FLUCTUATIONS IN MEAN-FIELD DYNAMOS

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Abstract: Conventional astrophysical dynamo models are usually formulated in terms of mean-field dynamos. This approach do not include in an explicit form magnetic field fluctuations as well as fluctuations of the dynamo governing parameters. Both types of fluctuations surely are presented in dynamos. We discuss how to include them in the mean-field description and what is the role of fluctuations in the astrophysical phenomenology.

1. Introduction

A traditional approach in treatment of various physical problems in random media is based on a consideration of mean-field equation for the physical field under investigation. This approach in dynamo theory was developed about 50 years ago and is known as mean-field dynamo. Of course, it is possible now to avoid mean-field approach and addresses a dynamo problem in the framework of direct numerical simulations only. The point however is that the mean-field description of a dynamo problem remains important for interpretation of results of direct numerical simulations. Particularly important roles plays mean-field dynamos in astrophysical problems because our knowledge concerning internal structure of remote celestial bodies is usually very limited and gives some mean quantities only. A specific feature of mean-field dynamos is that the number of the random turbulent or convective shells which are involved in the averaging in development of mean-field dynamo models is usually rather large (say, $N=10^4$) however is much lower than, say, Avogardo number which controls averaging in molecular physics. Correspondingly, fluctuations in mean-field dynamos play a much more important role in mean-field dynamos rather in other domains of statistical physics.

Here we present a review of a sequence of recent papers, which consider the role of fluctuations in mean-field dynamos. We argue that the most important kind of fluctuations is fluctuations of the α-coefficient, which plays a crucial role in many mean-field dynamos. The point is that the α-effect is usually quite weak (a naive estimate is $\alpha \approx 0.1\ v$ where v is the rms velocity) and one can expect 10% - 20% fluctuations of α for $N = 10^4$. We argue that this level of α-fluctuations in solar dynamos is sufficient to explain such phenomena as the famous Maunder minimum and other transient phenomena in solar cyclic activity. A possible role of the α-fluctuations in geodynamo is discussed.

We discuss a possible role of large-scale fluctuations of the mean-field velocities and their possible importance for advective dynamos as well as the large-scale manifestations of small-scale magnetic fluctuations in solar mean-field dynamos.

2. Mathematical implementation

Introducing fluctuations in a mean-field description we face some mathematical problems. The point is that the mean-field equations contain mean values of various microscopic quantities, i.e. non-random quantities only. In order to overcome this problem we have to include in the theory two kind of random quantities and two kind of averaging. The first averaging gives conventional mean-field equations while the second type of randomness remains to describe fluctuations of the dynamo governing parameters (mainly α) and fluctuations of the mean magnetic field. In practice these fluctuations are modeled by a proper random number generator and then are governed by the standard mean-field equations.

We consider solutions for individual realizations of the fluctuations and play with the random number generator to mimic the whole variety of the models. In principle, one could try to perform an additional averaging of the conventional mean-field equations taken over the ensemble of the fluctuations under discussion. We performed an exploratory investigation for this supplementary approach to recognize various mathematical difficulties arising. At least for the instant, we prefer to stay on the safe side and avoid any averaging of, say, α-fluctuations.

The format of mathematical description of the problem chosen follows conventional traditions of, say, molecular physics. In our opinion however mathematical foundation of this choice deserves a further discussion.

3. An example: Magnetic cycles in M-dwarfs.

Below we present an example [1] of the options which open the approach under discussion to explain astronomical phenomenology. The example deals with a particular type of stellar magnetic activity known for so-called M-dwarfs which are fully convective stars slightly smaller than the Sun.

M-dwarfs demonstrate two types of activity: 1) strong (kilogauss) almost axisymmetric poloidal magnetic fields; and 2) considerably weaker non-axisymmetric fields, sometimes including a substantial toroidal component.

Dynamo bistability has been proposed as an explanation. However it is not straightforward to obtain such a bistability in dynamo models. On the other hand, the solar magnetic dipole at times of magnetic field inversion becomes transverse to the rotation axis, while the magnetic field becomes weaker at times far from that of inversion. Thus the Sun resembles a star with the second type of activity. Paper [1] suggests that M-dwarfs can have magnetic cycles, and that M-dwarfs with the second type of activity can just be stars observed at times of magnetic field inversion. Then the relative number of M-dwarfs with the second type of activity can be used in the framework of this model to determine parameters of stellar convection near the surface.

Many solar observers have reported that the solar magnetic dipole does not vanish during the reversal while mean-field solar dynamo models predict an oscillating mean solar magnetic field whose magnetic dipole moment have to vanish at each activity cycle (11 years).

This apparent contradiction between expectations from dynamo modelling and observation can be resolved as follows [2]. The point is that a mean-field dynamo model deals with *mean* magnetic field and the averaging is performed over an ensemble of convective velocity cells, while the observational magnetic dipole data refer to *large-scale* magnetic field. Both quantities coincide for an infinitely large ensemble of convective cells, but in practice the number of cells is only moderately large. Because the convective cell ensemble contains a not extremely large number of cells, large-scale fluctuations of magnetic field arise which yield a fluctuating component $\delta\mathbf{d} = (b/B)\, N^{-1/2}(B_P/B_T)$ of the solar magnetic dipole $\mathbf{d}$. Here b is the rms value of small-scale magnetic field, i.e. the magnetic fluctuations, B is the typical value of the mean magnetic field which is determined mainly by the toroidal magnetic field B_T and the factor B_T/B_P takes into account that the magnetic dipole moment is determined by the poloidal magnetic field B_P.

The fluctuating part of the magnetic dipole is larger than the part determined by the mean magnetic field is about 4 months, i.e. about 3% of the solar magnetic cycle [3]. Assuming that the magnetic activity of M-dwarfs is more-or-less similar to that of the Sun, we can convert the relative time $\delta t/T$ during the magnetic activity cycle during which the magnetic dipole is determined by magnetic fluctuations and is strongly inclined to the rotation axis and expect that 3% of M-dwarfs should exhibit the second type of activity.

We can say also what physical parameters are required, e.g. N, to get a satisfactory correspondence with the observational data. In particular, if we want to explain that about 30% of M-dwarfs demonstrate the second type of activity, we have to assume that the number of convective cells at the surface of M-dwarfs is about two order of magnitudes lower than near the surface of the Sun, i.e. 10^2 instead of 10^4. Given the much greater relative depth of the convection zone in M-dwarfs compared to the Sun, an increase in the size of convection cells and corresponding decrease of N is not implausible.

4. Conclusion

The idea to include the dynamo governing parameters as well as magnetic field fluctuations in the mean-field dynamo models opens new perspectives to explain various features of magnetic activity known for various celestial bodies. Possibly, the idea can be useful as well for interpretation of laboratory dynamo experiments for which the dynamo generated magnetic field also can demonstrated a random behaviour (e.g. [4]).

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5. References

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