ORIENTATION, KINETIC AND MAGNETIC ENERGY OF PLANETARY DYNAMOS, THEIR INVERSIONS AND ASYMMETRIES

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Abstract: I derive, simplify and analyze integral evolutional laws of the kinetic, magnetic, and orientation energies in the liquid core of the Earth's type planets. These laws are reduced to the system of three ordinary differential equations for a given convection power. Estimates are obtained for the characteristic velocities, magnetic fields, periods and scales depending on the convection power at the stable states and near the inversion/excursion. That allows estimating how diffusion can determine the average period between geomagnetic reversals due to turbulent, thermal, electromagnetic and critical viscosity-compositional processes.

1. Introduction

I derive, simplify and analyze integral evolutional laws of the kinetic, magnetic, and original orientation energies in the liquid core of the Earth's type planets. These integral laws are reduced to the simplest system of three ordinary differential equations for a given convection power.

Estimates are obtained for the characteristic velocities, magnetic fields, periods and scales depending on the convection power at the stable states and near the inversion/excursion. It was shown that for the implementation of this short-time inversion/excursion the convection power should achieve some rare value, while a normal deviation from this value results in longer-time stable period. Here the inversion is a global process when the volume integral of the scalar product of convective velocity on the magnetic field changes sign.

So, the inversions and asymmetries are due to two types of stable states. Named as "lined" is a state with the magnetic field predominantly directed along velocity, while "contra lined" state is with their opposite direction. The lined state is characterized by smaller convection power and magnetic field in contrast to the contra lined state. The duration of the lined state is likely smaller than the duration of opposite state when the geodynamo power gradually increases with time, while for decreasing power it is vice versa.

Basing on the obtained results I estimate how diffusion can determine the average period between geomagnetic reversals due to turbulent, thermal, electromagnetic and critical viscosity-compositional processes. Predominant in this process, in many cases, can be identified from the dependence of the reversal frequency on the magnetic field intensity from paleomagnetic data. The data available to me suggest domination of the thermal processes.

2. Initial equations and hydromagnetic integrals

Initial equations with buoyancy acceleration **A**, velocity **V** and electromagnetic fields are from [1, 2] for the Earth's type planets:

$$\frac{\partial \mathbf{V}}{\partial t} - \mathbf{V}\times\nabla\times\mathbf{V} + 2\Omega\mathbf{1}_z\times\mathbf{V} + \nabla p + \nu\nabla\times(\nabla\times\mathbf{V}) = \mathbf{A} - \frac{\mathbf{B}\times\nabla\times\mathbf{B}}{\mu_0\rho},\ \nabla\cdot\mathbf{V} = 0. \qquad (1, 2)$$

$$\frac{\partial \mathbf{B}}{\partial t} = -\nabla\times\mathbf{E},\ \nabla\cdot\mathbf{B} = 0,\ \mathbf{E} + \mathbf{V}\times\mathbf{B} = \frac{\nabla\times\mathbf{B}}{\mu_0\sigma}. \qquad (3, 4, 5)$$

Notations are standard hereafter. Magnetic **B**, electric **E** and velocity **V** fields in (1-5) should satisfy jump (6, 7) and no-sleep (8) conditions on outside insulating $r = r_*$ and inside conducting $r = r_i$ co-rotating boundaries:

[B]=0, **[E]=0**, **V=0**. (6, 7, 8)

Integrating the scalar product of $\rho\mathbf{V}$ by (1) over the conducting planetary core one obtain the well-known equation for the kinetic energy:

$$\rho\frac{d}{dt}\left(\int_0^{r_*}\frac{V^2}{2}d^3r\right)=\int_0^{r_*}\left(\rho\mathbf{A}\cdot\mathbf{V}-\frac{\mathbf{V}\times\mathbf{B}\cdot\nabla\times\mathbf{B}}{\mu_0}-\rho\nu|\nabla\times\mathbf{V}|^2\right)d^3r. \quad (9)$$

