Numerical simulations for the DRESDYN precession dynamo

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Abstract: The next generation dynamo experiment currently under development at the Helmholz-Zentrum Dresden-Rossendorf (HZDR) will consist of a precessing cylindrical container filled with liquid sodium. We perform numerical simulations of kinematic dynamo action applying a velocity field that is obtained from hydrodynamic models of a precession driven flow. So far, the resulting magnetic field growth-rates remain below the dynamo threshold for magnetic Reynolds numbers up to Rm = 2000.

1. Introduction

Planetary magnetic fields are generated by the dynamo effect, the process that provides for a transfer of kinetic energy from a flow of a conducting fluid into magnetic energy. Usually, it is assumed that these flows are driven by thermal and/or chemical convection but other mechanisms are possible as well. In particular, precessional forcing has long been discussed as an at least additional power source for the geodynamo [1,2]. A fluid flow of liquid sodium in a cylindrical container, solely driven by precession, is considered as the source for magnetic field generation in the next gener-ation dynamo experiment currently under development in the framework of the project DRESDYN at the Helmholtz-Zentrum Dresden-Rossendorf (HZDR). In contrast to previous dynamo experiments no internal blades, propellers or complex systems of guiding tubes will be used for the optimization of the flow properties. However, rather large dimensions of the container are necessary in order to reach sufficiently large magnetic Reynolds numbers required for the onset of dynamo action, making the construction of the experiment a challenge (figure 1). At present a small scale water experiment is running in order to estimate the hydrodynamic flow properties in dependence of Reynolds number Re, precession angle α, and precession ratio $\Gamma = \Omega/\omega$ (figure 2). Flow measurements of axial velocity profiles at different radial posi-tions are done using Ultrasonic Doppler Velocimetry. First experimental results confirm that precession provides an efficient flow forcing mechanism which in the final realisation of the liquid metal experiment will allow magnetic Reynolds numbers up to $Rm \sim 700$. This value is rather close to the critical Rm reported by [3] for the onset of dynamo action in a precessing cylinder.

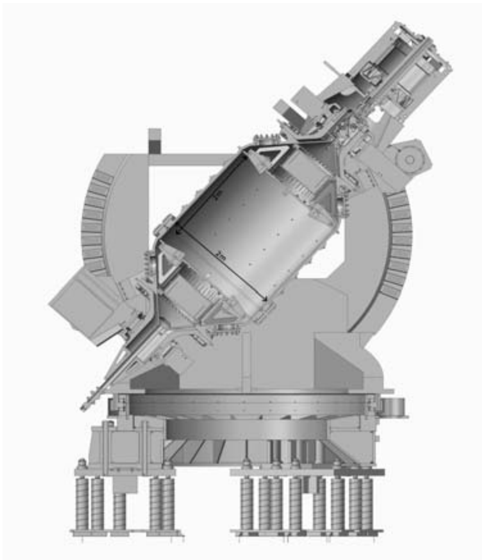

Figure 1: Sketch of the planned experiment. Diameter and height of the cylindrical container will be approximately 2 m.

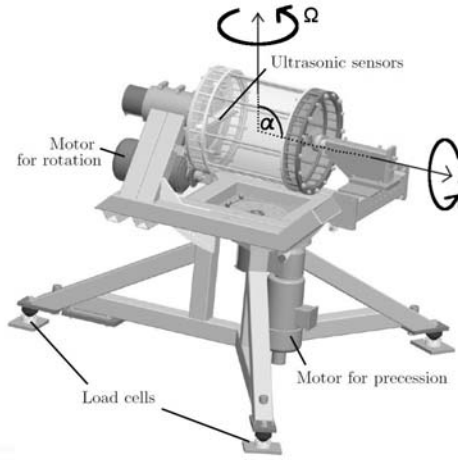

Figure 2: Sketch of the water mock-up. The dimensions of the cylinder are roughly 6 times smaller than the future liquid metal experiment.

