MIXED CONVECTION IN HORIZONTAL DUCTS WITH STRONG TRANSVERSE MAGNETIC FIELDS

ZHANG X., LV X., ZIKANOV* O.
University of Michigan – Dearborn, Dearborn, USA
*Corresponding author email: zikanov@umich.edu

Abstract: Mixed convection in liquid metal flows in horizontal ducts with strong transverse magnetic fields is analyzed numerically. Our goal is to understand the nature of the convection instabilities that may occur in liquid metal blankets for fusion reactors. High Hartmann ($Ha \leq 800$) and Grashof ($Gr \leq 10^9$) numbers are considered. In the case of bottom heating, roll-like structures aligned with the magnetic field and producing strong low-frequency temperature fluctuations at walls arise at high Ha and Gr. In the duct with sidewall heating, streamwise-uniform, laminar, steady-state flows are found at moderate and high Ha.

1. Introduction

In this paper, we consider the influence of very strong (high Hartmann numbers) magnetic fields on mixed (combined natural and forced) thermal convection in flows within horizontal ducts. The case of low magnetic Reynolds and Prandtl numbers is considered. The main motivation is the role of such flows in several possible designs of liquid metal (Li or PbLi) blankets for future nuclear fusion reactors (see, e.g. [1]). It is known that a strong magnetic field can dramatically change the flow's behavior. The effect is due to the Lorentz force and the conversion of the flow's kinetic energy into heat by the Joule dissipation of induced electric currents. An extensive discussion of the effect can be found, for example, in [2]. Since conventional turbulence is most likely fully suppressed by such a strong magnetic field, the flow structure and distribution of temperature are largely determined by the buoyancy force in its interaction with the Lorentz force, viscosity, and imposed pressure gradients. The immediate precursor of our work is the paper [3], where the linear stability and DNS analyses were conducted for the flow through a round horizontal pipe with transverse horizontal magnetic field and uniform constant-rate heating applied to the bottom half of the wall. The computational work was performed to explain the results of the experiments [4], where temperature fluctuations disappeared at moderately strong magnetic fields indicating suppression of turbulence, but reappeared at stronger magnetic fields in the form of high-amplitude low-frequency oscillations. The results of the numerical analysis of [3] were in a remarkably good quantitative agreement with the experimental data and produced the explanation that the oscillations are caused by the convection instability in the form of rolls aligned with the magnetic field. Our analysis is conducted in a manner similar to that of [3] but for the square duct geometry, two orientations of the magnetic field, and in a much broader range of Gr and Ha.

2. Theoretical and numerical model

As shown in fig. 1, we consider flows in horizontal ducts with imposed horizontal transverse magnetic field **B** and uniform constant-rate heating applied to either bottom or vertical wall. The other three walls are thermally insulated. All walls are electrically insulated. The liquid metal is modeled as an incompressible, electrically conducting Newtonian viscous fluid with constant physical properties. The Boussinesq approximation is applied for the temperature-related buoyancy force. Following the assumption of small magnetic Reynolds and Prandtl

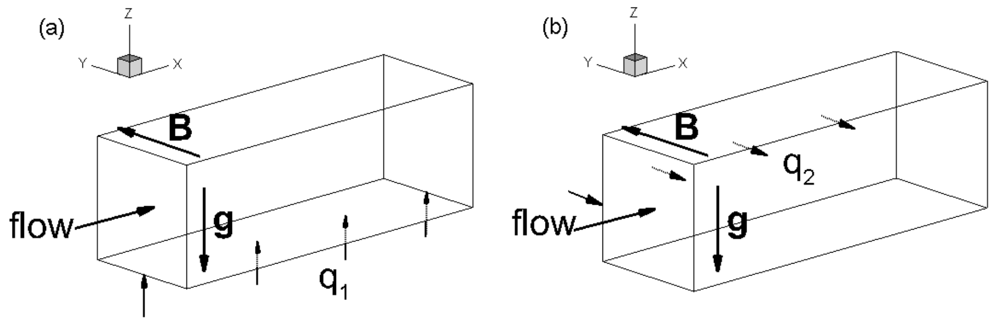

Figure 1: Flow geometry and coordinate system for bottom heating duct (a), sidewall heating duct (b).

numbers typically valid in the fusion reactor blankets, we use the quasi-static approximation (see, e.g. [2]). The governing equations are non-dimensionalized using the duct half-width d as the length scale, mean streamwise velocity U as the velocity scale, wall heating-based group qd/κ ,where κ is the thermal conductivity, as the temperature scale, B as the scale of the magnetic field strength, and dUB as the scale of electric potential. The non-dimensional governing equations are:

$$\partial_t \mathbf{u} + (\mathbf{u}\cdot\nabla)\mathbf{u} = -\nabla p - \nabla\hat{p} - \nabla\tilde{p} + \mathrm{Re}^{-1}\nabla^2\mathbf{u} + \mathbf{F}_b + \mathbf{F}_L,$$

$$\nabla\cdot\mathbf{u} = 0,$$

$$\partial_t\theta + \mathbf{u}\cdot\nabla\theta = \mathrm{Pe}^{-1}\nabla^2\theta - u_x\, dT_m/dx.$$

