A SIMULINK MODELIZATION OF AN INDUCTIVE MHD GENERATOR

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Abstract: In this paper an inductive MHD generator is proposed, which aims to overcome the typical drawbacks of the conventional MHD generators, such as the need to operate at high temperatures and to have superconducting coils for the generation of the external magnetic field. The conceptual idea is described and the energy conversion process is illustrated by means of a finite element method model, which describes the fluid dynamic aspects, joined to a Simulink® model, which describes the electromagnetic phenomena involved in the process. Some results of analysis are reported, putting in evidence the influence of design parameters of the machine.

Index Terms: inductive MHD generator, static energy conversion, numerical modeling

1. Introduction

Interest in magneto hydrodynamic (MHD) power generation was originally stimulated by the observation that the interaction of a plasma with a magnetic field could occur at much higher temperatures than were possible in a rotating mechanical turbine [1]. In spite of this, a number of drawbacks have limited a wide use of this technology that, after have been deeply studied for at least three decades, at the end of the past century it has been quite completely abandoned. Such technological limits concern essentially the difficulty to maintain a high level of ionization of the gas at low temperature, the needed of an intense magnetic field (> 5 T) which has to be generated by means of superconducting coils, the deterioration of electrodes by means of which the generated electrical current is extracted by the plasma. Even, thanks to the development in different research topics, great improvement have been obtained in the recent years on superconductors [3] and ionised gases [2], the complexity of the apparatus pushes to prefer other competitors, such as turbogas. Furthermore, until now no efficient solution has been found concerning the duration of electrodes [4]. In [5], an inductive gas fed generator has been proposed that allows one to overcome the above mentioned drawbacks of the conventional MHD generators, but at the same time it holds all their advantages, such as the static conversion of the energy and the capability to work with very high temperatures. This is obtained by substituting the external magnetic field with an electrostatic field, and by using eddy currents rather than electrodes to retrieve the generated electrical current. A key aspect of the proposed generator is the fact that the charge carriers are separated in order to obtain two separated flows of unbalanced charge carriers. Due to the fact that the ionization of the gas is time varying, also the flows of charge carriers vary during the time, so that they can give rise to an induced electromotive force in a magnetically coupled coil. The paper is organized as follows. In Section 2 the physics involved in the process is described. In Section 3 the energy conversion is qualitatively described and is treated more in details in Sections 4, 5 and 6 by means of its mathematical model. In Section 7 the Simulink® model and some results are reported. The last section provides some conclusions.

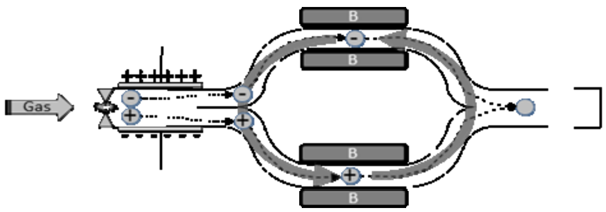

Figure 1: Inductive MHD generator functional scheme.

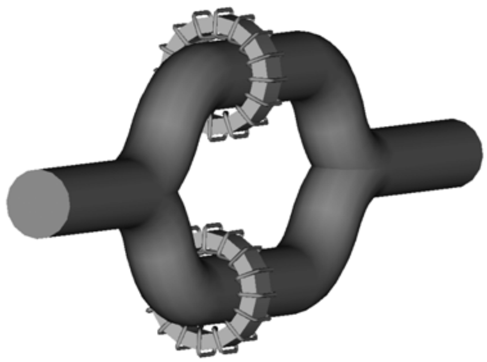

Figure 2: Inductive MHD generator layout.

2. System description

The working principle of the generator studied is quite simple (see Fig. 1). A high speed gas enters in the duct where in the first part it is ionized by a pulsating electric discharge. The charges of different sign are separate by an external electric field. Downstream of this process the flow is divided in two currents, one having an excess of positive charge, the other one of negative charge. In this way a time variant circulating electric current is generated, which involves the two branches of the duct. The current generated induces an electromotive force in two toroidal coils wrapped around the duct. The ionization of the fluid is obtained by means of electrodes powered by a high voltage generator to generate an electric pulsating discharge. After that the charges have been generated they have to be separated by means of an external electrical field. To this end, a capacitor externally powered by a DC high voltage generator is placed around the duct. In correspondence of the toroidal coils (Figs. 1 and 2) the energy conversion process is carried out, by inducing an electromotive force.

