Impact of surface viscosity upon an annular magnetohydrodynamic flow

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Using the matched asymptotic expansion method based on the small parameter $1/Ha$, this paper addresses an original analytical coupling between surface rheology of *e.g.* a gradually oxidizing liquid metal surface, and a supporting annular MHD flow. It is shown that the level of surface viscosity drives the electrical activation of the Hartmann layers, heavily modifying the MHD flow topology and leading to the emergence of a Lorentz force, for which interaction with the flow is not classical.

I. INTRODUCTION

In many industrial applications, there is rising concern over how to model the interactions between an electro-conductive fluid and a second phase, when both of them are subjected to an external magnetic field. Typically, the issue of how a magnetohydrodynamic (MHD) flow with a liquid/gas interface is affected when oxidation occurs is of prime interest. It potentially affects many fields, such as metallurgy (stirring by bubble plumes in reactors), microelectronics (MHD-driven metal cooling processes [1]), or nuclear fusion technology (two-phase MHD issues with the breeder blanket based cooling loop [2]).

To our knowledge, little is actually known about surface rheology of MHD flows, *e.g.* when a liquid metal is progressively contaminated through oxidation processes. On the one hand, the viscoelastic properties of liquid metals have been experimentally investigated [3], highlighting radically different mechanical behavior characteristics that depend on the level of oxidation, but these works are not related with MHD. On the other hand, the MHD of single-phase laminar flows exposed to strong uniform magnetic fields has been studied extensively for many years, for numerous layouts [4–6], but the oxidation occurring at the liquid surface of a free-surface MHD flow is most of the time not taken into account. Consequently, the two-phase MHD of a more or less oxidized interface coupled with a pure liquid metal bulk is worthy of investigation, which is, to our knowledge, an original approach coupling both MHD and surface rheology.

II. OUTLINES

The system under consideration is displayed in Fig. 1. The interest in this configuration, which is inspired by the deep channel viscometer [7], is that it is likely to generate strong velocity gradients along the $\vec{e}_z$ axis, whereas these gradients develop preferentially along the $\vec{e}_r$ axis in the more conventional case of the Taylor-Couette layout. As shown later in equation (8), the coupling term between the interface and the sub-phase flow brings a $\partial v_\theta/\partial z$ term into play,

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where v_θ is the azimuthal sub-phase velocity; the resulting shearing is expected to be more significant than in the Taylor-Couette layout, and the effects of varying boundary conditions at the liquid surface may be more easily highlighted. By considering the vertically applied magnetic field $\vec{B}_0$, it is shown in equation (1) that the generating term for the azimuthal magnetic induction B_θ is $\partial v_\theta / \partial z$ as well, which explains the interest in favoring gradients along $\vec{e}_z$.

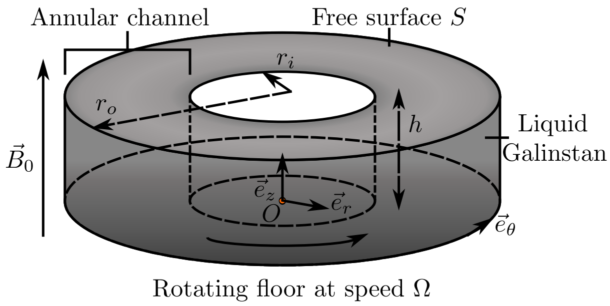

FIG. 1. Geometry under consideration.

The aim of this paper is to highlight the competitive effects between surface shearing and a strong transverse uniform magnetic field, especially with the emergence of an electrically active Hartmann layer along a gradually denser liquid surface, *e.g.* under oxidation processes.

