GENERATION OF HELICITIES IN THE ROTATING LAYER

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Magnetic fields are widely distributed in the Universe. Generation of the magnetic fields accompanied by the transformation of the kinetic energy of the conductive liquid to the energy of the magnetic field is a subject of the dynamo theory [1]. In general, dynamo equations include equations of thermal or (and) compositional convection as it is in planetary dynamo as well as the induction equation for the magnetic field **B**. These equations have a set of quantities (integrals of motion), which can conserve, provided diffusion effects as well as the external forces are absent. Thus in 3D the Navier-Stokes equation conserves the mean over the volume kinetic energy $E_K = \mathbf{V}^2/2$ and kinetic helicity $\chi = \langle\mathbf{V}\cdot\mathrm{rot}\mathbf{V}\rangle$, where $\langle\ldots\rangle$ denotes averaging over the domain and **V** is the velocity field. The Navier-Stokes equation with the induction equation conserves the magnetic helicity $\chi^M = \langle\mathbf{A}\cdot\mathbf{B}\rangle$ and the so-called cross-helicity $\chi^C = \langle\mathbf{V}\cdot\mathbf{B}\rangle$. Here **A** is the vector potential of the magnetic field $\mathbf{B} = \mathrm{rot}\mathbf{A}$.

For each of these quantities one can derive from the original dynamo system its own evolution equation. However, these new equations already are the simplified forms of the original equations; its study can be instructive and sometimes tell us something new without solving more sophisticated full dynamo equations. So as helicities by definition can change the sign, its conservation in the full volume of generation can be trivial. Moreover, some of these quantities can change its sign with transition from large-scales to small ones. That is why understanding of the structure of these quantities in the physical and wave spaces is important.

Here we consider this problem on an example of a rapidly rotating flat layer heated from below, where thermal convection of the conductive fluid takes place. The axis of rotation and the gravity direction coincide. The mathematical model is close to that in [2]. We used the Boussinesq approximation with the fixed temperatures at the boundaries $z = 0, 1$, the stress-free and non-penetrating boundary conditions for the velocity field **V**. The parameters of convection are chosen in such a way that to mimic the geostrophic state with the cyclonic cell forms typical to the planetary convection in the cores of the planets [3]: the Ekman number $E = 2\mathrm{x}10^{-5}$, the Prandtl number $Pr = 1$, the Roberts number $q = 2$, and the modified Rayleigh number $Ra = 300$.

The magnetic equation is written for the vector potential of the magnetic field, accompanied with the pseudo-vacuum boundary conditions. For all the considered physical fields the periodical boundary conditions in the both horizontal directions are applied. To solve these equation we used the control-volume method (SIMPLE algorithm by Patankar [4]) adopted to the cluster parallel computers using MPI. To solve equations we used the mesh grids 128^3 in all directions.

We started simulation for the pure convective regime, see evolution of the mean over the volume temperature fluctuations from the non-convective temperature profile and mean kinetic energy in Fig.1ab. The developed turbulent convection is anisotropic with cells elongated along the axis of rotation and with perpendicular scale $\sim E^{1/3}$. After the one diffusion time (based on the thermal diffusion) at the moment $T = T_B$ the seed of the magnetic field was introduced. The kinematic dynamo stage continued also about one diffusion time before the full dynamo state has developed, Fig.1c.

To follow evolution of the physical fields and its derivatives in the wave space we used the wavelet transform based on the complex Morlet wavelet for the space coordinates. Normalization of wavelets is chosen in such a way that to the periodic signals with different periods and equal amplitudes correspond the equal spectral amplitudes. Application of the wavelets for the non-periodical fields, like turbulence, seems more natural rather using the fast Fourier transform. Due to the rapid rotation it is important to distinguish two directions: along and perpendicular to the axis of rotation, so that one has two spectra on the wavenumbers $k_\perp$, and $k_\parallel$, correspondingly. Latter we consider only the transversal spectra, which are more effected by rotation.

