EXPERIMENTAL INVESTIGATION OF INERTIAL WAVES INSIDE A CYLINDRICAL LIQUID METAL COLUMN

VOGT Tobias, RÄBIGER Dirk and ECKERT Sven
Helmholtz-Zentrum Dresden-Rossendorf, PO Box 510119, 01314 Dresden, Germany
E-Mail address of the corresponding author: t.vogt@hzdr.de

Abstract: The dynamics of free inertial waves inside a cylindrical volume was investigated experimentally in this study. The liquid metal GaInSn was chosen as fluid in order to enable a contactless stimulation of the flow inside the cylinder by means of a rotating magnetic field which generates a supercritical rotating motion of the liquid. The experiment demonstrates that inertial waves may be excited spontaneously by turbulent structures in the rotating flow. The ultrasound Doppler velocimetry was used to record the flow structure and to identify the inertial waves occurring in the setup.

1. Introduction

Inertial waves appear to be a ubiquitous feature in rotating fluids. Consequently, this type of waves can be found almost everywhere in nature as well as in technical facilities where a fluid is in rotating motion. In nature, very large-scale inertial waves occur due to earth's rotation for example in the atmosphere [1] in oceans [2] and also deep inside the liquid earth's core [3]. Beside this, inertial waves can also be detected in a variety of technical applications. In all cases, the source for an inertial wave is a disorder in the centrifugal force balance of the rotating fluid. Such an imbalance leads inevitably to a radial motion of fluid and thus to a Coriolis force which acts in azimuthal direction on radially shifted fluid. Depending on the radial distribution of the angular momentum of the basic rotating flow, the radial motion of fluid can lead to two scenarios. If viscous damping is neglected, the border between the two possible scenarios is given by the Rayleigh (1917) stability criterion.

$$\frac{\partial}{\partial_r}\left(u_\varphi * r\right) \geq 0$$

If the azimuthal velocity decreases faster than this limit, the centrifugal force and the radial pressure gradient are out of balance and even small disturbances of the flow field will lead to turbulence. Otherwise, the flow is stable, but can oscillate via inertial waves when disturbed appropriately.

The aim of the present experimental work is to investigate the occurrence and properties of inertial waves in a magnetically driven swirling liquid metal flow. Flow measurements inside the closed cylindrical liquid metal column were carried out by means of the ultrasound Doppler technique.

2. Experimental Description

The flow measurements were conducted in the eutectic alloy $Ga^{68}In^{20}Sn^{12}$, which was filled into closed cylindrical vessels made of Perspex. The fluid vessels were placed concentrically inside the bore hole (diameter 350 mm, height 400 mm) of the MULTIpurpose MAGnetic field system (MULTIMAG) at HZDR. MULTIMAG is a compact magnetic coil system for the generation of rotating (RMF), vertically travelling (TMF) and pulsating magnetic fields and superposition thereof with a high accuracy [4]. Measurements of the fluid velocities were obtained by means of the ultrasound Doppler velocimetry (UDV), which is suitable to deliver

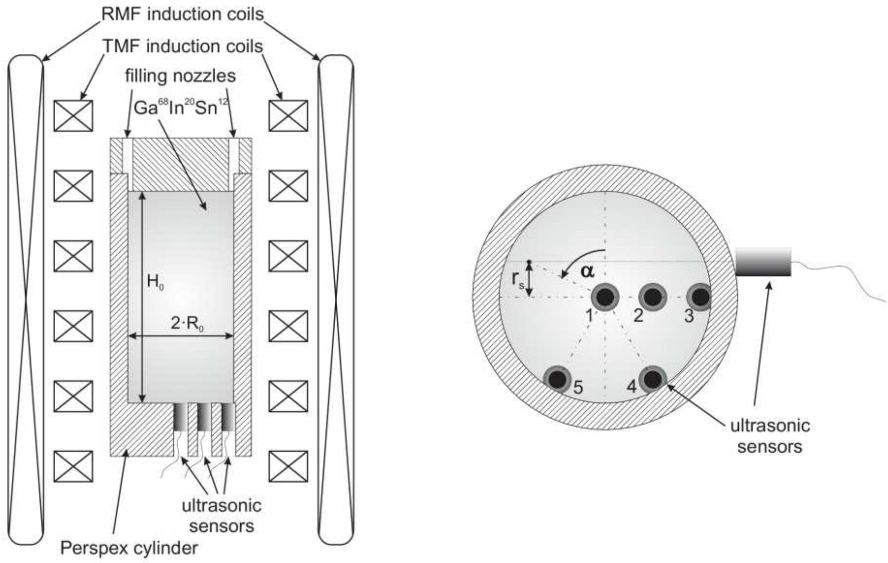

Figure 1**:** Schematic drawing of the experimental setup and applied sensor positions

