# MAGNETIC FIELD EFFECT ON THE ONSET OF HELICOIDAL INSTABILITY IN UP AND DOWN CONICAL FLOW SYSTEM

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**Abstract**: This work is concerned with the axial magnetic field effect on the onset of the helicoidal wave (spiral mode) in a rotating conical system flow defined by two truncated concentric conical cylinders, the inner cone is rotating and the external one is fixed. Two configurations are considered, down flow system and up flow system (figure.1.a and .1.b). Therefore, we introduce a new coordinates system to analyze the perturbed velocity field in each configurations. Firstly, we determine the mean velocity distribution corresponding to the steady laminar regime. Secondly, assuming the linear theory, we establish the equations system of stability with the associated boundary conditions. By means of the Galerkin method, we get the dispersion relation leading to determine the variation of Taylor number versus wavenumber. Then we interpret the stability diagram according to the onset of spiral mode in hydromagnetic stability in both cases (up and down conical flow system) compared with those obtained in the classical Taylor-Couette flow system (cylindrical case).

## Introduction

The hydrodynamics of rotating systems aims to study the mechanisms and properties prediction related to natural phenomena. Mainly in meteorology for understanding atmospheric phenomena and its application to weather prediction. In astrophysics, to investigate the atmospheric dynamics of the planets and to analyze the rotating stars heart as sun which is subjected to both hydrodynamic and hydromagnetic effects which is studied by Chandrasekhar problem [1].

As well as in industrial processes including liquid metal, this flow system is also of great importance, not only in the design of rotating machinery such as multiple concentric drives, turbine rotor, but also for the application to chemical equipment such as compact rotating heat exchangers and mixers. However, in this flow system the hydrodynamic equations associated to the electromagnetism equations presents a specific behavior that is difficult to predict theoretically and numerically.

To our best knowledge there has been no attempt on the theoretical and experimental approach to the effect of magnetic field on the nature and structure of the laminar-turbulent transition regime in conical Taylor-Couette flow system. However, in the cylindrical configuration it is well know that the magnetic field stabilized the stationary axial wave (Taylor vortex) by delaying it onset at the critical Taylor number $Ta = Tc_1$.

In this context, we propose an analytical approach to predict the helicoidal instability or spiral mode in conical Taylor-Couette flow system. We investigated the magnetohydrodynamic stability which is considered for the evaluation of the influence of an axial magnetic field, it is supposed to stabilize and delaying the onset of spiral mode as predicted by S.Chandrasekhar in the case of Taylor-Couette flow system.

## 1. Problem formulation

We consider a flow between two coaxial cones so as to the inner cone ($r = R_{1max}$) is rotating with angular velocity $\Omega_1$ and the outer cone ($r = R_{2max}$) is maintained at rest ($\Omega_2 = 0$). The cones have the same apex angle $\varphi = 12°$.

***1.1. Coordinates system.*** The coordinate system used in the study of the flow confined between two coaxial cylinders or two spheres does not apply here because the cone radius varies with the height which is related to the fluid confinement (Fig.1). Furthermore, the boundary conditions associated with the cone system is not constant along the generatrix. For this purpose the chosen orthogonal curvilinear coordinates system consists of three axes z, x and θ such as z is supported by the generatrix of the inner cone. Thus, x is the coordinate axis perpendicular to the surface (S) and θ is the third coordinate perpendicular to z and x which corresponds to the azimuthal coordinate [3, 4]. Therefore, the new coordinates are $X = (x\cos\varphi + z\sin\varphi)\cos\theta$, $Y = (x\cos\varphi + z\sin\varphi)\sin\theta$ *and* $Z = z\cos\varphi$

