SPACE TRIPS SUMMER SCHOOL

Theme 9: SPACE

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Coupling MHD/TA

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Introduction

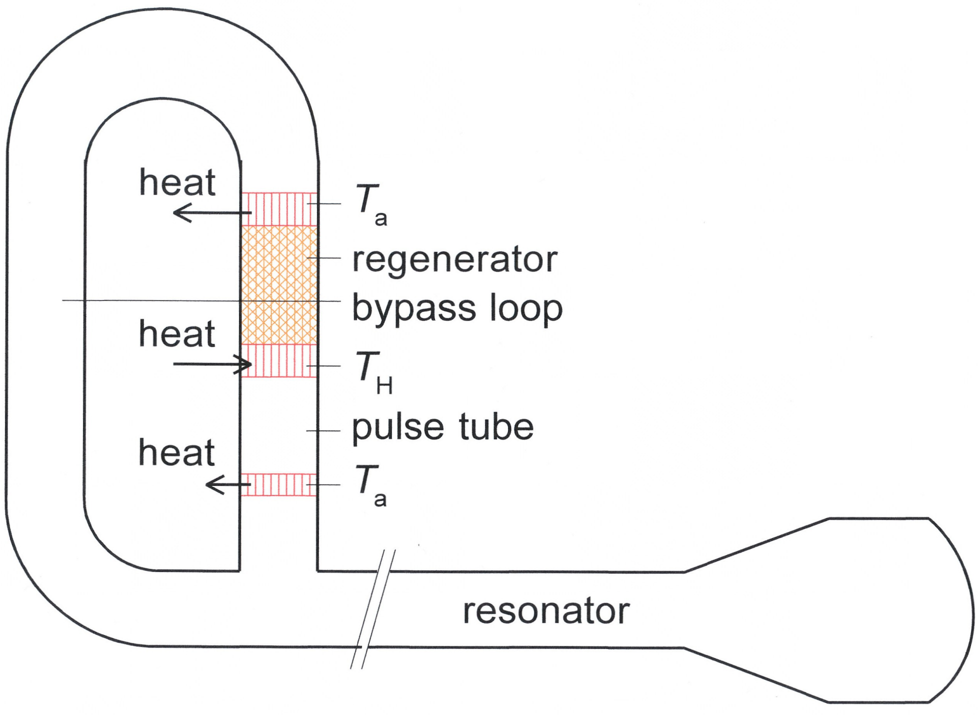
- Thermo-acoustic (TA) engines have been studied for the deep space exploration program:
  - Need of transforming heat produced by radioactive sources into electricity;
  - High efficiency rate;
  - Long service time (more than 10 years);
- TA engines produce only sound wave;
  - MHD converters can transform mechanical energy into electricity;
- A TA-MHD engine will not contain any moving parts, except for liquid sodium in the MHD converter

Introduction (cont.)

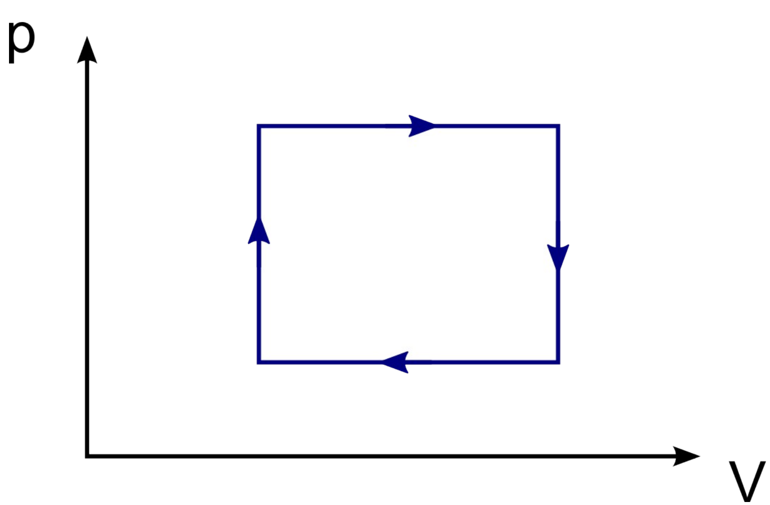
- Linearized equations systems are used to model both TA engines and MHD converters;
  - Wave (pressure, volume flow rate amplitudes) and temperature distributions;
- A full solution for TA-MHD engines requires coupling between TA and MHD equations