instantaneous velocity profiles in opaque fluids such as liquid metals. A schematically drawing of the experimental setup is shown in figure 1.
The RMF induces a Lorentz force in the liquid metal which is time-independent and has only one azimuthal component in the cylindrical configuration [5]:

$$F_{\emptyset} = \frac{1}{2}\sigma\omega B_r^2 r f(r,z)$$

here, $\sigma = 3.2 \times 10^6$ S/m is the electrical conductivity of GaInSn, $\omega/2\pi = 50$ Hz is the frequency of the RMF and r stands for the vessel radius. The shape function $f(r, z)$ reflects the influence of the finite cylinder length. The Lorentz force has its maximum near the sidewall of the vessel at the half-height and reduces to zero at the axis and the horizontal end walls. The magnetic Taylor number gives the dimensionless force magnitude that is used to characterize the RMF-driven flow:

$$Ta = \frac{\sigma\omega B_r^2 R_0^4}{\rho\vartheta^2}$$

The magnetic flux density (B_r) in both equations is given in terms of the root mean square value.
The classification of the different inertial wave modes is done in this work with the waveform vector (γ_i, n), where γ_i is the wavenumber of the i^{th} radial mode and n is the axial mode number.

3. Spontaneous occurrence of inertial waves

Inertial waves can straightforwardly be excited in a liquid metal column by distinct variations of the electromagnetic driving force. For instance, pronounced inertial waves were found during an RMF-driven spin-up, after the magnetic field has been suddenly switched on [6, 7]. These inertial waves are completely damped by viscous effects before the flow reaches the

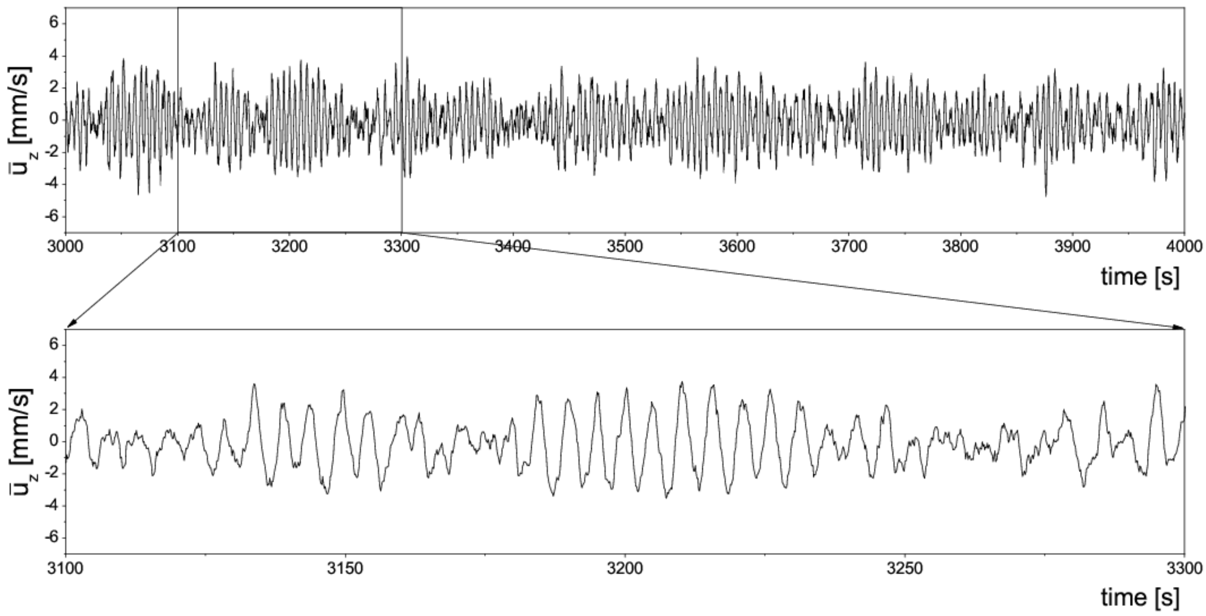

Figure 2: Mean axial velocity averaged along the cylinder height [$0 \leq z \leq H_0$]; recorded during a long-term measurement without magnetic disturbance $B_R = 1.3$ mT; $r/R_0 = 0$