3. Energy conversion process

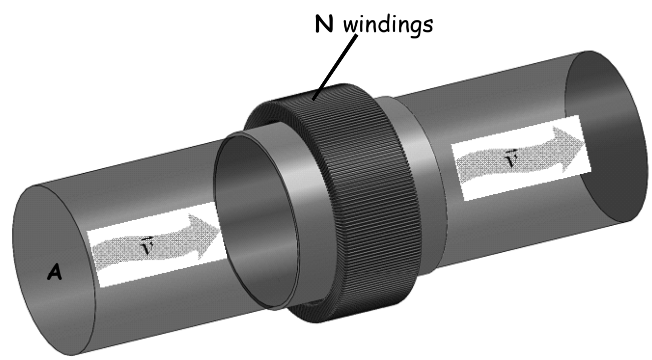

Figure 3: Detail of the generator corresponding to the section where the energy conversion takes place.

The energy conversion occurs in correspondence of the toroidal coils. Here the generator works as a conventional transformer, where the primary is represented by the plasma in the duct, while the secondary winding is the toroidal coil (Fig. 2). The electric current flowing in the electric load gives rise to an armature reaction, which in turn slows down the charge carriers into the plasma and then causing a gas expansion. The enthalpy jump one can obtain in the conversion depends on a great number of design parameters: the velocity of the gas, the geometry of the core, the number of coils of the windings and the section of the wire, and so on. In this work, the variation effect of some of these parameters has been investigated.

4. System modeling

First of all, the MHD problem has been split into a fluid dynamic part and the electromagnetic part. The former part is solved by means of the finite element analysis [5], while the latter one is solved by assigning to the charge carriers the same motion of the neutral particles. Such assumption can be justified because the charge carriers relative concentration is very low,

therefore they cannot affect the average motion of the fluid. Furthermore in analyzing the electromagnetic aspects, we consider a mono-dimensional model of the flow. This means to neglect the unavoidable variations of the flow across the section of the duct.
In Fig. 3 a particular of one branch of the duct is shown. The charges participate to the motion of the neutrals. In addition, other two components of the velocity have to be considered: one due to the repulsive force between charges of the same sign, and one due to the reaction of the armature. The repulsive force among charges of the same sign strongly affects the efficacy of the transfer of energy, because the more the charges are distributed along the axis of the duct, the less will be the induction produced in the coil. To reduce the effect of the reciprocal repulsive force of the charges, a supplementary electrical field is applied to the flow by means of a metallic sleeve around the duct (Fig. 3). Therefore, four components of the velocity have to be taken into account in calculating the distribution of the charges during the time:

$$v(x,t) = v_T + \mu_E \left[E_\rho(x,t) + E_C(x) + E_R(x,t) \right] \tag{1}$$

where v is the velocity of the particle of charge in the position x at the time t, v_T is the dragging component which is constant, μ_E is the electrical mobility of the charge carriers, E_ρ is the electrical field due to the charges distribution, E_C is the electrostatic field provided by the sleeve, E_R is the electrical field due to the armature reaction. At the beginning of the branch, the charge has a Gaussian distribution, but in evolving in the duct such distribution changes, because of the electrical fields E_C and E_R, becoming asymmetric. In order to take into account such asymmetry, the combination of two Maxwellian distributions has been used to model the charge density distribution:

$$\rho = \left(1 - \frac{x}{D}\right) \cdot \frac{4Q_{tot}}{\sqrt{\pi}} \cdot \frac{x^2}{x_m^3} \cdot e^{-\frac{x^2}{x_m^2}} + \frac{x}{D} \cdot \frac{4Q_{tot}}{\sqrt{\pi}} \cdot \frac{(D-x)^2}{(D-x_m)^3} \cdot e^{-\frac{(D-x)^2}{(D-x_m)^2}} \tag{2}$$

where $\rho(x)$ is the charge density, Q_{tot} is the total charge of the cloud, x_m is the position along the axis of the duct of the maximum of charge density.