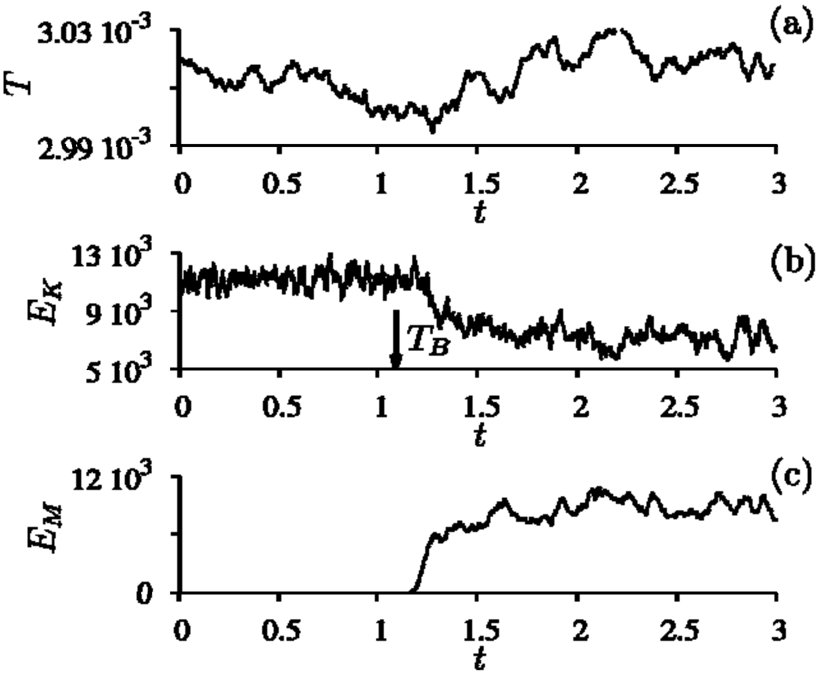

Figure 1: Evolution of the temperature fluctuations (a), kinetic (b) and magnetic (c) energies in the rotating layer. In the moment T_B magnetic field is switched on.

So as helicities can change its sign in the middle horizontal plane, for all the quantities we consider spectra at some point with the fixed horizontal coordinates (x_0, y_0) and integrate all the values in the vertical coordinate in the range z=[0, 0.5].

Evolution of the wavelet spectra is presented in Fig.2. The increase of the magnetic energy E_M to the nonlinear level is accompanied with the decrease of the kinetic energy E_K about 30%.

The cyclonic convection, developed due to the geostrophic state, is a source of the mean kinetic helicity in the system, which is believed to be a source of the large-scale magnetic fields in the planets. Equation for the kinetic helicity generation due to the Coriolis force can be derived from the following relation: $\frac{\partial \mathbf{V}}{\partial t} \sim -\mathbf{1}_z \times \mathbf{V}$. That leads to $\frac{\partial \chi}{\partial t} \sim \frac{\partial}{\partial x_i}(V_i V_z)$, where summation over the repeated indexes is assumed. Finally, due to the periodical boundary conditions in the horizontal directions, evolution of the mean kinetic helicity over the lower half-volume can be estimated as: $\frac{\partial}{\partial t}\int \chi dr^3 = \frac{1}{2}\Big[\int\int V_z^2 dxdy\Big]_{z=0}^{z=0.5}$. So as the normal velocity V_z is zero at z = 0, helicity is positive in the lower half-volume, where flow converges. In the upper half-volume z = [0.5, 1], where flow diverges, the sign of helicity is opposite. Note that these arguments can be used also for the certain wavenumebr $k_\perp^f$ provided the Rossby number for

$k_{\perp}^{f}$ is small. This estimate is supported by our simulations where χ is positive in the lower half-volume at all $k_{\perp}$.

The next one is the current helicity χ^J = <**J**·**B**>, where **J** is the electric current (Fig.2d). In contrast to the kinetic helicity, χ^J has different signs at the small and large scales. This phenomenon calls the separation in scales [5]. As well as the kinetic helicity, the current helicity is anti-symmetric in respect to the middle plane *z*=0.5. It is known that χ^J is closely connected to the magnetic helicity χ^M, see [5]. In the same time, it is based on the physical quantities **J** and **B**, which can be derived from observations. Information on the signs of χ^J at the different scales can be used for extrapolation of the observable fields to the invisible part of the spectra. This situation is common for the solar physics, e.g., for the solar active regions [6].