Temperature field is a sum of fluctuations θ and the mean-mixed temperature $T_m(x)$. Pressure is decomposed into the component $\hat{p}$(x) corresponding to spatially uniform streamwise gradient $d\hat{p}/dx$ adjusted at every time step to maintain constant mean velocity, component $\tilde{p}$ that balances the buoyancy force due to the x-dependent mean-mixed temperature $T_m(x)$ (see [3]), and perturbations p. The buoyancy and Lorentz forces are: $\mathbf{F}_b = -\mathrm{Gr}\mathrm{Re}^{-2}\mathbf{e}_z\theta$ and $\mathbf{F}_L = -\mathrm{Ha}^2\mathrm{Re}^{-1}\mathbf{j}\times\mathbf{e}_y$. The electric current is determined by the Ohm's law $\mathbf{j} = -\nabla\phi + \mathbf{u}\times\mathbf{e}_y$, where electric potential ϕ is a solution of the Poisson equation $\nabla^2\phi = \nabla\cdot(\mathbf{u}\times\mathbf{e}_y)$. The non-dimensional parameters of the problem are the Reynolds number $\mathrm{Re} = Ud/\nu$, Hartmann number $\mathrm{Ha} = Bd(\sigma/\rho\nu)^{1/2}$, Peclet number $\mathrm{Pe} = Ud/\chi$, and Grashof number $\mathrm{Gr} = g\beta q d^4/\nu^2\kappa$. The boundary conditions at the duct walls are the no-slip conditions **u**=0 for velocity, perfect electric insulation $\partial\phi/\partial n = 0$, conditions of constant-rate heating at the lower wall or side wall $\partial\theta/\partial n = -1$ and thermal insulation at the other walls $\partial\vartheta/\partial n = 0$. The inlet-exit conditions are those of periodicity of the velocity **u**, temperature fluctuations θ and pressure fluctuations p.

The numerical method is a version of the finite difference model first introduced in [5] and later used for various flows at strong magnetic fields (e.g. in [3]). The spatial discretization is of the second order and performed on a non-uniform collocated grid. The grid is clustered towards the walls according to the coordinate transformations: $y = \tanh(A_y\eta)/\tanh(A_y)$, $z = \tanh(A_z\xi)/\tanh(A_z)$. The spatial discretization is conservative with spatial derivatives evaluated using the velocity and current fluxes obtained by special interpolations to half-integer grid points (see [6], [7]). The time discretization uses the standard projection algorithm (see, e.g. [8]). The body forces and nonlinear convection terms are treated explicitly using the backward difference Adams-Bashforth scheme as described in [5]. The conduction and viscosity terms are treated implicitly in order to avoid the stringent stability limitations on the time step that arise at strong near-wall clustering. The elliptic equations for potential, pressure, three velocity components, and temperature are solved using the Fourier decomposition in the streamwise coordinate and the direct reduction solution of the two-dimensional equations for the Fourier components conducted on the transformed grid ($\eta-\xi$) (see [5]). The algorithm is parallelized using the hybrid MPI-OpenMP approach. The MPI memory distribution is along the y-coordinate in the physical space and along the streamwise wavenumber in the Fourier space.

The equations always have a solution in the form of a streamwise-uniform steady flow, which we take as the base flow. For every combination of Ha and Gr, we first compute the base flow by imposing streamwise-uniformity. The linear stability analysis of this flow is conducted next. We follow the evolution of small-amplitude perturbations added to the flow. For such perturbations, we can analyze instability to individual Fourier harmonics in the framework of a linearized problem. For this purpose, the DNS code is modified by applying Fourier filtering, which separates the zero mode and the mode, stability to which is studied from the rest (see [3] for a discussion). The computations are conducted in the domain of a desired wavelength. Finally, fully 3D DNS of the flow is conducted.

3. Results

We consider fully developed flows in long ducts (fig 1). Single values Pr = 0.0321 and Re = 5000 are used in the analysis. The first of them corresponds to the LiPb eutectic alloy at temperature around 570 K [9], while the second is selected arbitrarily. The Grashof number Gr varies in the range between 10^5 and 10^9. For the Hartmann number we select the range $50 \leq \mathrm{Ha} \leq 800$. The computational domain is a duct with cross-section 2×2 and periodic inlet-exit conditions. The streamwise size of the domain is 4π for DNS and equal to the wavelength of the tested Fourier mode for stability analysis. The computational grids are determined as functions of Ha and Gr in the grid sensitivity tests. We keep at least 6 points in each Hartmann layer and at least 12 points in each Shercliff layer. The DNS requires streamwise grid step not larger than 0.1. The maximum time step providing a numerically stable solution is used. Its value varies with Ha and Gr, but never exceeds 10^{-2}.