III. MATHEMATICAL MODEL

A. Bulk flow

Using the Maxwell and Navier-Stokes equations, and assuming that the Reynolds number is low enough so that the inertial effects can be neglected, we can derive the following set of equations that govern the MHD problem and traduce the balance between electromagnetic and viscous effects (see *e.g.* [5,6]), where the superscript * refers to non-dimensional quantities:

$$\frac{\partial^2 B_\theta^*}{\partial r^{*2}} + \frac{1}{r^*}\frac{\partial B_\theta^*}{\partial r^*} - \frac{B_\theta^*}{r^{*2}} + \frac{\partial^2 B_\theta^*}{\partial z^{*2}} + Ha\frac{\partial v_\theta^*}{\partial z^*} = 0, \tag{1}$$

$$\frac{\partial^2 v_\theta^*}{\partial r^{*2}} + \frac{1}{r^*}\frac{\partial v_\theta^*}{\partial r^*} - \frac{v_\theta^*}{r^{*2}} + \frac{\partial^2 v_\theta^*}{\partial z^{*2}} + Ha\frac{\partial B_\theta^*}{\partial z^*} = 0, \tag{2}$$

with $r^* = r/h$, $z^* = z/h$, $v_\theta^* = v_\theta/\hat{V}$, $B_\theta^* = B_\theta/\hat{B}$, $Ha = B_0 h\sqrt{\sigma/\eta}$, and with $\hat{B} = \mu\hat{V}\sqrt{\sigma\eta}$, μ being the magnetic permeability of the fluid, and $\hat{V} = h\Omega$, so that $v_\theta^*(r^*, z^* = 0) = r^*$ for the rotating floor. The associated boundary conditions are written as follows (see *e.g.* [4] for the condition $B_\theta^* = 0$ all around the bulk flow):

$$v_\theta^*\,(r^* = \frac{r_i}{h}, z^*) = 0\,, \qquad B_\theta^*(r^* = \frac{r_i}{h}, z^*) = 0, \tag{3}$$

$$v_\theta^*\,(r^* = \frac{r_o}{h}, z^*) = 0\,, \qquad B_\theta^*(r^* = \frac{r_o}{h}, z^*) = 0, \tag{4}$$

$$v_\theta^*\,(r^*, z^* = 0) = r^*\,, \qquad B_\theta^*(r^*, z^* = 0) = 0, \tag{5}$$

$$B_\theta^*\,(r^*, z^* = 1) = 0\,, \tag{6}$$

$$v_\theta^*\,(r^*, z^* = 1) = v_{\theta S}^*(r^*). \tag{7}$$

B. Surface flow

The boundary condition (7) brings a new unknown into play, which is the surface velocity $v_{\theta S}$. This stands as the first term of the two-way coupling between the surface and MHD bulk flow equations and the liquid surface conditions. The latter can be derived from a momentum balance written on an elementary heterogeneous volume that straddles a liquid surface of zero thickness, and by introducing the relevant surface "excess" viscous shear viscosity, modelled with a Boussinesq-Scriven constitutive law (see *e.g.* [8] for further details):

$$Bo\left(\frac{\mathrm{d}^2 v_{\theta S}^*}{\mathrm{d}r^{*2}}+\frac{1}{r^*}\frac{\mathrm{d}v_{\theta S}^*}{\mathrm{d}r^*}-\frac{v_{\theta S}^*}{r^{*2}}\right)=\left.\frac{\partial v_\theta^*}{\partial z^*}\right|_{z^*=1}. \tag{8}$$

The Boussinesq number $Bo=\eta_S/\eta h$ describes the balance between bulk (η is the Newtonian bulk shear viscosity) and surface (η_S is the surface excess shear viscosity along the liquid surface) viscous shearing. To solve for this jump of momentum balance (JMB), the following two Dirichlet end-point boundary conditions are required:

$$v_{\theta S}^*\left(r^*=\frac{r_i}{h}\right)=0,\quad v_{\theta S}^*\left(r^*=\frac{r_o}{h}\right)=0. \tag{9}$$

C. Two-way coupling

The overall coupling process between the sub-phase flow v_θ^* and the surface flow $v_{\theta S}^*$ stems from a somewhat tedious calculation process, which consists of a matched asymptotic expansion for the bulk solution and the determination of a Green function for the surface velocity. Details of calculations are not detailed here, but they are fully available in [9].

