MIXED CONVECTION IN VERTICAL DUCTS WITH STRONG TRANSVERSE MAGNETIC FIELDS

LIU L., SCHIGELONE A., ZHANG X., ZIKANOV* O.
University of Michigan – Dearborn, Dearborn, MI 48128, USA
*Corresponding author email: zikanov@umich.edu

Abstract: Fundamental features of mixed convection are investigated for upward and downward flows in long vertical ducts with one heated wall and strong imposed transverse magnetic field. It is found that the Q2D model accurate for the upward flow, but not always accurate for the downward one. Another feature of the downward flow is the exponentially growing streamwise-uniform elevator modes. Finally, the flows are prone to Kelvin-Helmholtz instabilities associated with inflection points of streamwise velocity profiles.

1. Introduction

This paper presents the first results of our computational study of mixed (combined natural and forced) convection in MHD flows of liquid metals in vertical ducts. Ducts of square cross-section with electrically insulated walls are considered. Three of the walls are thermally insulated, while the fourth wall is subject to constant-rate heating. A uniform steady-state magnetic field is imposed in the direction perpendicular to the flow and to the direction of the temperature gradient established by the wall heating. The focus of the study is on situations with strong heating and magnetic field (large values of Grashof and Hartmann numbers). The cases of upward and downward flows are considered. The study is relevant to the liquid metal blankets of fusion reactors, more specifically, to the blankets with long poloidal channels, in which electrical and thermal insulation of the walls is applied [1]. The system is simplified by assuming a fully developed flow, perfect electrical and thermal insulation of the walls, and constant physical properties of the liquid, as well as by replacing the volumetric heating near the first wall by the wall heating. At the same time, no a-priori assumptions about the flow's dimensionality and time-dependency are made. The computational model permits development of 3D and time-dependent structures. The study can be characterized as a theoretical analysis of basic features of convection in vertical ducts with strong transverse magnetic field.

Despite their evident importance for the operation of liquid metal blankets, the effects of natural and mixed convection in the presence of strong magnetic fields are still poorly understood. The only earlier computational work directly addressing the configuration of this paper is [2], where quasi-2D modeling was employed to study convection instabilities in upward flows in long vertical ducts. Instabilities of two types (the Kelvin-Helmholtz type associated with inflection points in the streamwise velocity profile and the boundary layer type) were found, both leading to oscillations of velocity and temperature fields. These results are consistent with the measurements [3] that showed high-amplitude oscillations of temperature in mercury flows in vertical pipes. We note that the effect of convection-generated oscillations is not limited to flows in vertical tubes. It is a general phenomenon likely to occur in tubes of almost all orientations. For example, the computational and experimental analysis of flows in horizontal pipes [4,5] and ducts [6] with bottom heating show that a sufficiently strong transverse horizontal magnetic field results in the convection instability in the form of growing rolls aligned with the field and transported by the main flow.

2. Theoretical and numerical model

We consider a flow of an incompressible, Newtonian, electrically conducting fluid (a liquid metal) in a vertical duct of square cross-section. Heating of constant uniform rate q is applied to one of the walls. Constant magnetic field **B** oriented perpendicularly to the duct and to the direction of the heating-induced temperature gradient is imposed in the entire flow domain. The flow with the mean velocity U directed either upwards or downwards is driven by an applied pressure gradient $\nabla\hat{p}$. Using U as the velocity scale, the duct's half-width d as the length scale, qd/κ, where κ is the thermal conductivity, as the temperature scale, and the B-derived scales for the electromagnetic fields, we can write the non-dimensional equations as

$$\partial_t \mathbf{u} + (\mathbf{u}\cdot\nabla)\mathbf{u} = -\nabla p - \nabla\hat{p} + \mathrm{Re}^{-1}\nabla^2\mathbf{u} + \mathbf{F}_b + \mathbf{F}_L, \tag{1}$$

$$\nabla\cdot\mathbf{u} = 0, \quad \partial_t\theta + \mathbf{u}\cdot\nabla\theta = \mathrm{Pe}^{-1}\nabla^2\theta - u_x \, {dT_m}/{dx}. \tag{2}$$