Basics of a traveling-wave TA engine

Analogy with a Stirling engine: a sound wave instead of pistons



Basic scheme of a traveling-wave TA engine. *Source: Wikipedia*

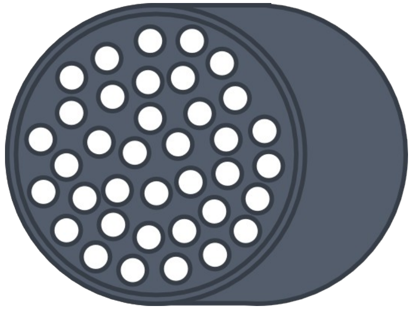


Idealized Carnot cycle

Main parts of a TA engine

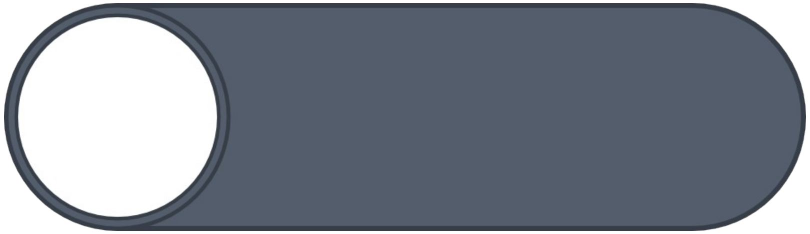
- Regenerator;

- Thermo-buffer;
- Heat exchanger;



- Duct

  Compliance,
  inertance



Equations governing the TA systems

- Navier-Stokes equation;
- Mass conservation law;
- Gas state equation;
  - Ideal gas;
- Enthalpy transfer equation

Assumptions and simplifications

- Small-amplitude pressure fluctuations;
- Convection is neglected;
- Harmonic fluctuations

$$p(x,t) = p_0 + Re\left[p_1(x)e^{i\omega t}\right]$$

$$u(x,y,z,t) = Re\left[u_1(x,y,z)e^{i\omega t}\right]$$

$$U_1(x) = \iint\limits_A u_1(x,y,z)ds$$

$$T(x,t) = T_0(x) + Re\left[T_1(x)e^{i\omega t}\right]$$

Duct equations

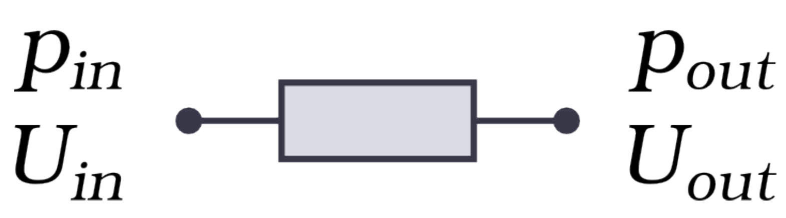
- Users Guide of DeltaEC, version 6.3b11;
- 1D equations for pressure and volume flow rate amplitudes

$$\frac{dp_1}{dx} = -\frac{i\omega\rho_m}{(1-f_\nu)A}U_1$$

$$\frac{dU_1}{dx} = -\frac{iA\omega}{\rho_m a^2}\left(1 + \frac{\gamma - 1}{1+\epsilon_s}f_k\right)p_1$$

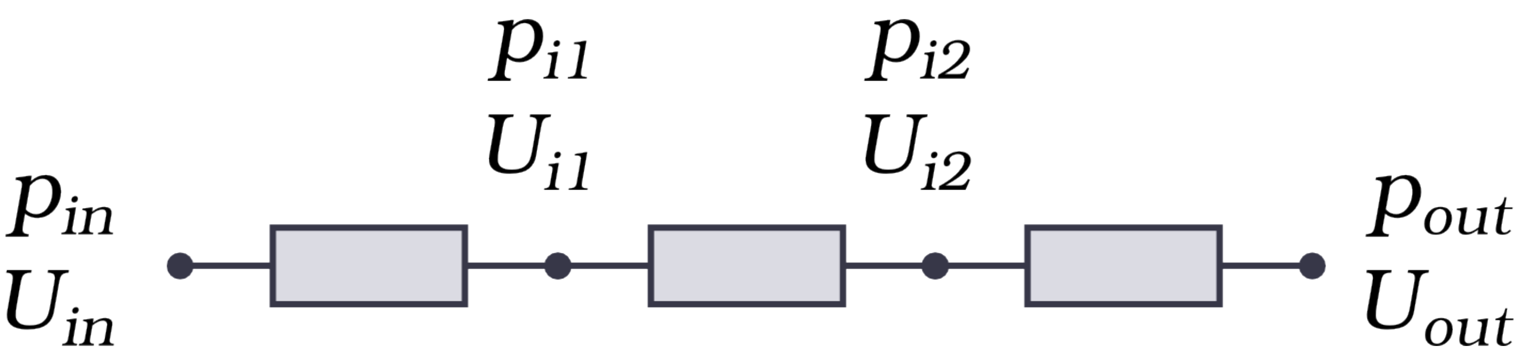
Integration of the duct equations

- Straightforward integration (Runge-Kutta methods);
- Output values are determined only by the input values and chosen frequency ω



Chaining equations

- Output of one element can be input for another;
- Final output is determined only by the input values and the frequency ω