3.1 Duct with bottom heating [12]. The summary of transverse circulation structures is shown in fig. 2. At each value of Ha, the Gr has to increase above a certain limit to overcome the suppression by the magnetic field and to cause development of secondary circulations in transverse plane. The heating also creates a profile of mean temperature linearly growing along the duct and the associated buoyancy force leads to significant top-bottom asymmetry of mean velocity [6]. The linear stability analysis shows the formation of convection rolls aligned with the magnetic field as a common feature of the flow invariably observed at Ha > 200 and sufficiently high Gr. The rolls are localized in the lower half of the duct. At high Ha, the rolls approach Q2D form, while increase of Gr, via the modification of mean flow, localizes the rolls near the heated wall (see fig. 3). Transport of the rolls by the mean flow causes high-amplitude low-frequency temperature fluctuations at walls.

The DNS results agree with the stability analysis and confirm that the secondary flow regimes can be of one of the two types depending on whether Gr is smaller or larger than a certain threshold Gr*(Ha). The low-Gr type observed at Gr < Gr* is characterized by complete domination of the instability-generated spanwise rolls and quasi-two-dimensional distributions of velocity and temperature fields. In the high-Gr flows at Gr > Gr*, the spanwise rolls are combined with a streamwise roll similar to the circulation roll in the streamwise-uniform base flow. Significant flow and variations of temperature along the magnetic field lines are present. This classification is important when we consider that 3D computations are unfeasible at the parameter values typical for a fusion reactor. It is commonly assumed that the Q2D modeling [10] can be utilized at high Ha (see [11]). Our results, however, demonstrate that even at high Ha the applicability of the model is not a-priori certain. The model is applicable to the low-Gr regimes, but would be based on incorrect assumptions and produce erroneous results at the values of Gr, at which high-Gr regimes are realized.

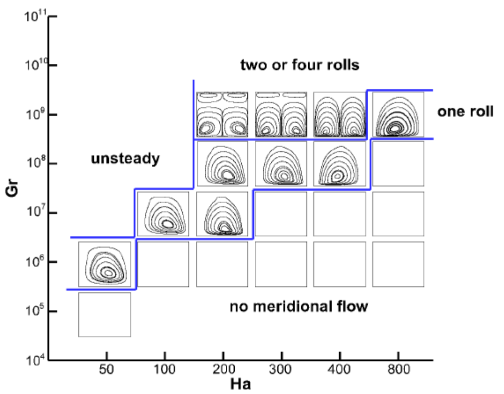

Figure 2. Bottom heating. Transverse plane circulation patterns found in computed base flows. Blank squares indicate flow regimes, in which the kinetic energy of transverse velocity components is less than 10^{-6}.

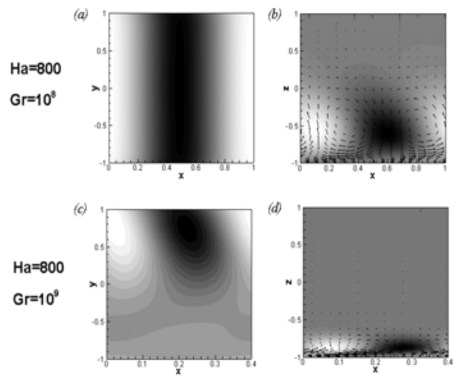

Figure 3. Bottom heating. Convection modes in the horizontal plane (left), vertical streamwise mid-plane (right), at Ha = 800 and Gr = 10^8 (upper) and Gr = 10^9 (lower). Vectors and contours show velocity and temperature perturbations, respectively.

3.1 Duct with sidewall heating [13]. Fig 4 summarizes the observed structures of the transverse circulation in the streamwise-uniform flows with sidewall heating duct. Circulations with the average kinetic energy of transverse velocity components below 10^{-6} are not shown. In the DNS analysis, only the flows at Ha = 50 and 100 with Gr = 10^9 demonstrate three-dimensionality. All the other flows are found to be laminar, steady-state, and 2D (uniform in the streamwise direction). This is fully confirmed by the stability analysis that finds no growth of perturbations at the wavelengths between 0.05π and 20π. An illustration of a 2D flow is presented in fig. 5.