In the equations and the following discussion, the coordinates x,y,z are, respectively, in the streamwise direction, direction of the magnetic field, and direction orthogonal to the heated wall. The pressure field is the sum of $\hat{p}$(x) with spatially uniform gradient $d\hat{p}/dx$ adjusted at every time step to maintain constant mean velocity and fluctuation $p(x,t)$. The temperature field T is the sum of the mean-mixed temperature $T_m(x) = A^{-1}\int u_x T\, dA = 2Pe^{-1}x$ and fluctuations $\theta(x,t)$. The buoyancy force is computed using the Boussinesq approximation as $\mathbf{F}_b = \mathrm{Gr}\mathrm{Re}^{-2}\alpha\mathbf{e}_x\theta$. Here, we assume that the buoyancy due to the mean-mixed temperature is fully balanced by a vertical pressure gradient and use the coefficient α to account for the two possible flow directions: α=1 in upward flow (x facing upwards) and α=-1 in downward flow (x facing downwards). The Lorentz force is $\mathbf{F}_L = \mathrm{Ha}^2\mathrm{Re}^{-1}\mathbf{j}\times\mathbf{e}_y$, where the current density is determined by the Ohm's law $\mathbf{j} = -\nabla\phi + \mathbf{u}\times\mathbf{e}_y$ and the electric potential is a solution of the Poisson equation $\nabla^2\phi = \nabla\cdot(\mathbf{u}\times\mathbf{e}_y)$. The non-dimensional parameters are the Reynolds number $\mathrm{Re} = Ud/\nu$, Hartmann number $\mathrm{Ha} = Bd(\sigma/\rho\nu)^{1/2}$, Peclet number $\mathrm{Pe} = Ud/\chi$, and Grashof number $\mathrm{Gr} = g\beta q d^4/\nu^2\kappa$. The boundary conditions at the walls are: **u**=0, $\partial\phi/\partial n = 0$, $\partial\vartheta/\partial n = 0$ at the thermally insulated walls, and $\partial\theta/\partial n = -1$ at the heated wall. Inlet-exit periodicity of ϑ, p, **u** and ϕ is assumed.

We use a version of the finite difference model introduced in [7] and later adapted to flows with mixed convection in [4,6]. The method uses the second-order time discretization, in which viscous and conductive terms are treated implicitly and the incompressibility is satisfied by the standard projection algorithm (see, e.g. [8]). The spatial discretization is of the second order on a structured collocated grid with points clustered near the walls via the tanh coordinate transformation (see [6,7] and the discussion of various clustering schemes applied to the MHD convection in [9]). The discretization uses fluxes of velocity and electric current interpolated to half-integer points. In the non-viscous, non-conductive limit, it exactly conserves mass, momentum, electric charge, and internal energy, while kinetic energy is conserved with the dissipative error of the 3rd order.

In the computations, we keep constant Re=5000 and Pr=0.0321, while Ha and Gr vary in broad ranges. The computational domain is a duct of 2x2 cross-section and sufficient streamwise length. The computational grids are tested in a thorough sensitivity analysis so that the smallest grids providing accurate solutions are determined (see [6,9] for details). Such grids typically have about 7 points within each Hartmann boundary layer and 10-12 points within each Shercliff (sidewall) boundary layer.

3. Results

3.1 Analytical Q2D solutions in comparison with computations. In high-Ha flows of certain geometries, one can apply the Q2D model [10], in which the flow fields are averaged wall-to-wall in the magnetic field direction and the electromagnetic effects are reduced to linear friction at the Hartmann walls. The model can be applied in our case, where the averaging is in the y-direction. We do that for the streamwise-uniform steady-state flows:

$$\mathbf{u} = u(y,z)\mathbf{e}_x, \qquad \theta = \theta(y,z), \qquad p = 0 \,. \tag{3}$$

Such solutions always exist in our system. As discussed below, they can be stable or unstable depending on the values of Ha and Gr.