Equations for the regenerator and thermo-buffer tube

- Non-linear temperature equation derived from the energy conservation law

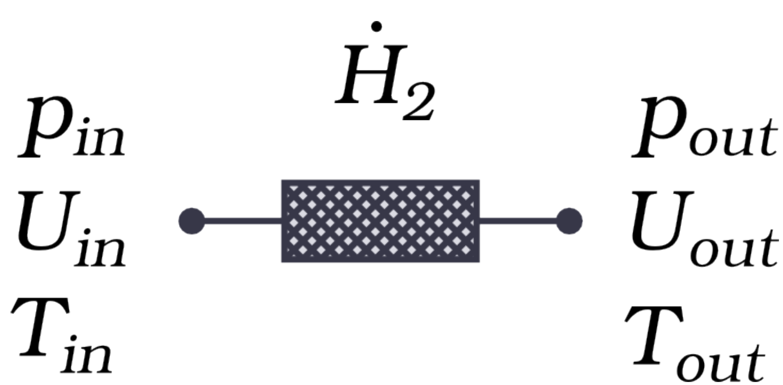
$$\frac{dp_1}{dx} = -\frac{i\omega\rho_m}{(1-f_\nu)A_g}U_1$$

$$\frac{dU_1}{dx} = -\frac{i\omega A_g}{\rho_m a^2}\left(1 + \frac{(\gamma-1)f_k}{1+\epsilon_s}\right)p_1 + \\ + \frac{\beta(f_k - f_\nu)}{(1-f_\nu)(1-\sigma)(1+\epsilon_s)}\frac{dT_0}{dx}U_1$$

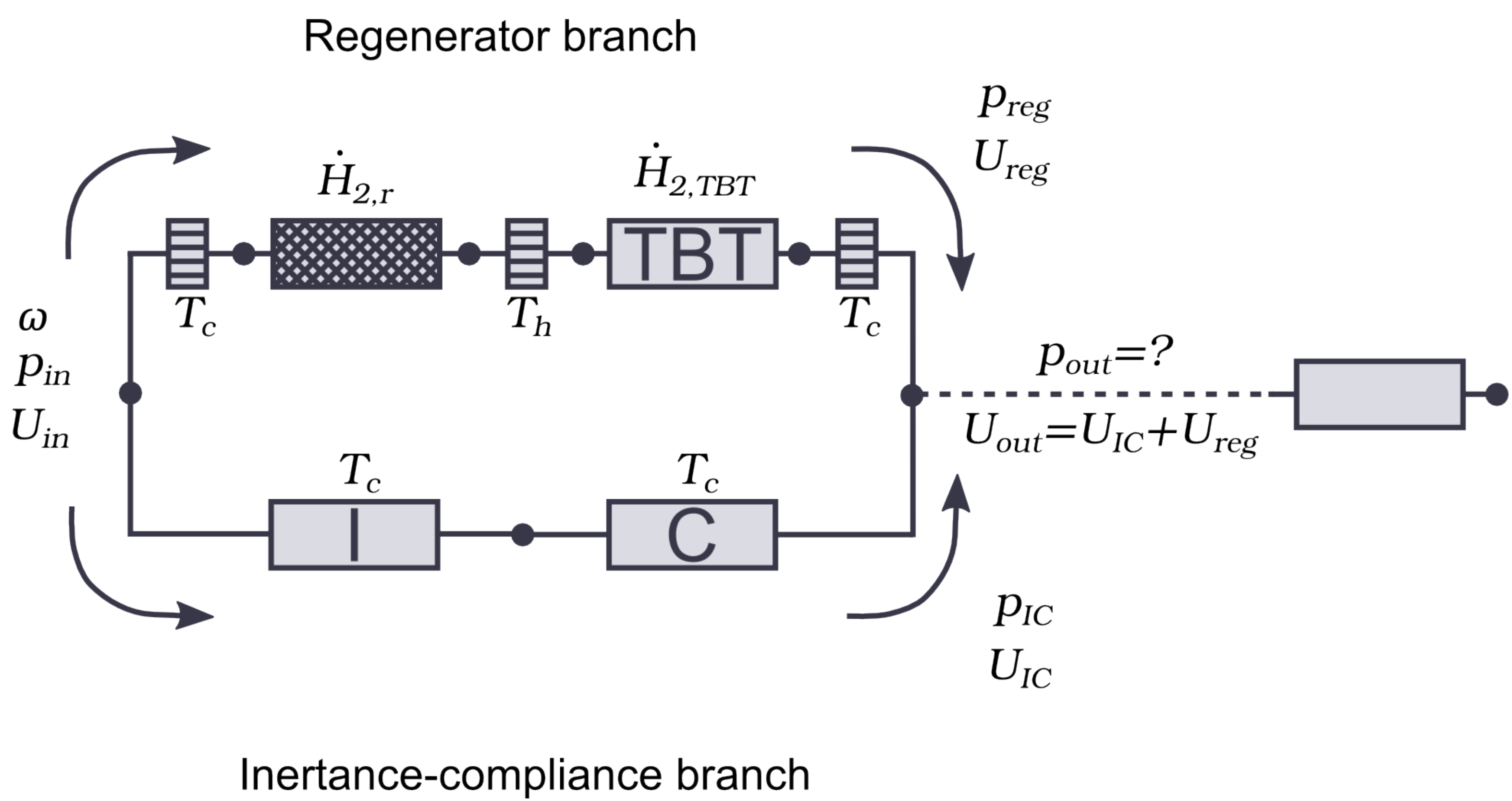
$$\frac{dT_0}{dx} = \frac{\dot{H}_2 - \frac{1}{2}Re\left[p_1\tilde{U}_1\left(1 - \frac{T_0\beta(f_k - \tilde{f}_\nu)}{(1+\epsilon_s)(1+\sigma)(1-\tilde{f}_\nu)}\right)\right]}{\frac{\rho_m c_p |U_1|^2}{2\omega A_g(1-\sigma)|1-f_\nu|^2}Im\left[\tilde{f}_\nu + \frac{(f_k - \tilde{f}_\nu)(1+\epsilon_s f_\nu/f_k)}{(1+\epsilon_s)(1+\sigma)}\right] - A_g\kappa - A_s\kappa_s}$$