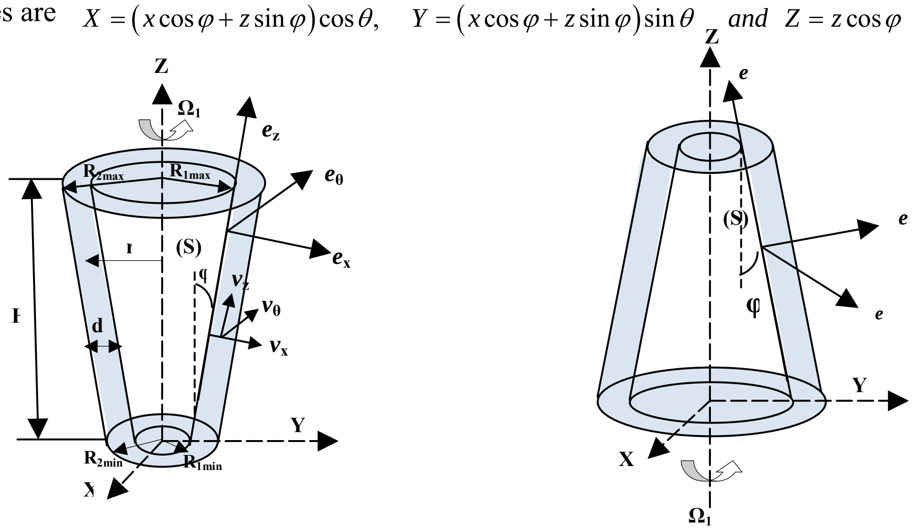


Figure 1: a**:** Up flow system configuration, b: down flow system configuration.

***1.2. Governing equations.*** In order to predict the onset of the spiral mode of a fluid flow characterized by a density ρ, a kinematic viscosity ν, a magnetic permeability μ and an electrical conductivity σ, we consider the Navier Stokes and Maxwell equations as follows

$$\begin{cases} \dfrac{\partial V}{\partial t} + V.\nabla V = -\dfrac{1}{\rho}\nabla p + \nu\Delta V + \dfrac{1}{\rho} J\times B \qquad with \quad J = \sigma(E + V\times B) & (1)\\ \nabla\cdot V = 0 & (2)\\ \dfrac{\partial B}{\partial t} = \nabla\times(V\times B) + \dfrac{1}{\mu\sigma}\Delta B & (3)\\ \nabla\cdot B = 0 & (4)\end{cases}$$

In view to simplify the system of equations (1-4) we suppose the following assumptions as the physical properties (ρ, ν, μ and σ) of the fluid are constants, the problem is axisymmetric $\partial/\partial\theta = 0$, the magnetic field is steady in the axial direction $B = B_0 e_z$ and we assume insulating walls.

***Boundary conditions:*** The boundary conditions associated to the problem are given as

$x = 0$: $\quad V_x = V_z = 0,\ V_\theta = \Omega z\sin\phi,\ B_x = B_\theta = 0$ and $B_z = B_0$

$x = d$: $\quad V_x = V_z = V_\theta = 0$

***1. 3 Flow control parameters.*** For simplicity, the governing equations are writing in dimensionless form using the following reduced variables and functions given by:

$$t^* = \frac{\upsilon}{d^2}t, \eta = x/d, \zeta = z/d, r^* = r/d = \zeta\sin\phi + \eta\cos\phi, V_x^* = V_x/V_\theta^0, V_\theta^* = V_\theta/V_\theta^0, V_z^* = V_z/V_\theta^0 \text{ and } p^* = p/(\rho V_\theta^{0\,2})$$

The flow is characterised by several control parameters, the annular gap $\boldsymbol{\delta = d/R_{1max} = 0.12}$, the Reynolds number $\mathbf{Re = R_{1max}\Omega d/\nu}$ which is the ratio between the viscose force and inertial forces, the Taylor number $\mathbf{Ta = (R_{1max}\Omega d/\nu)\delta^{1/2}}$ the magnetic Reynolds number $R_m = 4\pi\mu\sigma d V_\theta^0$ It shows the relationship between the terms of convection and diffusion in a magnetic fluid and the Hartmann number $\mathbf{Ha = R_m B_0 / V_\theta^0 \sqrt{1/2\pi\rho\mu}}$ which is the ratio of the Lorentz force and viscous forces.

**2. Solving problem.**

***2.1. Mean hydromagnetic field***. By expending the equations system (5-8), we notice that the previous system is independent of the imposed magnetic field. For this, we note that the axial magnetic field doesn't affect the mean velocity component in both configurations up and dawn. The same result is obtained by Chandrasekhar in the classical Taylor-Couette flow [1].

