

Feasible homopolar dynamo with sliding liquid-metal contacts

AVALOS-ZÚÑIGA, R.,¹ PRIEDE, J.²

¹*CICATA-Qro, Instituto Politécnico Nacional, Cerro Blanco 141,
Colinas del Cimatario, Querétaro, Mexico; ravalosz@ipn.mx*

²*Applied Mathematics Research Centre, Coventry University,
Coventry, CV1 5FB, UK; j.priede@coventry.ac.uk*

Abstract: We present a feasible homopolar dynamo design consisting of a flat, multi-arm spiral coil, which is placed above a fast-spinning metal ring and connected to the latter by sliding liquid-metal electrical contacts. Using a simple, analytically solvable axisymmetric model, we determine the optimal design of such a setup. For small contact resistance, the lowest magnetic Reynolds number, $Rm \approx 34.6$, at which the dynamo can work, is attained at the optimal ratio of the outer and inner radii of the rings $R_i/R_o \approx 0.36$ and the spiral pitch angle 54.7° . In a setup of two copper rings with the thickness of 3 cm, $R_i = 10$ cm and $R_o = 30$ cm, self-excitation of the magnetic field is expected at a critical rotation frequency around 10 Hz.

1 Introduction

The homopolar dynamo is one of the simplest models of the self-excitation of magnetic field by moving conductors which is often used to illustrate the dynamo action that is thought to be behind the magnetic fields of the Earth, the Sun and other cosmic bodies [1, 2]. In its simplest form originally considered by Bullard [3], the dynamo consists of a solid metal disc which rotates about its axis, and a wire twisted around it and connected through sliding contacts to the rim and the axis of the disc. At a sufficiently high rotation rate, the voltage induced by the rotation of the disc in the magnetic field generated by an initial current perturbation can exceed the voltage drop due to the ohmic resistance. At this point, initial perturbation starts to grow exponentially leading to the self-excitation of current and its associated magnetic.

Despite its simplicity no successful implementation of the disc dynamo is known so far. The problem appears to be the sliding electrical contacts which are required to convey the current between the rim and the axis of the rotating disc. Electrical resistance of the sliding contacts, usually made of solid graphite brushes, is typically by several orders of magnitude higher than that of the rest of the setup. This results in unrealistically high rotation rates which are required for dynamo to operate [4]. To overcome this problem we propose to use liquid-metal sliding electrical contacts similar to those employed in homopolar motors and generators [5]. The aim of this paper is to develop a feasible design of the disc dynamo which could achieve self-excitation at realistic rotation rates.

2 Physical and mathematical models

The principal setup of the proposed disc dynamo shown in Fig. 1(a) consists of a stationary coil (1) made of a copper disc sectioned by spiral slits and a fast-spinning disc (2) placed beneath it, which

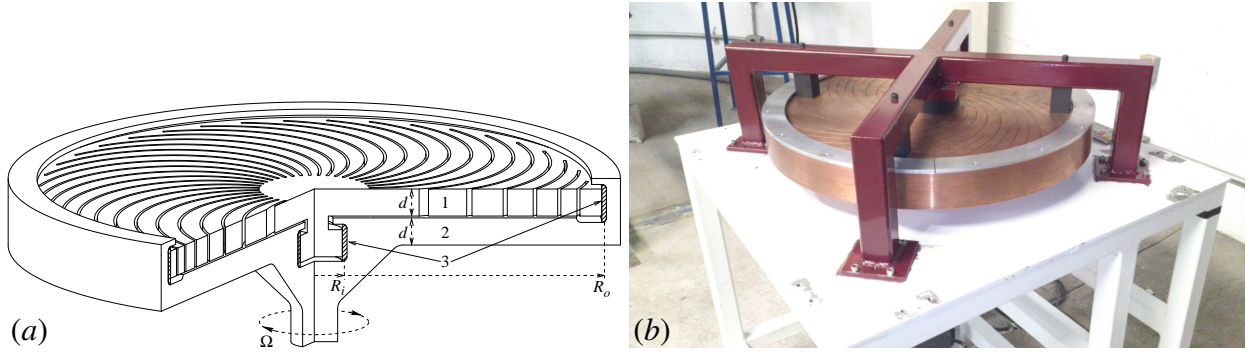


Figure 1: Schematic view of the disc dynamo setup consisting of a stationary coil (1) made of a copper disc sectioned by spiral slits and a fast-spinning disc (2), which is electrically connected to the former by sliding liquid-metal contacts (hatched) (3) (a) and the experimental device (b).

is electrically connected to the former by sliding liquid electrical contacts (3). The coil is supported by the holders which are not shown in this basic view. The liquid metal is hold in vertical state by the centrifugal force. Such a compact and symmetric design not only minimizes the electrical resistance in the system but also makes it amendable to simple analysis which is carried out in the following.