Integration of the regenerator and thermo-buffer tube equations

- A set of first order equations;
  - Numerical integration with Runge-Kutta method (for example);
- Output temperature is determined by inputs and by an unknown enthalpy flow rate;
- Run calculations with different enthalpy flow rates, until such a value is found that ensures the required output temperature;
    - Newton's method for root finding;
- Matlab, octave, ...



Finding a solution of TA equations for a traveling wave engine



Finding a root

- Consider two variables and two functions:

$$X_1, X_2$$

$$f_1(X_1, X_2), f_2(X_1, X_2)$$

- Find such variable values which ensure that functions are both zero:

$$X_1^*, X_2^*$$

$$f_1(X_1^*, X_2^*) = 0$$

$$f_2(X_1^*, X_2^*) = 0$$

Newton's method for root finding

- Choose initial variable values;

$$f_i(X_1, X_2) \neq 0$$

- X_1 and X_2 must be corrected:

$$f_i(X_1 + \Delta X_1, X_2 + \Delta X_2) \to 0$$

- Expansion into Taylor series:

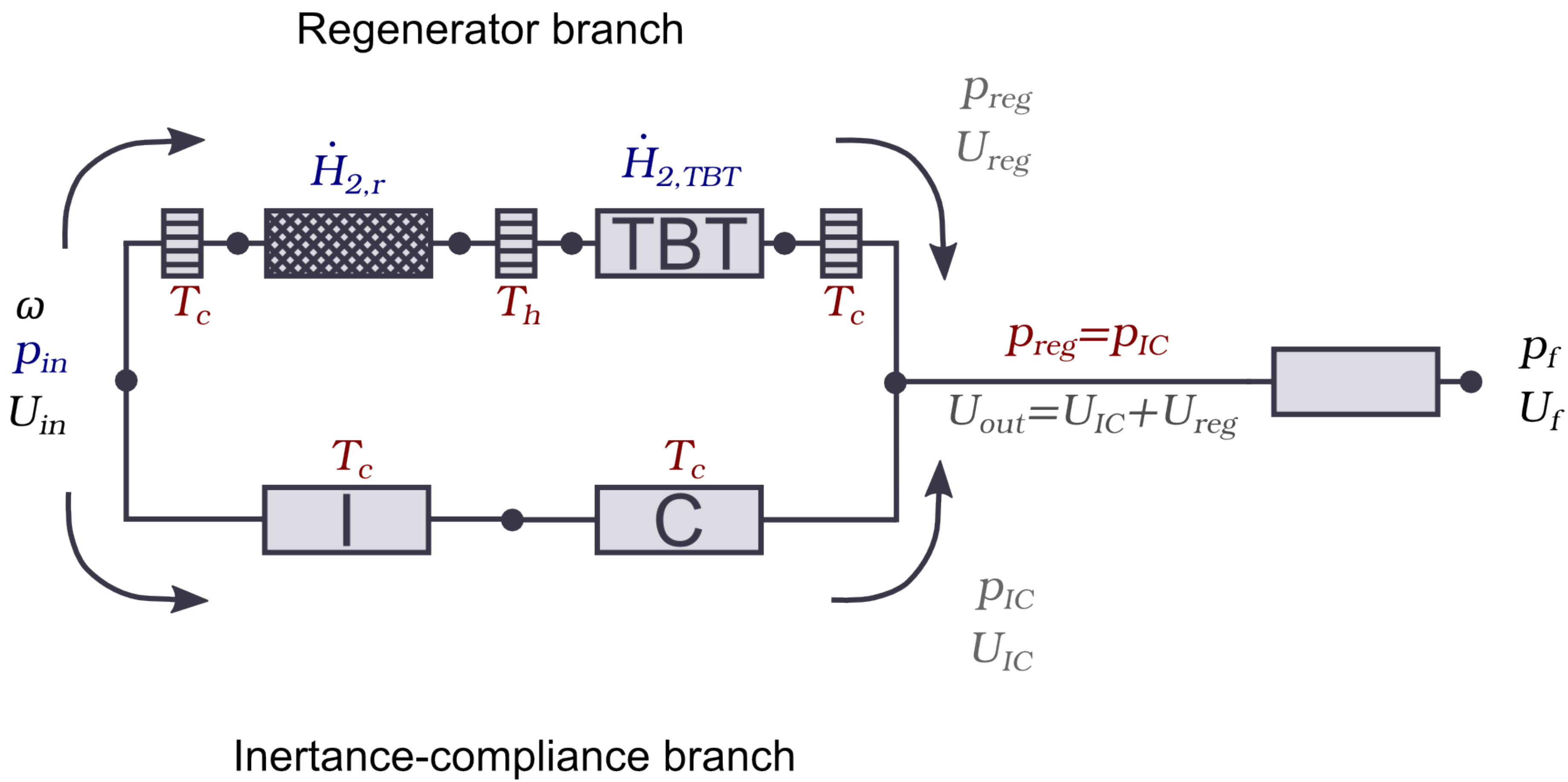
$$f_i(X_1, X_2) + \frac{\partial f_i}{\partial X_j} \Delta X_j = 0$$