***22. Perturbed hydrodynamic field.*** In order to solving the previous equations system (1-4), by means of the Galerkin method which led to establish an eigenvalues problem. For that, we propose the solutions that showing the axial periodicity depending on the ζ component and satisfying the spiral wave nature given by

$$v_x^* = v_x^*(\eta)e^{j(\lambda\zeta-\beta\tau)} \qquad v_z^* = v_z^*(\eta)e^{j(\lambda\zeta-\beta\tau)} \qquad v_\theta^* = v_\theta^*(\eta)e^{j(\lambda\zeta-\beta\tau)} \qquad p^* = p^*(\eta)e^{j(\lambda\zeta-\beta\tau)}$$

$$b_x^* = b_x^*(\eta)e^{j(\lambda\zeta-\beta\tau)} \qquad b_z^* = b_z^*(\eta)e^{j(\lambda\zeta-\beta\tau)} \qquad b_\theta^* = b_\theta^*(\eta)e^{j(\lambda\zeta-\beta\tau)}$$

With injecting those solutions in the previous system and we used the operator D defined by $D = d/d\eta$ in order to simplify the partial differential equations where we must eliminate the pressure term in order to obtaining the stability equations versus the magnetic field component $b_x^*$ and $b_\theta^*$ .

$$L_1 b_x^*(\eta) + L_2 b_\theta^*(\eta) = 0 \qquad \text{(I)}$$

$$L_1' b_x^*(\eta) + L_2' b_\theta^*(\eta) = 0 \qquad \text{(II)}$$

***2.3. Implementation of the Galerkin method.*** One must choose a basic approximation of the solution as follows

- Radial component: $b_x^* = \sum_{n=1}^{N} \alpha_n b_{xn}^*$ with $b_{xn}^* = \eta^2 (1-\eta)^2 \eta^{n-1}$

- Tangential component: $b_\theta^* = \sum_{n=1}^{N} \beta_n b_{\theta n}^*$ with $b_{\theta n}^* = \eta(1-\eta)\eta^{n-1}$

The magnetic field must satisfy the boundary conditions of problem. This will then imposes

$$b_{xn}^* = b_{\theta n}^* = Db_{xn}^* = 0 \quad at \quad \eta = 0 \ \ and \ \ \eta = 1$$

By injecting expressions in equation (I) and (II), we establish the evaluation of the error associated with two velocity components as Follows

$$L_1 \sum_{n=1}^{N} \alpha_n b_{xn}^* + L_2 b_{\theta n}^* = \varepsilon_{n\alpha}^{(1)} \ , \quad L_1 b_{xn}^* + L_2 \sum_{n=1}^{N} \beta_n b_{\theta n}^* = \varepsilon_{n\beta}^{(1)} \qquad (I')$$

The total error committed in the equation (19) is the sum of previous errors

$$\varepsilon_n^{(1)} = \varepsilon_{n\alpha}^{(1)} + \varepsilon_{n\beta}^{(1)} \quad , \qquad \varepsilon_n^{(1)} = L_1 \sum_{n=1}^{N} \alpha_n b_{xn}^* + L_2 \sum_{n=1}^{N} \beta_n b_{\theta n}^*$$

It is now optimized values to minimize errors in each of the previous system of equations. To do this, it is necessary to make a point based on the property which follows from the properties related to the integral inner product, namely:

$$\langle f | g \rangle = 0 \quad if \quad f = 0, \qquad \int_0^1 f.g\,dx = 0 \quad if \quad f = 0 \quad x \in [\ 0,\ 1\ ]$$

The application of this property to the result of the error $f = \varepsilon_n^{(1)}$ of order n committed in the first equation gives the following relation

$$\langle \varepsilon_n^{(1)} | b_{xm}^* \rangle = \int_0^1 \varepsilon_n^{(1)} b_{xm}^* d\eta = 0$$

This is reflected by the fact that the corresponding error $\varepsilon_n^{(1)}$ becomes perpendicular to the base of decomposition chosen $b_{xm}^*$ . Under these conditions, we $\int_0^1 \left( \int_0^{2\pi/\lambda} \sum_{n=1}^{N} \left( L_1 \alpha_n b_{xn}^* + L_2 \beta_n b_{\theta n}^* \right) d\zeta \right) b_{xm}^* d\eta$

$$a_{nm}=\int_0^1\left(\int_0^{2\pi/\lambda}L_1 b_{xn}^* d\zeta\right)b_{xm}^* d\eta,\ b_{nm}=\int_0^1\left(\int_0^{2\pi/\lambda}L_2 b_{\theta n}^* d\zeta\right)b_{xm}^* d\eta,\ c_{11}=\int_0^1\left(\int_0^{2\pi/\lambda}L_1' b_{xn}^* d\zeta\right)b_{\theta m}^* d\eta,\ d_{11}=\int_0^1\left(\int_0^{2\pi/\lambda}L_2' b_{\theta n}^* d\zeta\right)b_{\theta m}^* d\eta$$