Similarly $\mathbf{B}/\mu_0$ times (3) with making use of (4-8) and **E=0** on the insulating boundary give us known magnetic energy equation (10) together with Lorentz force power representation (11) that will be useful for further simplification:

$$\frac{d}{dt}\left(\int_0^{r_*}\frac{B^2}{2\mu_0}d^3r\right)=\int_0^{r_*}\left(\frac{\mathbf{V}\times\mathbf{B}\cdot\nabla\times\mathbf{B}}{\mu_0}-\frac{|\nabla\times\mathbf{B}|^2}{\mu_0^2\sigma}\right)d^3r,\ \frac{\mathbf{V}\times\mathbf{B}\cdot\nabla\times\mathbf{B}}{\mu_0}=\sigma|\mathbf{V}\times\mathbf{B}|^2-\sigma\mathbf{V}\times\mathbf{B}\cdot\mathbf{E}. \quad (10, 11)$$

To obtain the equation for cross-helicity $\mathbf{V}\cdot\mathbf{B}$, I multiply (1) with **B**, (3) with **V**, add the results, integrate, simplify and denoting $\eta=\nu+1/\mu_0\sigma$, obtain:

$$\frac{d}{dt}\left(\int_0^{r_*}\mathbf{V}\cdot\mathbf{B}d^3r\right)=\int_0^{r_*}(\mathbf{A}\cdot\mathbf{B}-2\Omega\mathbf{1}_z\cdot\mathbf{V}\times\mathbf{B}-\eta\nabla\times\mathbf{V}\cdot\nabla\times\mathbf{B})d^3r$$

$$+\int_{r=r_*}(\nu\mathbf{1}_r\times\mathbf{B}\cdot\nabla\times\mathbf{V}-pB_r)d^2r. \quad (12)$$

Multiplying the first integral in (12) by $\sqrt{\rho/\mu_0}$, I define orientation energy:

$$\mathrm{O}\equiv\sqrt{\frac{\rho}{\mu_0}}\int_0^{r_*}\mathbf{V}\cdot\mathbf{B}d^3r. \quad (13)$$

Let's as well universally define a given convection power as

$$W\equiv\int_0^{r_*}\mathbf{A}\cdot\mathbf{V}d^3r. \quad (14)$$

3. Simplified system

I define time-dependent kinetic Y, magnetic X and orientation Z values as

$$Y=\sqrt{\int_0^{r_*}V^2d^3r}\approx V_*,\ X=\sqrt{\int_0^{r_*}B^2d^3r}\approx B_*,\ Z=\int_0^{r_*}\mathbf{V}\cdot\mathbf{B}d^3r, \quad (15)$$

while stars are marking slowly varying with time typical values.

Using given $W(t)$ from (14), simply approximating vectors production and derivatives and neglecting by small terms I obtain from (9-12) the simplest system:

$$Y\frac{dY}{dt}=W-\frac{c}{\rho}(X^2Y^2-Z^2),\ X\frac{dX}{dt}=\mu_0c(X^2Y^2-Z^2)-\frac{X^2}{\tau},\ \frac{dZ}{dt}=CW-\omega(XY-Z). \quad (16)$$

Here c, τ, C and ω cold slowly change with time and will be estimated below.

All the derivatives of the system (16) are zero in stationary point that is denoted by s:

$$Y_s=\frac{1}{2}\frac{W_0+W}{\sqrt{\tau\mu_0cW_0W}},\ X_s=\sqrt{\tau\mu_0\rho W},\ \frac{Z_s}{Y_sX_s}=\frac{W_0-W}{W_0+W}\approx\cos o,\ W_0\equiv\frac{\rho}{c}\left(\frac{\omega}{C}\right)^2. \quad (17)$$

Here o is an average between velocity and magnetic field. This angle $o\approx\pi$ in the stable state, while $o\approx\pm\pi/2$ during inversion/excursion when $W=W_0$. Supposing $C=B_0/V_0$, $X_s=B_0$, $Y_s=V_0$ for this moment I obtain typical inversion/excursion values:

$$V_0=\sqrt{\frac{W_0}{\omega}}\approx 0.7\,\text{mm/s},\ B_0=\sqrt{\frac{\omega\rho}{c}}\approx 1\,\text{mT},\ \tau_0=\frac{\omega}{\mu_0cW_0}\approx 2\cdot10^8\,\text{s}\Rightarrow d_0\approx 10\,\text{km}. \quad (18)$$

Those estimates are for $W_0 = 3\cdot10^{-13}$ Вт/кг from [Braginsky, Roberts, 1995; Starchenko, Jones, 2002; Starchenko, Pushkarev, 2013; Olson et al., 2013], well-known $\rho = 10^4$ kg/m^3, while $\omega = 10^{-6}$/s and $c = 10^4$ Sm/m are from the chapter 5 below.