- Solution:

$$\Delta X_j = -\left(\frac{\partial f_i}{\partial X_j} \right)^{-1} f_i$$

Final result

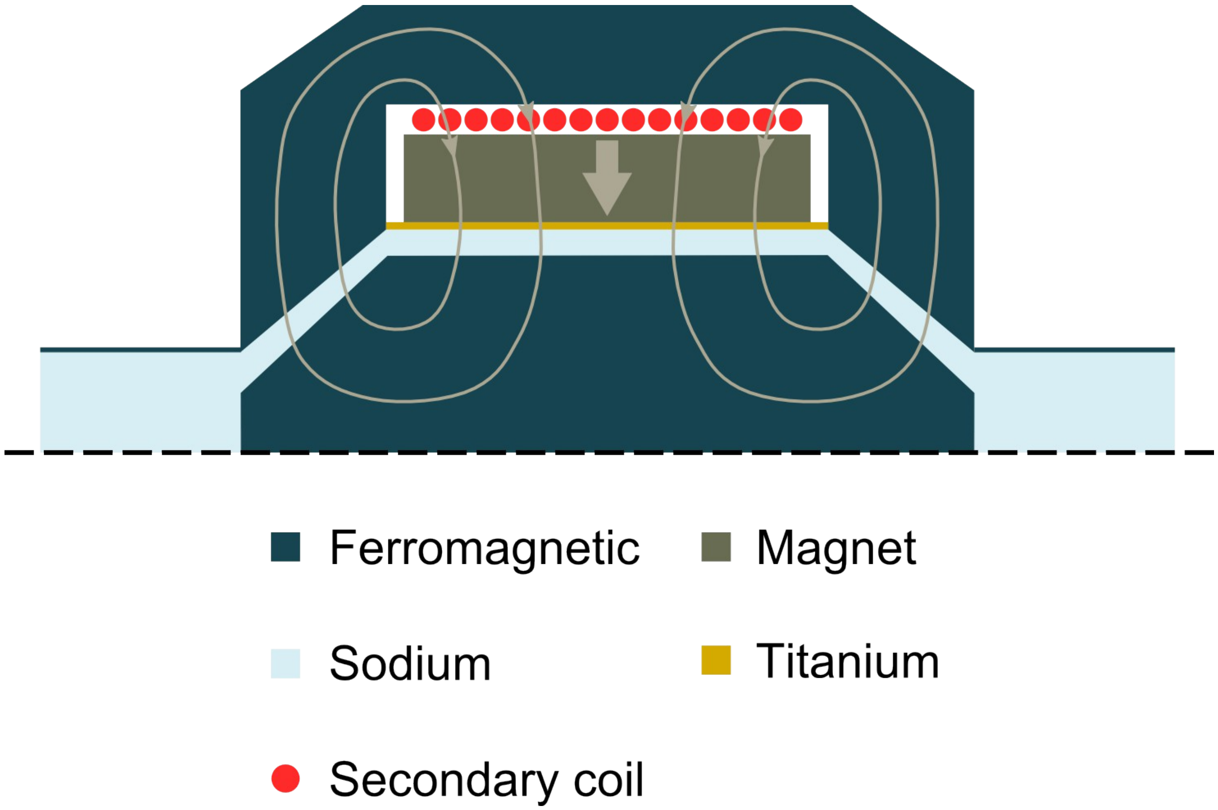
- What determines frequency and input volume flow rate?
  - Conditions at the output, i.e., MHD converter.