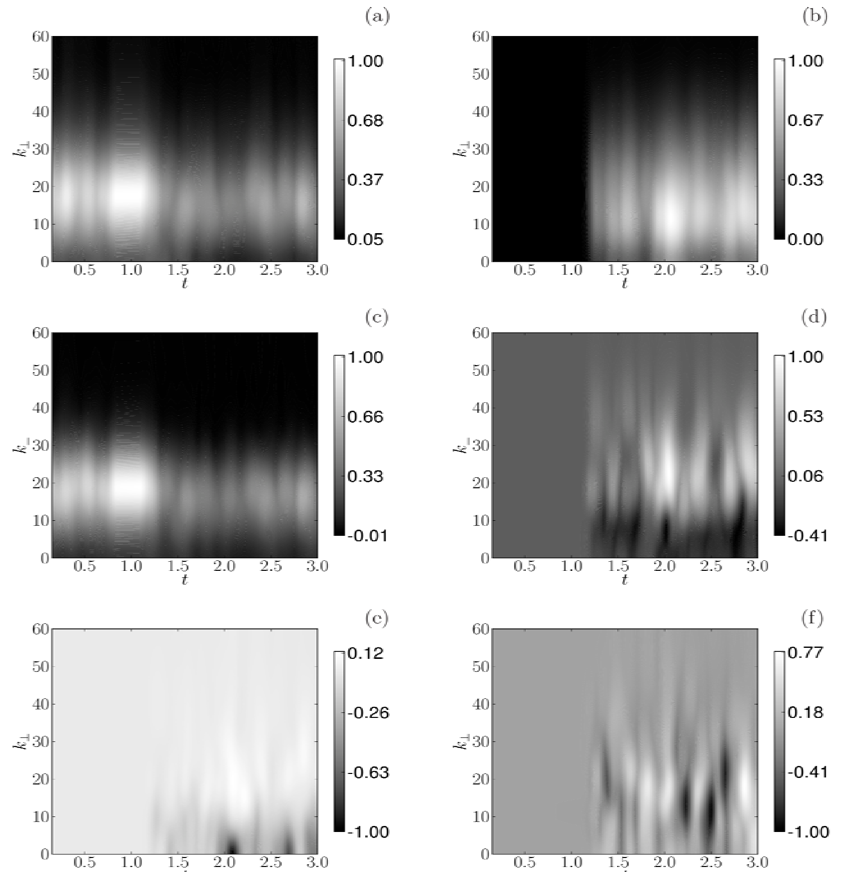

Figure 2: Evolution in time of the transversal wavelet spectra of the kinetic (a) and magnetic (b) energies, kinetic (c), current (d), magnetic (e), and cross- (f) helicities.

Magnetic helicity χ^M is the integral of the induction equation, based only on the magnetic quantities **A** and **B.** Note, the vector potential **A** is defined up to the gradient of some arbitrary function. This fact requires careful treatment of the magnetic helicity, see [5]. Spectrum of χ^M, presented in Fig.2.e also demonstrates the effect of the scales separation. The rough estimate leads to a simple relation between χ^J and χ^M: $\chi^J \sim k_{\perp}^2 \chi^M$.

The last kind of helicity is the cross-helicity χ^C, which is also the invariant of the dynamo equation in the limit of zero dissipation and absence of the external forces. Its evolution and structure of the spectra (Fig. 2f) differ from the previous kinds of helicities. It has no separation in scales but the sign of χ^C, which is the same at all the scales, changes in time. The additional integration in *x*-*y* plane leads to the very negligible estimate of χ^C over the half-volume. Note that due to the quadratic dependence of the Lorentz force on the magnetic field **B**, the hydromagnetic states with **B** and –**B** are equivalent. It means that existence of the non-zero χ^C with the rapidly alternating magnetic field in time, like it happens in the turbulent flow, would contradict this statement.

References

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