A necessary condition for the existence of a solution (nontrivial solution) is to impose that the determinant of the Cramer previous system is zero. This condition leads us to solve the associated eigenvalue problem. In other words, it is set according to λ*, σ, α, ω, Ha and $R_m$ on physical point of view this corresponds to a dispersion relation.

Is performed solving the eigenvalue problem by performing a first order approximation by setting n = m = 1. Under these conditions, the Galerkin matrix G of order 2 has the form:
To the solution is not trivial is imposed: det G = 0 is: $a_{11}d_{11}-c_{11}b_{11}=0$

## 3. Results and discussion

***3.1. Theoretical results.*** By imposing a null determinant associated with the eigenvalues problem, it is established a relationship which leads to the following solutions. The Taylor number evolution according to the wave number λ* and the magnetic Reynolds number Rm = $10^{-3}$ follows a parabolic behavior law characterized by a maximum in the set (Ta=24.5, λ* = 0.01) in the cas of up conical system but in the down configuration Taylor number increases and reaches a maximum value (35000-100000) . The magnetic Reynolds number Rm = $10^{-3}$ corresponding to the industrial applications such as the MHD generator-gaz and the diffuse discharge.
By varying the Hartmann number Ha and keeping the magnetic Reynolds number constant, we notice that the critical Taylor number decreases when the magnetic Reynolds number increases about 10 %. The previous curves indicate us that the variation of the Hartmann number produces any visible change on this type of flow in the both cases of Rm = $10^{-3}$ and Rm = $10^{-2}$.

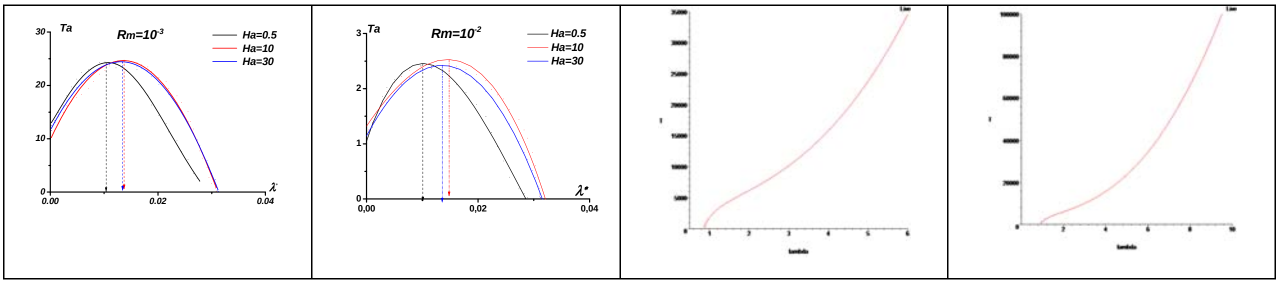


Figure 2: Taylor number evolution versus the wave number λ*.

From Rm = 10, we observe a change of the previous behavior laws which is characterized by a maximum for (Ta=35, λ* = 0.4 and Ta=3.06, λ* = 0.4) the Hartman number has an important effect in this case for the Down configuration.

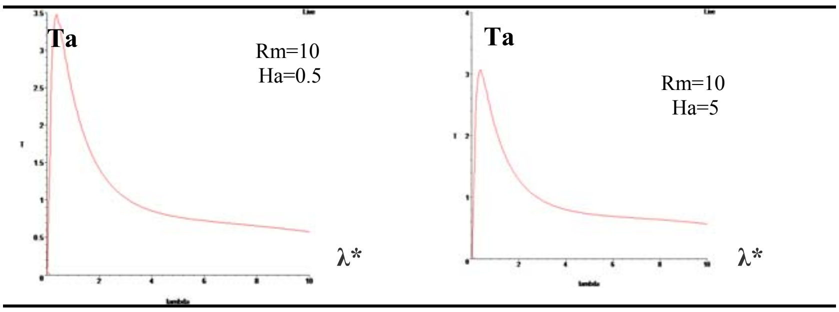


Figure3: Taylor number evolution versus the wave number λ* in the case of Rm = 10.