5. Electrical fields acting on the charge distribution

Given the charge distribution, it is possible to calculate the electric field $E_\rho(x,t)$ in the generic point x on the axis of the duct and then the component of velocity due to such field in (1). The $E_C(x)$ can be determined by solving an electrostatic problem. The $E_R(x,t)$ is the armature reaction. In the ideal case the magnetic field generated by the current circulating in the coil is completely contained within the core, but it gives rise to a vector potential in the duct, which derivative is proportional to an electric field opposite to the motion of the charges. In order to calculate said vector potential, we can use the same mathematical steps which allow to define the Biot-Savart law, obtaining the following relation:

$$d\vec{A} = \Phi \cdot \frac{d\vec{l} \times \vec{r}}{r^3} \tag{3}$$

Eq. (3) is formally identical to the Biot-Savart equation, as a consequence we will obtain that the force lines of electrical field generated by the flux Φ have the same shape of the magnetic field generated by a turn of electrical current, and it allows one to calculate the electric field along the axis of the duct, which represents the armature reaction. For symmetry the resultant potential vector $\vec{A}(x)$ is directed as the axis of the duct, so that in integrating the contributions $d\vec{A}$ due to the flux Φ along the torus, we have to consider their projection along the axis x:

$$\vec{A}(x)=\int_0^{2\pi}\Phi\cdot\frac{d\vec{l}\times\vec{r}}{r^3}=\left(\int_0^{2\pi}\frac{\Phi\cdot\sqrt{R^2+x^2}}{\left(\sqrt{R^2+x^2}\right)^3}\cdot\cos\varphi\cdot Rd\theta\right)\cdot\vec{i}=\frac{\Phi\cdot\sqrt{R^2+x^2}}{\left(\sqrt{R^2+x^2}\right)^3}\cdot\frac{R}{\sqrt{R^2+x^2}}\cdot R\int_0^{2\pi}d\theta=\frac{2\pi\cdot\Phi\cdot R^2}{\left(\sqrt{R^2+x^2}\right)^3}\cdot\vec{i} \tag{4}$$

Finally, it is possible to calculate the armature reaction as the time derivative of the vector potential:

$$\vec{E}_R=-\frac{\partial\vec{A}}{\partial t}=-\frac{2\pi\cdot R^2}{\left(\sqrt{R^2+x^2}\right)^3}\cdot\frac{d\Phi}{dt}\cdot\vec{i} \tag{5}$$

The (5) can be substituted in (1) to calculate the velocity of each charge of the distribution during the time, and then to determine the evolution of the charge distribution.

6. Power transfer to the electric load

The energy conversion process can be described by resorting to the Ampère equation, written for the plane which is orthogonal to the axis of the duct and which is symmetry plane for the torus:

$$\oint_L \vec{H}\cdot d\vec{l}=I-N\cdot i+\varepsilon\frac{d}{dt}\iint_S\left(E_\rho+E_R\right)dS \tag{6}$$

where H is the magnetic field evaluated along the torus of length L, I is the plasma electric current flowing in the duct, N is the number of turns of the secondary winding, i is the electric current circulating in such winding, ε is the permittivity. As it can be seen, field E_C does not appear in the displacement current term, because it is null in the origin for symmetry reasons. On the other hand, it implicitly affects the current I by influencing the charge distribution $\rho(x,t)$. Let us now to separately consider the terms of the (6). The circulation integral at the left-hand side can be so re-written:

$$\oint_L \vec{H}\cdot d\vec{l}=\oint_L\frac{\Phi}{\mu\cdot\Sigma}dl=\frac{\Phi L}{\mu\cdot\Sigma} \tag{7}$$