IV. ASYMPTOTIC RESULTS AND INTERPRETATION

After relevant additional post-processing scaling [9] traduced by the * superscript, the choice is made to discuss only the asymptotic cases $Bo \gg Ha$ and $Bo \ll Ha$, as shown in Fig. 2. The reader may refer to [9] for further physical insight. The aim here is to indicate two radically different MHD regimes, and to see how the overall flow topology can be strongly modified.

When considering the velocity, the topology evolves from an exclusively radial dependence, with velocity gradients that are perfectly aligned with $\vec{e}_r$ (apart from those near the walls) for $Ha \gg Bo$ in figure 2a), to a motionless configuration for $Ha \ll Bo$ in figure 2b), where the velocity is mainly concentrated near the bottom right corner. This behavior is linked to significant physical phenomena. For figure 2a), the electromagnetic blocking is responsible for the dissipation of the secondary vortices, and for the fluid alignment with the rotating floor (rigid body motion). For figure 2b), the motionless topology is partly because of an inert interface for $Bo \gg Ha$, which imposes matching with a vanishing velocity at the interface, but other mechanisms are also brought into play, and are highlighted through analysis of the electromagnetic quantities.

The latter are obviously confined in the Shercliff layers when $Bo \ll Ha$, with two electrical loops near the side-walls, as shown in figures 2c) and 2e), with the core and the Hartmann layers