2. Hydrodynamic simulations of precession in a cylindrical container

Hydrodynamic simulations of a precession driven flow are performed using SEMTEX [4]. The code applies a spectral element method on the meridional planes and Fourier decomposition in the azimuthal direction for the numerical solution of the Navier-Stokes equation which in the precessing frame reads:

$$\partial \mathbf{u} / \partial t + (\mathbf{u} \cdot \nabla)\mathbf{u} + 2(\boldsymbol{\omega} + \boldsymbol{\Omega}) \times \mathbf{u} = \nu \nabla^2 \mathbf{u} + \nabla \Phi \, . \tag{1}$$

Here $\mathbf{u}$ denotes the velocity field, $\boldsymbol{\omega}$ the rotation around the symmetry axis of the container, $\boldsymbol{\Omega}$ the rotation around the precession axis, ν the viscosity and Φ the modified pressure that includes the centrifugal contributions. In the precessing frame the boundary conditions are given by $\mathbf{u} = \boldsymbol{\omega} \times \mathbf{r}$. For small precession ratios and Reynolds numbers, $Re = \omega R^2 / \nu$, we obtain a good agreement between simulations and measurements with the flow being dominated by the fundamental Kelvin mode with an azimuthal wavenumber m=1 (figure 3). The agreement is less convincing for larger precession ratios (right column in figure 3). A slight tilt of the fundamental mode with respect to the rotation axis is apparent in the simulations (top row) as well as in the experimental data (bottom row).

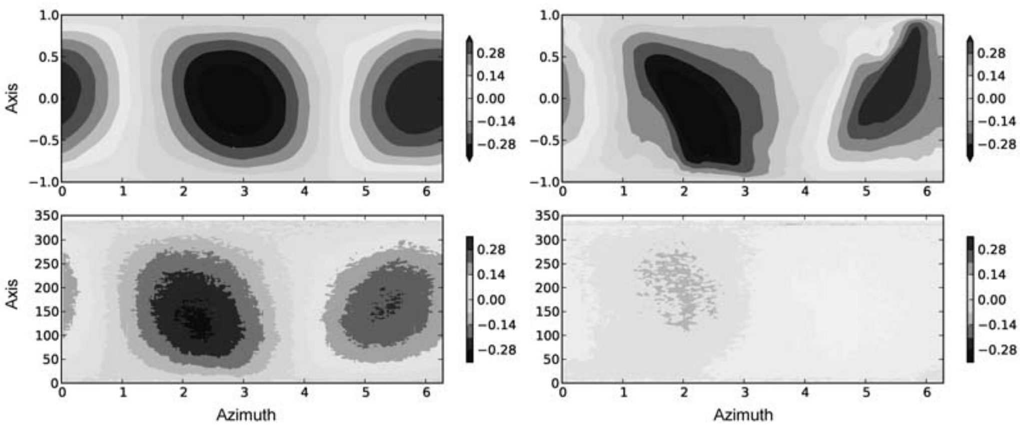

Figure 3: Comparison of the axial velocity at r=0.74 from hydrodynamic simulations (top) and experimental measurements (bottom). Left: Γ = 0.06, right: Γ = 0.10.

The most striking feature found in the water experiment is an abrupt transition from a laminar state to a disordered chaotic behavior at a critical precession ratio. This transition is less pronounced in the numerical simulations, which, however, are carried out at a much smaller Reynolds number. An indication of the change of state in the flow simulations is reflected in the re-orientation of the main fluid rotation axis which in the disordered state is aligned parallel to the precession axis (right side figure 4).