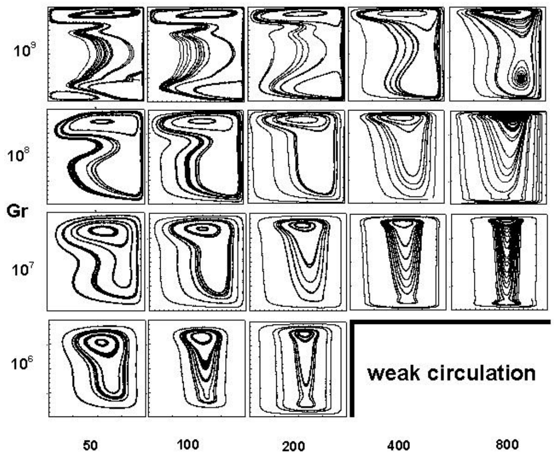

Figure 4: Sidewall heating. Transverse plane circulation patterns found in computed base flows. The flow regimes, in which the kinetic energy of transverse velocity components is less than 10^{-6} are not shown.

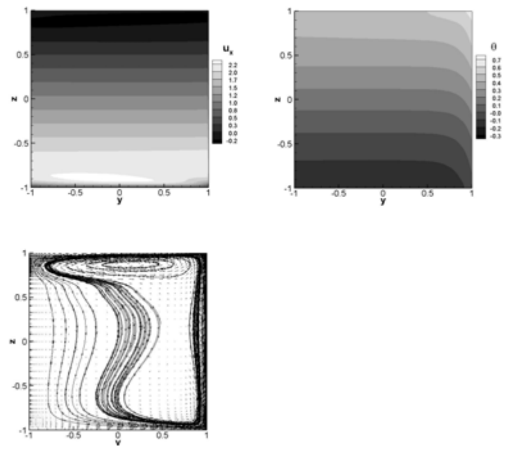

Figure 5: Sidewall heating. 2D flow at Ha = 400, Gr = 10^9. Streamwise velocity u_x, temperature θ, and vector field and streamlines of the transverse circulation u_y, (u_y, u_z) are shown.

Acknowledgments: Authors thank Dmitry Krasnov for continuing assistance with the numerical model. Financial support was provided by the US NSF (Grant CBET 1232851).

4. References

[1] Smolentsev, S., Moreau, R., Bühler, L. & Mistrangelo, C. MHD thermofluid issues of liquid-metal blankets: Phenomena and advances. Fusion Eng. Design **85** (7-9), 1196 -1205 (2010).
[2] Davidson, P. A., An Introduction to Magnetohydrodynamics, (2001).
[3] Zikanov, O., Listratov, Ya. & Sviridov, V. G. Natural convection in horizontal pipe flow with strong transverse magnetic field, J. Fluid Mech. **720**, 486–516 (2013).
[4] Genin, L. G., Zhilin, V. G., Ivochkin, Y. P., Razuvanov, N. G., Belyaev, I. A., Listratov, Y. I. & Sviridov, V. G. Temperature fluctuations in a heated horizontal tube affected by transverse magnetic field. Fundamental and Applied MHD, Proc. the 8th PAMIR Conference, pp. 37–41. Borgo, Corsica, France (2011).
[5] Krasnov, D., Zikanov, O. & Boeck, T. Comparative study of finite difference approaches to simulation of magnetohydrodynamic turbulence at low magnetic Reynolds number, Comp. Fluids. **50**, 46–59 (2011).
[6] Morinishi, Y., Lund, T. S., Vasilyev, O. V. & Moin, P. Fully conservative higher order finite difference schemes for incompressible flow, J. Comp. Phys. **143**, 90–124 (1998).
[7] Ni, M. J., Munipalli, R., Huang, P., Morley, N. B., Abdou, M. A. A current density conservative scheme for incompressible MHD flows at low magnetic Reynolds number. Part I: On a rectangular collocated grid system, J. Comp. Phys. **227,** 174204 (2007).
[8] Zikanov, O. Essential computational fluid dynamics. Wiley (2010).
[9] Mas de les Valls, E., Sedano, L.a., Batet, L., Ricapito, I., Aiello, a., Gastaldi, O. &Gabriel, F. Lead-lithium eutectic material database for nuclear fusion technology, J. Nucl. Mater. 376 (3), 353–357 (2008).
[10] Sommeria, J. & Moreau, R. Why, how and when MHD-turbulence becomes two-dimensional, J. Fluid Mech. **118**, 507–518 (1982).
[11] Vetcha, N., Smolentsev, S., Abdou, M. & Moreau, R. Study of instabilities and quasi-two-dimensional turbulence in volumetrically heated magnetohydrodynamic flows in a vertical rectangular duct, Phys. Fluids **25** (2), 024102 (2013).
[12] Zhang, X., Zikanov, O. Mixed convection in a horizontal duct with bottom heating and strong transverse magnetic field (submitted to J. Fluid Mech. (2014).
[13] Lv, X., Zikanov, O. Mixed convection in horizontal duct flow with transverse magnetic field and heating of side wall (submitted to Phys. Fluids (2014).