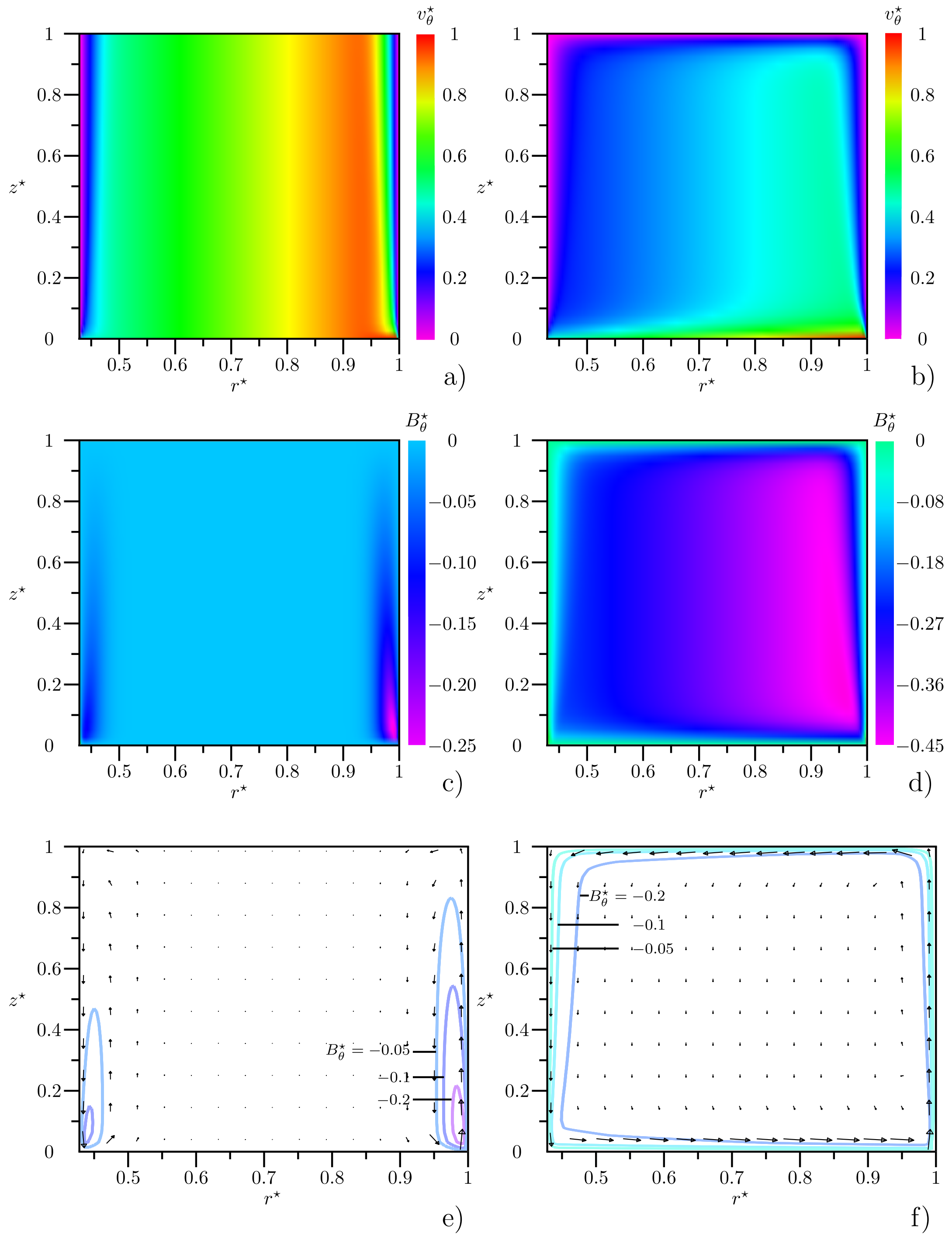

FIG. 2. Bulk MHD quantities for the two extreme cases $Bo \ll Ha$ (left-hand column) and $Bo \gg Ha$ (right-hand column). a) and b) represent $v_\theta^\star$, c) and d) represent $B_\theta^\star$, and e) and f) represent the vector current density $\vec{j}^\star$ with $B_\theta^\star$ streamlines. For a given velocity $\Omega = 0.25\,\mathrm{rpm}$, with $r_o = 7\,\mathrm{cm}$, $h = 1\,\mathrm{cm}$, $\sigma = 2.3 \times 10^6\,\mathrm{S{\cdot}m^{-1}}$ and $\eta = 2.4 \times 10^{-3}\,\mathrm{N{\cdot}m^{-1}}$, $\bar{J} = 4.1 \times 10^2\,\mathrm{A{\cdot}m^{-2}}$ for $Ha = 30$ (right-hand side) and $6.8 \times 10^2\,\mathrm{A{\cdot}m^{-2}}$ for $Ha = 50$ (left-hand side). $\vec{j}^\star$ is log-scaled by the magnitude $\exp\left(\left(\ln\left(\|\vec{j}^\star\|/\bar{J}\right)\right)/(1+p)\right)$, $p = 3$ for e) and $p = 1$ for f).

making a negligible contribution. The topology is dramatically different for $Bo \gg Ha$, where the core seems to be more involved, but the Hartmann layers in particular are now electrically active, with the setting of an ”electric bridge” between the two side layers that is established through the Hartmann layers, as shown in figures 2d) and 2f). This is linked to the velocity topology mentioned earlier, with no particular gradients of $v_\theta^\star$ along the $\vec{e}_z$ axis in the case where $Ha \gg Bo$, other than near the side walls, whereas these gradients arise elsewhere and especially arise near the interface in the case where $Ha \ll Bo$. Depending on the regime, this means that the generation term of $B_\theta^\star$ in equation (1) either exists or does not, which could explain the Hartmann layers are electrically active or not.

Note also that the bottom Hartmann layer is only activated when the top layer is activated, whereas the ”dynamic” configuration remains the same near the rotating floor. This is because of the current continuity equation $\nabla \cdot \vec{j} = 0$, which causes the electric current to close up inside the fluid. When the Hartmann layer at the top is activated, the current then flows across the boundary layers.

V. CONCLUSION

The competitive effects of surface rheology and supporting annular MHD flow have been highlighted: surface rheology is indeed found to monitor the generation of the Hartmann layers, and therefore a change in the topology of the electrical circuit, which dramatically affects overall MHD core flow.

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