steady state at the end of the spin-up. Having this in mind, one would hardly expect that inertial waves occur during a stationary rotation without any external perturbation. In a recent work [8], Sauret et al. analyzed the flow in a librating cylinder by means of numerical simulations. The authors reported a new mechanism of spontaneous excitation of inertial waves in the center of a librating cylinder ensuing from turbulent sidewall boundary layers. To check whether turbulent fluctuations may act as source of such inertial waves, an experiment was performed under stationary flow conditions with a continuously rotating liquid. Any geometrical or electromagnetic perturbation was avoided. The measurements were performed inside a cylinder of the aspect ratio $A = H_0/2R_0 = 3$ ($H_0 = 180$ mm and $R_0 = 30$ mm). The rotating flow inside the cylindrical vessel was driven by an RMF at a constant field strength of $B_R = 1.3$ mT which corresponds to a value of the magnetic Taylor-number of $Ta = 1.79 \times 10^6 \gg Ta^{cr}$. The nature of RMF-driven flows at about $100 \times Ta^{cr}$ can be considered as supercritical to certain instabilities. Figure 2 displays a section of a long-term measurement lasting over a period of about one hour. In this figure, the evolution of the vertical velocity (u_z) averaged along the axis of the liquid metal column is shown. An almost periodic oscillation of the mean axial velocity becomes visible collapsing from time to time. Because any external forcing was avoided here, the inertial wave must be triggered by the flow itself. A frequency and wave-mode analysis of this inertial wave was done in terms of a 2D-FFT, which is shown in figure 3. In this figure, the vertical axis is determined by the axial mode-number which is defined as $n = 2H_0/\lambda$, whereby λ is the wavelength. The abscissa in this figure represents the frequency. The normalized amplitude of the respective wave modes is represented by the gray scale. This 2D-FFT was performed within the measuring time interval of $t = 3000...4000$ s. The diagram clearly reveals the dominating (γ_1, 1) mode with a frequency of $f \approx 0.2$ Hz. Due to the fact that the wave-length of the $n = 1$ inertial wave is twice the cylinder height and thus, the FFT does not identify a complete wave, the mode-number information of the FFT is projected to $n = 0$ (representing a uniform component) and to various positive and negative integer wave length (a likewise decomposition occurs for all odd modes). The $n = 2$ signature at $f = 0$ Hz is a signature of the Bödewadt recirculation. Several other inertial wave modes can be identified in this FFT plot in addition to the dominant (γ_1, 1) mode. One example therefor is the $n = 2$ mode at $f \approx 0.4$ Hz. The interesting question is how these inertial wave modes are excited. A typical feature of an RMF-driven

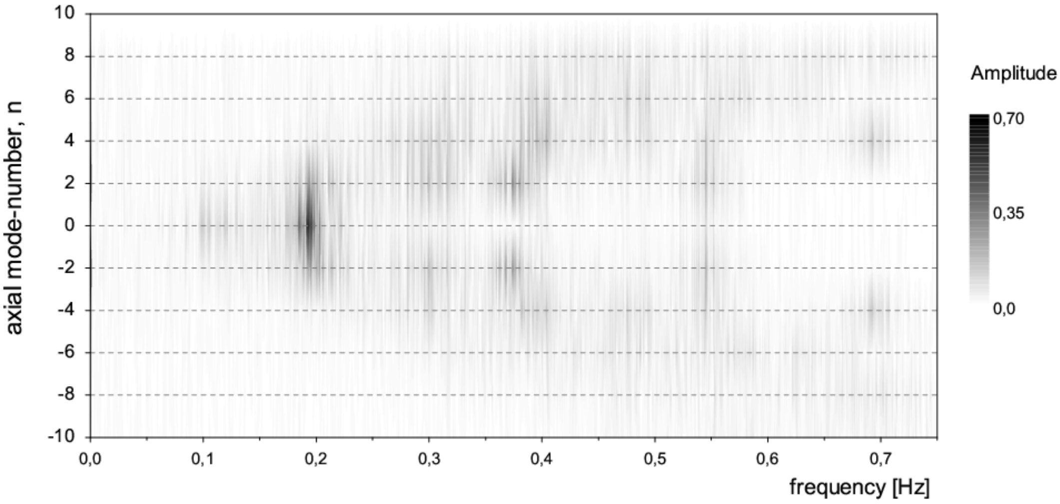

Figure 3: 2D-FFT of the spatio-temporal flow structure recorded during a long-term measurement without magnetic disturbance (corresponds to the measurement shown in fig. 2, top) $B_R = 1.3$ mT; $r/R_0 = 0$