Applying the Q2D approximation and the just assumed state of the flow, we derive:

$$C + \mathrm{Gr}\mathrm{Re}^{-2}\alpha\bar{\theta} + \mathrm{Re}^{-1}\bar{u}'' - \mathrm{Ha}\mathrm{Re}^{-1}\bar{u} = 0, \quad \bar{u} = \mathrm{Re}^{-1}\bar{\theta}, \tag{4}$$

$$\bar{u}(\pm 1) = 0, \qquad \bar{\theta}(-1) = -1, \qquad \bar{\theta}(1) = 0 \,, \tag{5}$$

where $\bar{u}(z)$ and $\bar{\theta}(z)$ are the y-averaged fields, and $C = -\frac{d\bar{p}}{dx}$. The system can be reduced to a boundary-value problem for a 4th-order ordinary differential equation with constant coefficients and solved analytically (a similar solution for the case of internal heating is available in [2]).

00The results are presented in Fig. 1, where the analytically found profiles $\bar{u}(z)$ are compared with the computed solutions of the full problem. In order to obtain the streamwise-uniform numerical solutions, we apply x-averaging at every time step and compute the evolution of the flow until convergence to a steady state is achieved. After that, the computed profiles $u(y,z)$, $\theta(y,z)$ are averaged in y. We see in Fig. 1a that in the case of the upward flow the agreement between the Q2D and full solutions is very good. This has been found in all our computations conducted at Ha=50, 100, 200, 400, 800 and Gr=10^6, 10^7, 10^8, 10^9. Quite different results are obtained for the downward flows. Here, the Q2D model allows us to accurately calculate u and θ only at low Gr. At high Gr, as illustrated in Figs. 1c,d, the Q2D solutions are inaccurate or even unphysical. The upper boundary of the interval of acceptable accuracy increases with Ha, but even at Ha=400 and 800, high values of Gr render the Q2D model inaccurate.

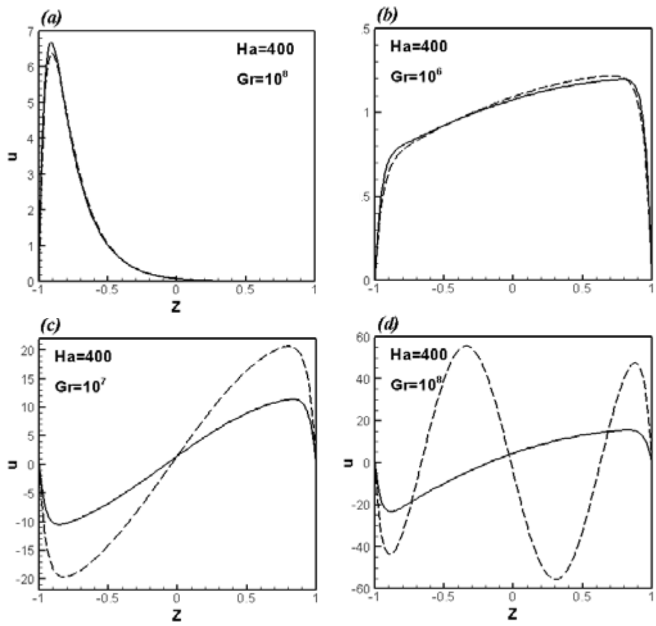

Figure 1: Comparison between analytical Q2D (dashed lines) and computed (solid lines) solutions for streamwise-uniform steady-state flows. *(a)* – upward flow, *(b)-(d)* – downward flow. Heated wall is at z =-1.