To simplify the analysis both discs are subsequently assumed to be thin coaxial rings of thickness d , the outer radius $R_o \gg d$, and the inner radius $R_i = \lambda R_o$, where $0 < \lambda < 1$ the ratio of the inner and outer radii. The rings are separated by a small axial distance and connected to each other at their rims through the sliding liquid-metal electrical contacts. The design of the stationary top ring, which forms a compact coil consisting of spiral sections is described in detail below. The bottom ring is mounted on an axle which is driven by an electric motor with the angular velocity Ω . The electric current I_0 is induced by the rotation of the bottom ring in the magnetic field generated by the same current returning through the coil formed by the top ring. In the solid rotating ring, the current is assumed to flow radially with the linear density $J_r = \frac{I_0}{2\pi r}$, which decreases due to the charge conservation inversely with the cylindrical radius r . Current returns through the top ring where it is deflected by the spiral slits that produce an azimuthal component proportional to the radial one:

$$J_\phi = -J_r \beta = \frac{I_0 \beta}{2\pi r}, \quad (1)$$

where $\arctan \beta$ the pitch angle of the current lines relative to the radial direction. The shape of slits following the current lines is governed by $\frac{J_\phi}{J_r} = \frac{r d\phi}{dr} = -\beta$ and given by the logarithmic spirals

$$\phi(r) = \phi_0 - \beta \ln r, \quad (2)$$

where ϕ is the azimuthal angle. The electric potential distribution in the coil ring follows from Ohm's law

$$\vec{J} = \frac{I_0}{2\pi r} (-\vec{e}_r + \beta \vec{e}_\phi) = -\sigma d \vec{\nabla} \varphi_c, \quad (3)$$

as $\varphi_c(r, \phi) = \frac{I_0}{2\pi \sigma d} (\ln r - \beta \phi)$. Thus, the potential difference along the current line between the rims of the ring is

$$\Delta \varphi_c = [\varphi_c(r, \phi(r))]_{R_i}^{R_o} = -\frac{I_0}{2\pi \sigma d} (1 + \beta^2) \ln \lambda. \quad (4)$$

The potential difference across the bottom ring, which rotates as a solid body with the azimuthal velocity $v_\phi = r\Omega$, is defined by the radial component of Ohm's law for a moving medium

$$J_r = \frac{I_0}{2\pi r} = \sigma d(-\partial_r \varphi_d + v_\phi B_z),$$

where B_z is the axial component of the magnetic field. Integrating the expression above over the ring radius we obtain

$$-\frac{I_0}{2\pi} \ln \lambda = \sigma d(-\Delta \varphi_d + \Omega \Phi_d), \quad (5)$$

where $\Delta \varphi_d = [\varphi_d(r)]_{R_i}^{R_o}$ is the potential difference across the rotating ring and $\Phi_d = \int_{R_i}^{R_o} B_z r dr$ is the magnetic flux through it. Using the relation $B_z = r^{-1} \partial_r (r A_\phi)$, the latter can be expressed in terms of the azimuthal component of the magnetic vector potential A_ϕ as

$$\Phi_d = [r A_\phi]_{r=R_i}^{R_o}. \quad (6)$$

In the stationary state, which is assumed here, the potential difference induced by the rotating ring in Eq. (5) is supposed to balance that over the coil defined by Eq. (4) as well as the potential drop over the liquid-metal contacts with the effective resistance \mathcal{R} : $\Delta \varphi_d = \Delta \varphi_c + \mathcal{R} I_0$. This equation implicitly defines the marginal rotation rate at which a steady current can sustain itself.