flow at supercritical Ta numbers is the formation of Taylor-Görtler (TG) vortices, which is a result of the instability of the side wall layers. These TG-vortices develop typically in the boundary layer at the side wall of the cylindrical vessel. After formation, the TG-vortices are conveyed by the secondary flow towards the horizontal end walls of the vessel, where they dissipate in the Bödewadt layers [9]. The impingement of the TG-vortex on the Bödewadt layer provokes a temporary disturbance of the boundary layer. Velocity measurements made during an RMF-driven spin-up [10] showed how such a disturbance propagates inside the boundary layer and excites an inertial wave appearing in the center of the vessel, as observed in figure 2. These findings suggest the assumption that the phenomenon of the spontaneous generation of inertial waves might be caused by the dissipation of TG-vortices in the Bödewadt-layers. With the exception of the (γ_1, 1) mode all inertial waves detected in the velocity measurements do not become prevalent with respect to the turbulent motion. Thus, we want to focus now a bit more in detail on the dominant (γ_1, 1) mode. Figure 2 demonstrates that the amplitude of the (γ_1, 1) mode remains more or less stable for many periods, although it suddenly decays intermittently. Thus, there must be a mechanism sustaining the (γ_1, 1) wave. We suppose that this interesting phenomenon can be ascribed to an interaction between the (γ_1, 1) mode and the pronounced Bödewadt layers at the top and bottom wall of the vessel. According to the velocity recordings in figure 2, the inertial wave implicates an alternatingly up- or downwards directed flow along the vessel axis. This alternating flow leads to periodic variations of the Bödewadt layer thickness and thus to a periodic modulation of the Ekman-transport. Let us consider a moment, when the inertial wave in the center of the vessel is directed upwards. At that instant the Ekman-transport beneath the lid of the fluid cylinder is repressed, while the liquid metal is still pumped inwards in the bottom Bödewadt layer. As a result an upwards directed jet is formed in the core of the fluid vessel. A half cycle later, the situation becomes inverted. This alternating Ekman-transport is necessarily in phase with the inertial wave. In this way, a continuous energy transfer may be enabled from the primary swirling flow over the Ekman-transport toward the inertial wave. Such a mechanism, which is speculative at present, could be responsible for the enduring persistence of the (γ_1, 1) inertial mode. This mechanism would also explain the observations reported by Zhang et al. [11] for the situation of an RMF-driven flow in a liquid metal column, where the free surface of a liquid metal was covered by a distinct oxide layer. In that case, an intense distortion of the bulk flow occurred without external stimulation. The authors suggest the friction forces between the rigid oxide layer and the side wall to be responsible for the development of pronounced flow oscillations in the bulk of the melt. However, they did not associate their findings with the occurrence of inertial waves. A closer analysis of the flow pattern found in the reported experiment and a comparison with the results of the present study reveals the existence of the (γ_1, 1) mode in that experiment. The strong interaction between this inertial wave and the oxide layer, which causes strong and persistent flow oscillations, appears to be similar to the mechanism described here.

4. Conclusion

In this work, the occurrence of spontaneously excited inertial waves was studied inside a cylindrical vessel. A rotating magnetic field (RMF) generates a swirling motion inside the liquid metal filled in the cylinder. A prominent feature of our experimental configuration is the interaction between the inertial modes and the secondary flow arising from the Ekman transport. We observed the formation of an (γ_1, 1) inertial wave mode even without any external triggering in form of deliberate disturbances of the rotating flow field. The reason for such a spontaneous excitation of (γ_1, 1) inertial waves can be explained by the existence of Taylor-Görtler vortices at the sidewall of the vessel. These TG-vortices are conveyed by the secondary flow towards the top and bottom of the vessel where they dissipate in the Bödewadt layer. Such a vortex dissipation in the Bödewadt layer leads to a perturbation of the Ekman pumping resulting in the excitation of an inertial wave.

5. References

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