Table 1 - Design parameters of the MHD generator

Physical quantity	Symbol	Value [units]
Radius of duct	R	0.20 [m]
Permittivity	ε	$10^{-9}/36\pi$ [F·m^{-1}]
Charge in the duct		10^{-5} [C]
Permeability	μ	0.0628 [H·m^{-1}]
Gas velocity	v_T	150-200 [m·s^{-1}]
Length of the sleeve		0.20 [m]
Charge on the sleeve		10^{-5} [C]
Ions mobility	μ_E^+	$2\cdot10^{-4}$ [m^2·V^{-1}·s^{-1}]
Electrons mobility	μ_E^-	$3\cdot10^{-2}$ [m^2·V^{-1}·s^{-1}]
Length of the torus	L	1.4 [m]
Cross area torus	Σ	0.02 [m^2]
Number of coil turns	N	2 000
Electric load	R_{el}	100-2 000 [Ω]

The current I in the right-hand side, is given by the concentration of charge ρ given by the (2), evaluated in the origin and then multiplied by the velocity, given by the (1):

$$I=\rho(t)\big|_{x=0}\cdot\left\{v_T+\mu_E\left[E_\rho(t)+E_R(t)\right]_{x=0}\right\} \tag{8}$$

The current i circulating in the winding is related to both voltage v_2 and electric load R_{el}. Let us assume, for sake of simplicity, the electric load to be resistive. Then the current i can be so expressed:

$$i=\frac{v_2}{R_{el}}=\frac{1}{R_{el}}\cdot\frac{d\left(N\cdot\Phi\right)}{dt}=\frac{N}{R_{el}}\cdot\frac{d\Phi}{dt}$$

and then the second term of the right-hand side of (6) becomes:

$$-N \cdot i = -\frac{N^2}{R_{el}} \cdot \frac{d\Phi}{dt} \tag{9}$$

Finally, for the third term, corresponding to the displacement current is:

$$\varepsilon \frac{d}{dt} \iint_S \left(E_\rho + E_R\right) dS = \varepsilon \frac{d\left(E_\rho + E_R\right)\big|_{x=0}}{dt} \iint_S dS = \varepsilon \cdot S \cdot \frac{d\left(E_\rho + E_R\right)\big|_{x=0}}{dt} \tag{10}$$

By substituting (7-9) in (6) we obtain an equation of the second order in the unique variable Φ.

7. Results

The equations of the model have been solved by using the Simulink® environment. In Table 1 the design parameters of the generator are reported. Among that, two parameters, namely velocity of the working gas and electrical load has been swept in order to evaluate the corresponding sensitivity of voltage and transferred power. The results show the high velocities and high loads allow to obtain higher power. As a feature to evaluate the suitability of the gas velocity, we can check if the whole cloud of charges crosses the toroidal winding in spite of the mutual repulsive force among the charges. The simulations show that a gas velocity of 150 m·s^{-1} is sufficient to drag the charges beyond the torus. This time depends on both gas velocity and total charge, therefore it has to be stated on the basis of design parameters. The duration of the pulse of the induced voltage indicates the minimum time interval between two consecutive discharges.

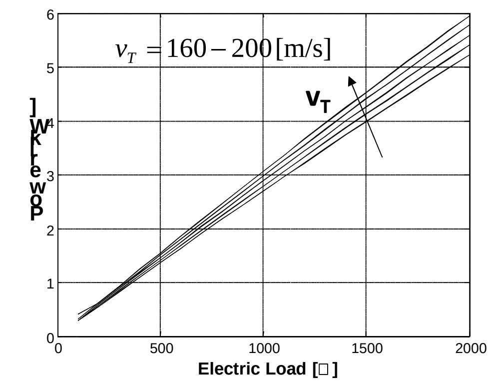

Figure 6: Trend of power with respect to load and velocity.

8. Conclusion

A conceptual study of an inductive MHD generator has been presented, which aims to overcome the typical drawbacks of the conventional MHD generators, such as the need to operate at high temperatures and to have superconducting coils for the generation of the external magnetic field. The process is described and analyzed by means of a mathematical model that has been solved by using the Simulink® environment. A parametric evaluation of the velocity of the working gas and electrical load has been done in order to evaluate the corresponding sensitivity of voltage and transferred power.

9. References

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