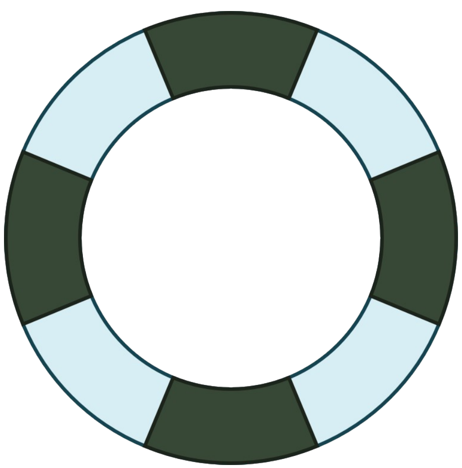
Converting mechanical power into electric power

- Mechanical energy of the gas must transformed into more useful form;
  - Electricity;
- Electromechanical converters contain moving mechanical parts;
- MHD converters contain only moving liquid.

MHD converter

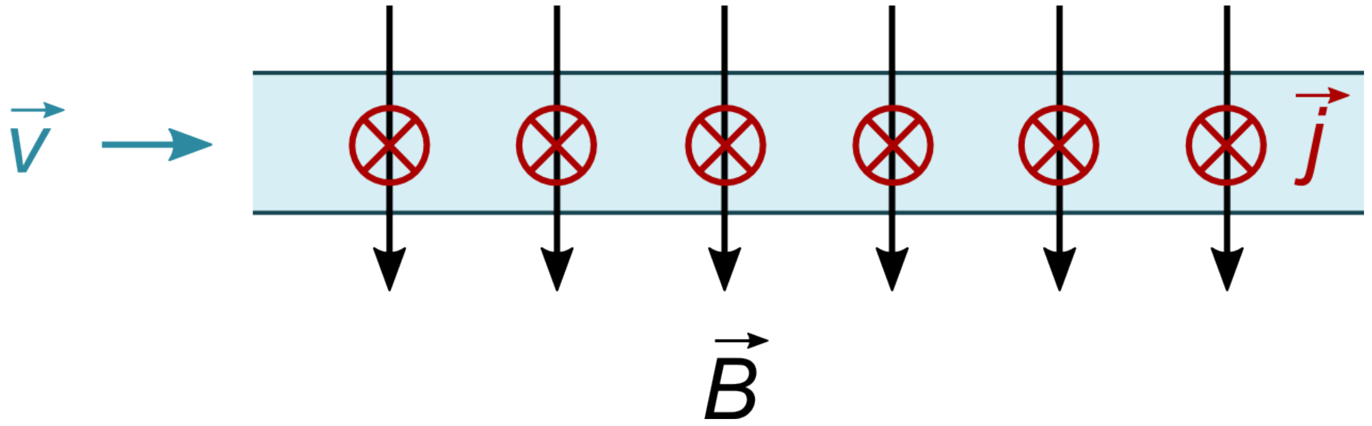


- Ring-shaped permanent magnet
- Magnetic field in the channel
- Liquid sodium fluctuates under the influence of the sound waves