In the case of the controlled thermonuclear reactions (Rm=100), the variation of the Taylor number versus the wave number evolves according to two different laws. In the range 0 < $\lambda^*$<2, the law is parabolic characterized by a maximum at $\lambda^*$m = 0.99, Ta = Tm =14.9. For 2 < $\lambda^*$ <4 the behavior law crosses an inflection point of coordinate $\lambda^*_c$ = 2.7, Ta = Tc1 = 38.1 whose position is invariant in the interval $0.5 \leq Ha \leq 30$, followed by a maximum $\lambda^*_M = 3.18$, $Ta = T_M = 50.4$ for the Hartmann number $Ha = 0.5$ and $Ha = 10$.

In the case $Ha = 5$, it is observed a significant change of the behavior law except in $0 < \lambda^* < 2$. Beyond $\lambda^* = 2$, the behavior law, the inflection point position and the maximum point remain invariables for any Hartmann number.

The analysis of these results led us to note that the magnetic field advances the onset of the spiral mode (helicoidal wave).

For higher Reynolds magnetic number $Rm = 10^3$ it is noticed that the Taylor number evolution according to the wave number $\lambda^*$ changes its behavior in the range $2 < \lambda^* < 4$.

By varying the Hartmann number between $Ha = 0.5$ and 30, it is noted that the critical Taylor number conversely increases with the wavelength $\lambda^*_c$.

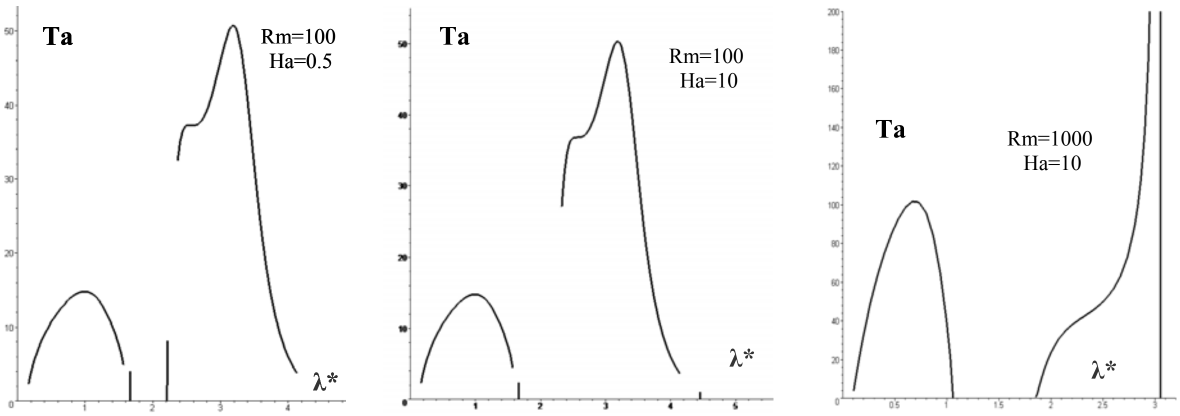


Figure 4: Evolution of Taylor number versus the wave number $\lambda^*$ in the case of $Rm=10^2$, $Rm=10^3$ in the Up conical flow system.

## Conclusion

Following a new coordinates system it was possible to solve the equations of motion in the approximation of the small annular gap configuration in the rotating conical flow system. It was found that the imposed magnetic field does not affect the mean hydrodynamic field. Therefore, there is a change in the behavior laws by varying the magnetic Reynolds number and is placed in different areas such as gas-generator-MHD, controlled thermonuclear reactions, plasmas and astrophysics. The study of the stability problem of the effective magnetic field shows the existence of a delay in the onset of spiral mode which becomes more important when the Hartman number increases. Conversely, the spiral mode wavenumber decreases when the Hartman number increases this is consistent with the result of Chandrasekhar established in classical Taylor-Couette flow system.

## References


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