4. Inversion frequencies due to diffusion

Well-known value of the average time between inversions is about half million years [3, 4, 5]. Here I consider different diffusion processes matching this value.

Turbulent diffusion coefficient I estimate as $V_* d/3$ giving the average time $T = 3(r_*)^2/V_* d$. For that using (17) and ($C=B_*/V_*$, $X_s=B_*$, $Y_s=V_*$) I obtain as required:

$$T \approx 3r_*^2 \frac{\mu_0}{B_*}\sqrt{\frac{2W\omega\rho c}{W_0+W}} \approx 1.5\cdot10^{13}\text{s}. \qquad (19)$$

for different realistic parameters from [6, 2, 1]. Obviously this time T is diminishing with a growing of magnetic field that could be tested with paleomagnetic data.

Using up to date determination of conductivity σ from [6] one immediately obtain desired half-million years as a typical magnetic diffusion time $(r_*)^2\mu_0\sigma$ that depends on relatively well-known material properties only.

Thermal diffusion time on magnetic size d could be represented from (16-17) using $X_s= B_*$ as

$$T \approx B_*^2/(\mu_0^2 c\rho W k)\,. \qquad (20)$$

This again gives half-million years with thermal diffusion $k=2\cdot10^{-5}$ m^2/s from [6]. That (20) contrary to (19) is growing with a growing of magnetic field and that is indeed rudely confirmed with paleomagnetic data of [4].

5. Scaling of the energy equations

Here using (15) and planetary dynamo numerical [7] and analytic [8] scaling laws I estimate coefficient of the energy system (16) as functions of the given convection power $W(t)$.

Using well-known after Prof. Christensen magnetic scaling law for B_* and (17) I obtain an excellent representation for the first coefficient from (16) τ that has a meaning (and real value!) of the secular variation

$$B_* = (\mu_0\rho)^{1/2}(WH)^{1/3} = X_s \Rightarrow \tau = (H^2/W)^{1/3}\,. \qquad (21)$$

Here $H = (r_* - r_i)$ and τ, as it is directly observed for centuries, is of order decades for geodynamo typical W~10^{-13} W/kg [7, 2, 1].

Scaling law for velocity V_*, (17) and (21) gives quadratic equation for $\sqrt{W_0}$:

$$V_* = \left(W^2H/\Omega\right)^{1/5} = Y_s \Rightarrow W_0 - 2\Omega^{-1/5}W^{11/15}H^{8/15}\sqrt{\mu_0 c W_0} + W = 0\,. \qquad (22)$$

That gives W_0 via given W as

$$W_0 = \left(\Omega^{-1/5}W^{11/15}H^{8/15}\sqrt{\mu_0 c} \pm \sqrt{(\Omega^{-1/5}W^{11/15}H^{8/15})^2\mu_0 c - W}\right)^2. \qquad (23)$$

Here « + » for $W_0 > W$ and « - » for $W_0 < W$, while $W = W_0$ during inversion or excursion and

$$W = W_0 = \Omega^{-3/2}H^4(\mu_0 c_0)^{15/4},\; c = c_0 = (\Omega^6 W_0^{-7}H^{-16})^{1/15}/\mu_0 \qquad (24)$$

that had been already used in (18) above.

Sinus of the average angle between magnetic and velocity field from [Starchenko, Pushkarev, 2013] s could be presented from cosines from (17) as

$$\frac{W_0 - W}{W_0 + W} = \cos o = \pm\sqrt{1-s^2} = \pm\sqrt{1-\left(\frac{W}{H^2\Omega^3}\right)^{4/15}}\,. \qquad (25)$$

From (25) one can obtain W_0 via W. That in (23) determines $c(t)$ from (16). Two remained coefficients of (16) are determined by (17) as

$$C\sqrt{cW_0/\rho} = \omega, \tag{26}$$

that with (23) determine all the coefficients with up to only one fitting parameter.

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