Cross section of the conical channels

Main sodium channel

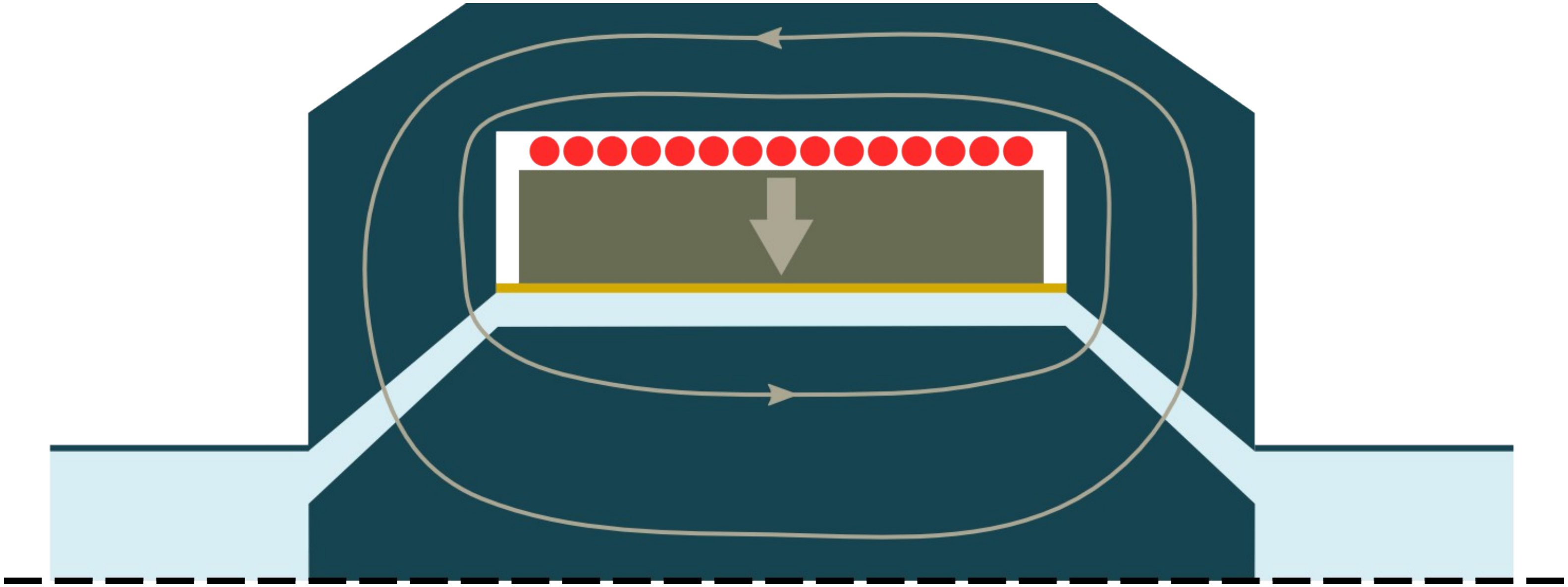


- Permanent magnet generates a magnetic field in the main channel;
- Electric current is induced in the moving, electrically-conducting media (sodium):

$$\vec{j} = \sigma\left(\vec{E} + \vec{v} \times \vec{B}\right)$$

- If velocity fluctuates, the current fluctuates, too.

Oscillating magnetic field



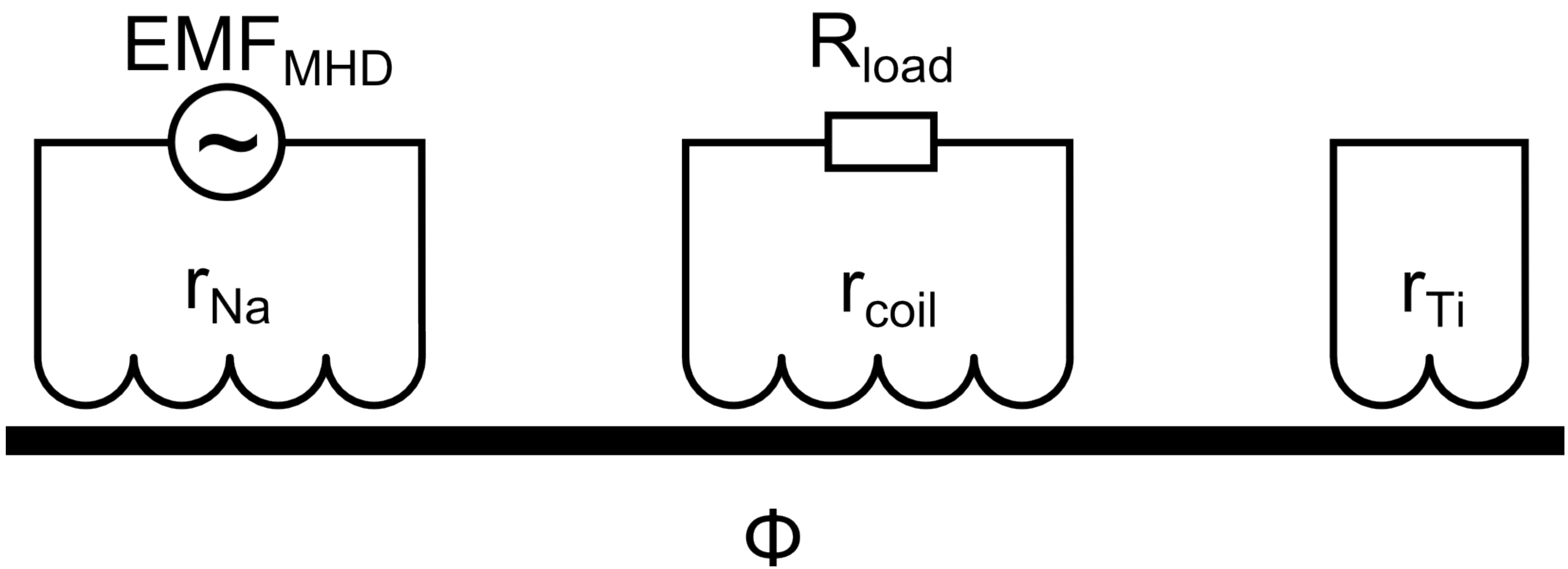
- Electric current in the main channel induces fluctuating magnetic field;
- The flow of this field is enclosed in the ferromagnetic material;
- The fluctuating magnetic field induces EMF in conducting loops (the main channel, the secondary coil, and so on).