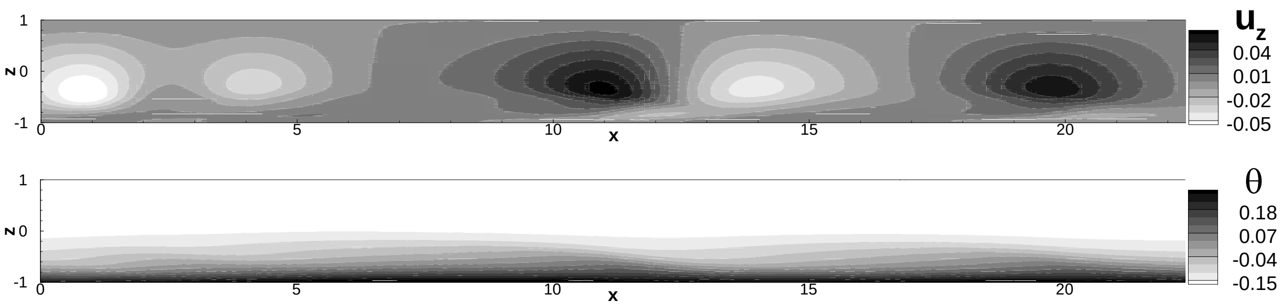

Figure 2: 3D DNS of upward flow at Ha=200, Gr=10. Instantaneous distributions of transverse velocity component u_z and temperature θ in fully developed flow are shown for the mid-plane normal to the magnetic field.

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3.2 Elevator modes in downward flows. In the case of downward flow, solutions of type (3) could not be computed at high values of Gr. Instead of converging to steady states, the calculated fields demonstrated exponential growth as in:

$$u = e^{\gamma t} u_0(y,z) e_x, \qquad \theta = e^{\gamma t}\theta_0(y,z), \qquad p = 0, \qquad C = e^{\gamma t} C_0, \qquad \gamma > 0 \quad . (6)$$

Substituting (6) into the governing equations, we obtain the eigenvalue problem, the mathematical solution of which is yet to be found. From the physical viewpoint, existence of (6) is not surprising. In the downward flow, the balance between wall heating and streamwise convective transport results in the mean-mixed temperature $T_m(x)$ increasing downwards, i.e. unstable stratification. In the absence of top and bottom boundaries, unstably stratified systems are known to have exponentially growing vertically uniform solutions called the 'elevator modes' (see, e.g. [11]). Under normal circumstances, such solutions are not observed, since the growing upward and downward jets typically present in them quickly become unstable and succumb to turbulence. In MHD flows, however, a sufficiently strong magnetic field stabilizes the jets and make the elevator modes actually realized numerical solutions. This was demonstrated in the periodic box computations [12], where, to our knowledge, the elevator modes were first identified and described.

It is hard to say how relevant the solutions (6) to the processes in channels of liquid metal blankets. The top and bottom boundaries of a channel would make the clear-cut elevator modes impossible. At the same time, we may hypothesize that in a long channel the unstable stratification cause strong upward and downward jets. The jets, possibly, grow to large amplitudes and break down to inflection point instabilities creating sporadic turbulence-like bursts of velocity and temperature fluctuations. This scenario and its role as an explanation of strong temperature fluctuations observed in [4] will be explored in our future work.

3.3 Stability analysis and DNS. For the cases, where the steady-state streamwise-uniform solutions (3) could be obtained we conducted their stability analysis. This was done using a modification of the 3D DNS method, in which we followed the evolution of small-amplitude perturbations added to (3) and restricted to a given streamwise wavelength λ using FFT filtering (see [4,6] for details of the method). We have found that, with exception of flows at Gr=10^6, the solutions (3) are unstable in wide ranges of λ for both upward and downward flows. The instability modes are the typical Kelvin-Helmholtz rolls oriented along the magnetic field lines and associated with inflection points in the base flow profiles. Similar modes are found in the Q2D analysis of the upward flow [2].

The flow regimes arising from the instabilities were investigated in DNS. The computational domains sufficiently long to include all or nearly all the unstable modes were used. Typical results are illustrated in Fig. 2.

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4. References

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