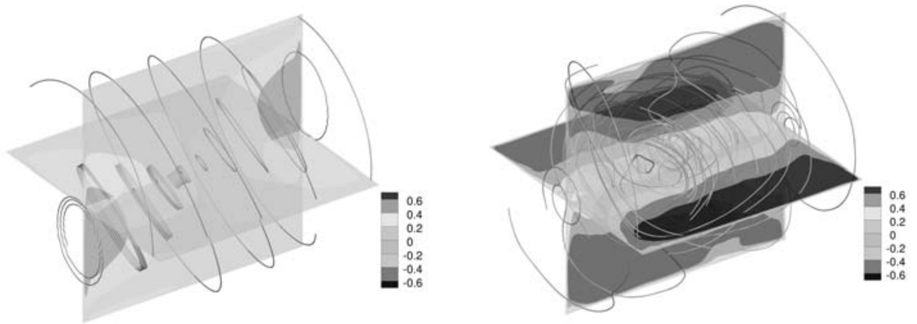

Figure 4: Streamlines from simulations with weak precession (Γ =0.03, left) and strong precession (Γ = 0.15, right). Note the bulk fluid rotating around the precession axis in the latter case.

3. Kinematic dynamo action from a simulated precessional flow

In the following the velocity field obtained from the hydrodynamic simulations is used as an input in a kinematic solver for the magnetic induction equation which reads

$$\partial \mathbf{B} / \partial t = \nabla \times \left(\mathbf{u} \times \mathbf{B} - \eta \nabla \times \mathbf{B} \right), \tag{2}$$

with the magnetic flux density **B** and the magnetic diffusivity η. The resulting growth-rates and the corresponding critical magnetic Reynolds numbers will provide a restriction of the useful parameter regime and will allow an optimization of the experimental configuration. We start with a flow obtained at a low Reynolds number of Re=1500 and a precession ratio Γ=0.1. At these parameters the flow in the precessing frame is (more or less) stationary with a rather simple pattern that is very close to the fundamental m=1 Kelvin mode (left column in figure 3). The temporal behavior of the magnetic energy is shown in the left panel of figure 5. We do not find growing solutions up to a magnetic Reynolds number of $Rm=\omega R^2/\eta$=2000, and from the behavior of the corresponding growth-rates (right panel in figure 4) it seems unlikely that a crossing of the dynamo threshold occurs within reasonable Rm.

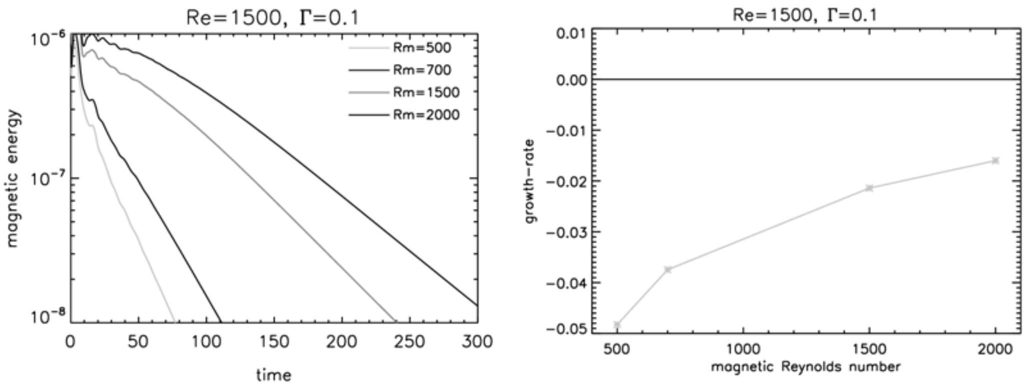

Figure 5: Left: temporal behavior of the magnetic energy for a velocity field obtained at Re = 1500 and Γ = 0.10. Right: Corresponding growth-rates of the fundamental dynamo eigenmode versus Rm.

Next, we take a velocity field from the more turbulent regime at Re = 6500. At this value, the flow is time-dependent but still dominated by the m=1 mode (figure 6). Again, we do not find any growing solutions for the magnetic field, however, the behavior of the growth-rates indicates a critical magnetic Reynolds number in the range of Rm =3000...4000 (figure 7), which unfortunately would be far out of reach in the forthcoming dynamo experiment.