To complete the solution we need to evaluate the magnetic flux (6) through the rotating disc. The azimuthal component of the vector potential appearing in Eq. (6) is generated by the respective component of the electric current which is present only in the coil. Thus, we have

$$A_\phi(r, z) = \frac{\mu_0}{4\pi} \int_0^{2\pi} \int_{R_i}^{R_o} \frac{J_\phi(r') \cos \phi' r' dr' d\phi'}{\sqrt{r'^2 - 2r'r \cos \phi + r^2 + z^2}},$$

where z is the axial distance from the coil ring carrying the azimuthal current J_ϕ defined by Eq. (1). Note that the poloidal currents with radial and axial components circulating through the rings and liquid-metal contacts produce purely toroidal magnetic field, which is parallel to the velocity of the rotating ring and, thus, do not interact with the latter. In the plane of the ring ($z = 0$), the double integral above can be evaluated analytically as $A_\phi(r, 0) = \frac{\mu_0 \beta I_0}{8\pi^2} [F(R_i/r) - F(R_o/r)]$, where the function $F(x) = (1-x)K(m_+) + (1+x)E(m_+) + \text{sgn}(1-x)[(1+x)K(m_-) + (1-x)E(m_-)]$, which is produced by the computer algebra system Mathematica in terms of the complete elliptic integrals of the first and second kind, $K(m_\pm)$ and $E(m_\pm)$, of the *parameter* $m_\pm = \frac{\pm 4x}{(1 \pm x)^2}$ [6]. Taking into account that $F(1) = 4$, the magnetic flux (6) can be written as

$$\Phi_c = \frac{\mu_0 \beta I_0 R_o}{8\pi^2} \bar{\Phi}(\lambda), \quad (7)$$

where $\bar{\Phi}(\lambda) = F(\lambda) + \lambda F(\lambda^{-1}) - 4(1 + \lambda)$ is a dimensionless magnetic flux and $\lambda = R_i/R_o$ is the radii ratio.

In the following, we assume the axial separation between the rings to be so small that the magnetic flux through the rotating ring is effectively the same as that through the coil, i. e., $\Phi_d \approx \Phi_c$. Substituting the relevant parameters into the equation defining the marginal rotation rate we eventually obtain

$$Rm = \mu_0 \sigma d R_o \Omega = \frac{4\pi(\bar{\mathcal{R}} - (2 + \kappa \beta^2) \ln \lambda)}{\kappa \beta \bar{\Phi}(\lambda)}, \quad (8)$$

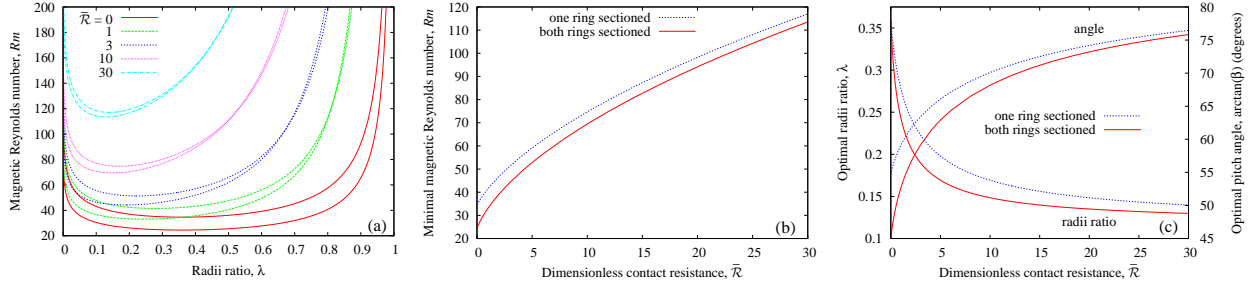


Figure 2: Marginal Rm versus λ for only one ring sectioned ($\kappa = 1$) at various dimensionless contact resistances $\bar{\mathcal{R}}$ and the optimal β , (a). Minimal magnetic Reynolds number Rm (b), optimal radii ratio λ , and the coil pitch angle $\arctan \beta$ (c) versus the dimensionless contact resistance $\bar{\mathcal{R}}$.

which is the marginal magnetic Reynolds number defining the dynamo threshold depending on the spiral pitch angle $\arctan \beta$, the radii ratio λ , and the dimensionless contact resistance $\bar{\mathcal{R}} = 2\pi\sigma d\mathcal{R}$.

The case of solid rotating ring considered above corresponds to $\kappa = 1$, whereas $\kappa = 2$ corresponds to the rotating ring sectioned similarly to the stationary one except for the opposite direction of the spiral slits. As seen from the expression above, the latter case is equivalent to the former with both Rm and β reduced by a factor of $\sqrt{2}$.

