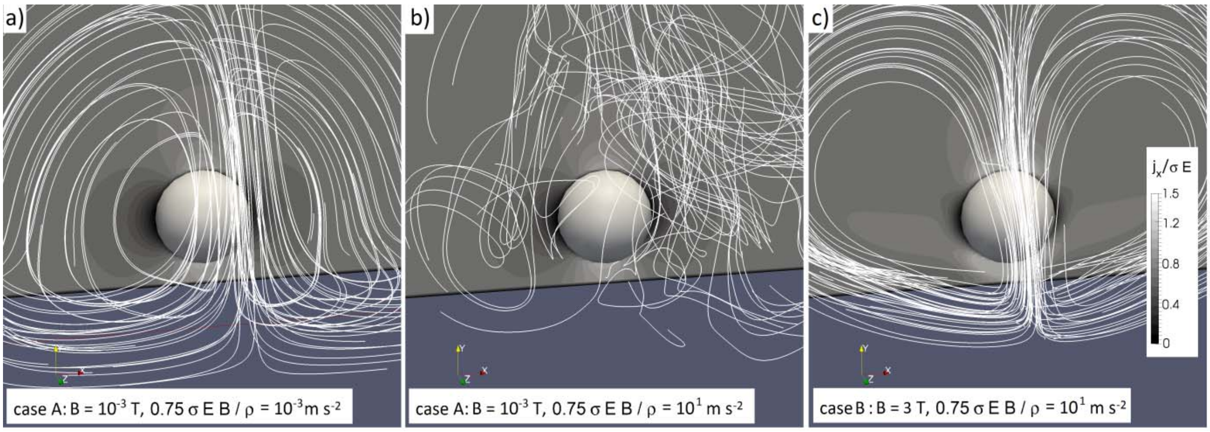

Figure 3: The plots show a contour of the horizontal component of the electric current j_x and arbitrarily chosen instantaneous fluid stream lines for the cases specified in the insets. The colour scale is the same for all plots.

5. Results for a moving sphere

Now the sphere is moved parallel to the wall. For absence of electric and magnetic fields (Case C) the force experienced by the sphere is parallel to the wall and depends nonlinearly on the velocity (fig. 4a). A purely magnetic field (Case D) can increase the force due to induced currents which create Lorentz forces that in turn damp the velocity. This is in agreement with Lenz’s law.

The electromagnetic field, mentioned above, creates a stationary vortex structure around the sphere (fig. 3a), resulting in nonlinear inertia effects. Increasing the velocity of the sphere (Case E), the flow more and more tends towards a dipole-type flow structure as shown in fig. 4b. This results in a reduction of the nonlinear effects on the force normal to the wall. With increasing velocity of the sphere the force normal to the wall tends towards the results for negligible induction and inertia. At the same time, the force in tangential direction is increased slightly by adding an electromagnetic field (Case E). This is shown in fig. 4 a.

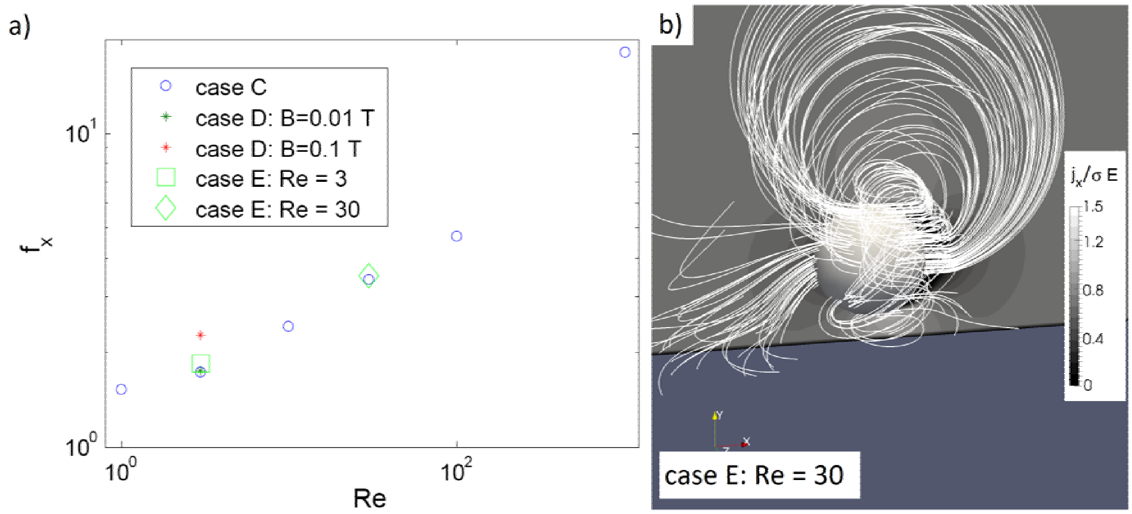

Figure 4: Force on a sphere moving tangentially to a wall. (a) Force without electric and magnetic fields (Case C), with a magnetic field only (Case D) and with an electromagnetic field (Case E). (b) Flow structure around the moving sphere in an electromagnetic field for Case E (Re = 30).

6. Concluding remarks

The magnetohydrodynamic flow around a fixed and a moving sphere in the vicinity of a wall has been simulated. The results agree very well with values from the literature, in cases where these are available. Due to the electrical insulation of a sphere, a curling vortex system is induced in its vicinity. With increasing field strength, nonlinear effects resulting from inertia and induction come into play. A magnetic field can damp the fluid motion and reduce turbulence, as it is well known from Hartmann flow and other configurations. Increasing the magnitude of the magnetic field at constant external electromagnetic field strength (i.e. simultaneously reducing the electric field), the force on the sphere is reduced.

7. References

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