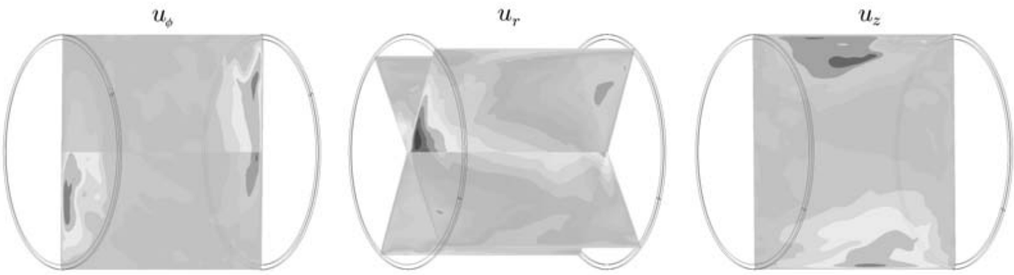

Figure 6: Snapshot of the time-dependent velocity field at Re = 6500 and Γ = 0.08.

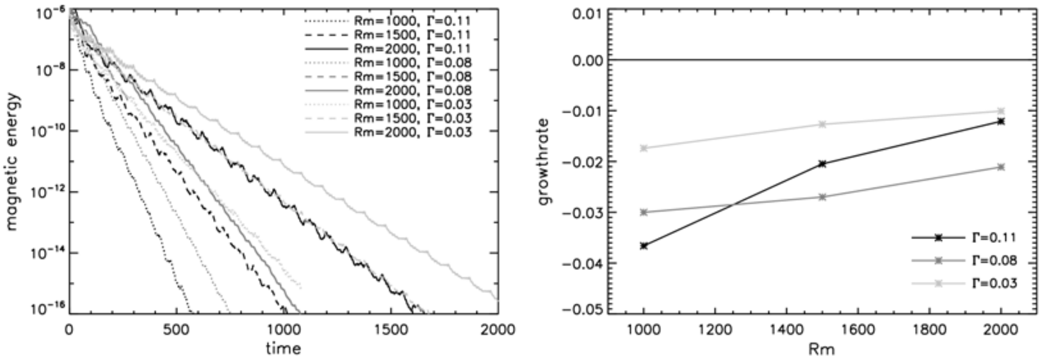

Figure 7: Left: temporal behavior of the magnetic field amplitude for a velocity field obtained at Re = 6500. Right: Corresponding growth-rates of the fundamental dynamo eigenmode versus Rm.

4. Conclusion

Using velocity fields obtained from simulations at *Re*=1500 and *Re*=6500, our kinematic simulations do not show growing magnetic fields. Hence, for the slow rotation rates examined so far, the flow structure most probably is too simplistic to provide for dynamo action. However, for larger Reynolds numbers, we expect more complex structures to emerge. There are two promising candidates from which we expect better properties regarding their ability to drive a dynamo. So- called triadic resonances, that consist of the forced fundamental (*m*=1) mode and two free inertial modes with larger azimuthal wavenumber, have repeatedly been observed in experiments and simulations of precessing flows [5,6]. A subclass of these modes have a close similarity to the columnar convection rolls that are responsible for dynamo action in geodynamo models and there is little reason to believe that this should not be the case with a precession driven flow field. The second possibility relies on observations of cyclones in the French precession experiment ATER [7]. In that experiment large scale vortex-like structures emerge for intermediate precession ratios.

Figure 8: Idealized model for cyclones observed in the ATER experiment [7].

These vortices are oriented along the rotation axis of the cylindrical container, and, depending on the parameter regime, their number varies between one and four (figure 8). The vortices are cyclonic, i.e., their sense of rotation is determined by the rotation orientation of the cylindrical container. We suspect that these vortices provide a significant amount of helicity, but so far, the axial dependence of their contribution and their interaction with the fundamental m=1 mode is unknown. Furthermore, cyclones were neither observed in the HZDR experiment (so far, no appropriate velocity measurements in a horizontal plane are available) nor in any simulations which probably must run at much higher Reynolds number in order to reveal these modes.

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