Now, let us determine the optimal β and λ that yield the lowest Rm for a given $\bar{\mathcal{R}}$. In the simplest case of a negligible contact resistance, which corresponds to $\bar{\mathcal{R}} = 0$, Eq. (8) yields $Rm \sim 2/(\kappa\beta) + \beta$. It means that Rm attains a minimum at $\beta_c = \sqrt{2/\kappa}$, which corresponds to the optimal pitch angles of 54.74° and 45° for only one and both rings sectioned. The respective lowest values of Rm , 34.63 and 24.49, are attained at the same optimal radii ratio of $\lambda = 0.3602$. (see Fig. 2(a) for the case of only one ring sectioned). The minimal Rm increases with $\bar{\mathcal{R}}$, which also causes a steep reduction of the optimal radii ratio and a comparably fast rise of the pitch angle (see Fig. 2(b,c)).

3 Feasible setup

Finally, let us evaluate the rotation rate required for self-excitation in a setup with the outer radius of $R_o = 30\text{cm}$ and the ring thickness of $d = 3\text{cm}$. First, we need to estimate electrical resistance of sliding liquid-metal contacts. A suitable metal for such contacts may be the eutectic alloy of GaInSn [5], which is liquid at room temperature with the kinematic viscosity $\nu = 3.5 \times 10^{-7}\text{m}^2/\text{s}$, electrical conductivity $\sigma_{\text{GaInSn}} = 3.3 \times 10^6\text{S/m}$. Assuming the contact gap width of $\delta = 0.5\text{cm}$ and the inner radius $R_i \approx 10\text{cm}$, we have $\mathcal{R}_i \approx \frac{\delta\sigma_{\text{GaInSn}}^{-1}}{2\pi d R_i} \approx 0.02\mu\Omega$. The resistance of the outer contact is by a factor of $\lambda = R_i/R_o = 0.33$ lower than \mathcal{R}_i . Then the dimensionless contact resistance can be estimated as $\bar{\mathcal{R}} = 2\pi\sigma_{\text{Cu}}d\mathcal{R}_i(1+\lambda) \approx \frac{\sigma_{\text{Cu}}}{\sigma_{\text{GaInSn}}} \frac{\delta}{R_i}(1+\lambda) \approx 0.2$. If only one disc is sectioned, which is easier to manufacture, the respective magnetic Reynolds number is $Rm \approx 40$. This corresponds to the rotation frequency $f = \frac{\Omega}{2\pi} = \frac{Rm}{2\pi\mu_0\sigma_{\text{Cu}}dR_o} \approx 10\text{Hz}$, which is well within the operation range of standard AC electric motors. The respective linear velocity of the outer edge of the ring is around $v \approx 20\text{m/s}$. At this velocity the tensile stress at the rim of the ring, $\rho_{\text{Cu}}v^2 \approx 4\text{MPa}$, is more than by an order of magnitude below the yield strength of annealed Copper [7]. The optimal inner radius $R_i \approx 0.3R_o \approx 9\text{cm}$ following from Fig. 2(c) is not far from the value assumed above. The respective

pitch angle for $\beta \approx 1.6$ is about 58° .

The number of spiral arms is determined by the following arguments. The current distribution defined by Eq. (1) can hold only in the inner parts of the ring which are radially confined between the spiral slits. This ideal distribution is expected to break down at the rims, which are radially exposed to the edges of the ring located at the nearly equipotential metal liquid contacts. In order to confine this perturbation to the outer rim with $r/R_o \gtrsim 0.9$, Eq. (2) suggests that $\frac{-2\pi}{\beta \ln 0.9} \approx 40$ equally distributed spiral slits are required.

The last critical issue is the viscous power losses associated with the turbulent drag acting on the outer sliding contacts at high shear rates. These losses can be estimated as $Q = S\tau v \approx 7 \text{ kW}$, where $S \approx 2\pi dR_o$ the area of the outer sliding contact, $\tau = \frac{c}{4} \frac{\rho v^2}{2}$ is the turbulent shear stress, and $c \approx 0.02$ is the Darcy friction factor for turbulent pipe flow with the Reynolds number $Re \sim 10^5$ [8]. Note that the relatively large setup size is due to the turbulent energy dissipation which scales as $Q \sim (dR_o)^{-2} \sim \Omega^2$. Namely, reducing the system size by one third would require about five times higher power input to achieve self-excitation of the magnetic field.

In conclusion, the proposed disc dynamo design appears feasible in terms of both the disc spinning rate and the power required to drive it. A corresponding device, which is shown in Fig. 1(b), has been built and is to undergo first tests in May 2014.

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