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

General MHD converter scheme

- Knowing geometric and material properties, we can calculate induced currents for a given magnetic field flow Φ

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$



Maxwell equations

- Consider low frequency (50 – 500 Hz), and small dimensions (about 1 m)
  - Ignore the displacement current

$$\nabla \cdot \vec{E} = 0 \qquad \nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\nabla \cdot \vec{B} = 0 \qquad \nabla \times \vec{H} = \vec{j}$$

- Constitutive relations

$$B = B(H) \qquad \vec{j} = \sigma\left(\vec{E} + \vec{v} \times \vec{B}\right)$$

Magnetic vector potential and electric scalar potential

$$\nabla \times \vec{A} = \vec{B} \qquad \vec{E} = -\frac{\partial \vec{A}}{\partial t} - \nabla \varphi$$

$$\nabla \cdot \vec{A} = 0$$

$$\Delta \varphi = 0$$

$$\nabla \times \left(\frac{\nabla \times \vec{A}}{\mu(B)\mu_0} \right) = \sigma \left(-\frac{\partial A}{\partial t} - \nabla \varphi + \vec{v} \times \vec{B} \right)$$

Linearization of the equations

- Divide into two fields: constant and harmonic
- Solve equations separately
  - Linearize equations for the harmonic field

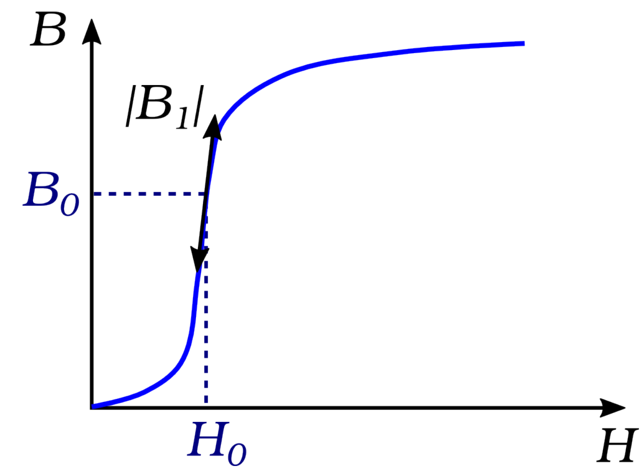
$$\vec{A} = \vec{A}_0 + Re\left[\vec{A}_1 e^{i\omega t}\right]$$

$$\varphi = \varphi_0 + Re\left[\varphi_1 e^{i\omega t}\right]$$

Linearizations of equations

Equation for the constant field

$$\nabla \times \left(\frac{\nabla \times \vec{A}_0}{\mu(B_0)\mu_0} \right) = \vec{j}_{mag}$$



Equations for the harmonic field

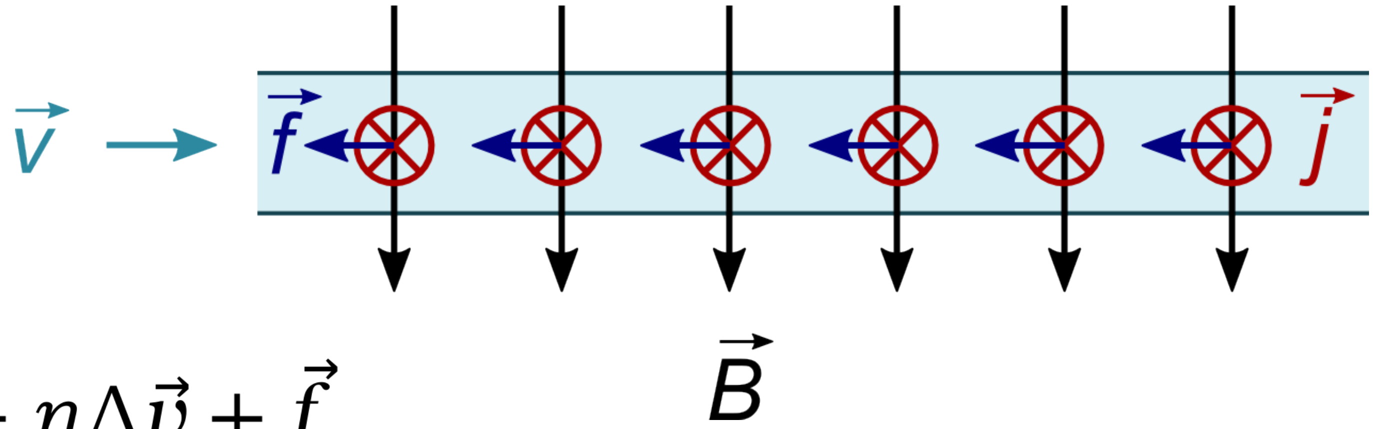
$$\nabla \times \left(\frac{\nabla \times \vec{A}_1}{\mu' \mu_0} \right) = \sigma \left(-i\omega \vec{A}_1 - \nabla \varphi_1 + \vec{v} \times \vec{B}_0 \right)$$

$$\mu' = \left. \frac{dB}{dH} \right|_{B=B_0}$$ (Actually the dependence is anisotropic)

Acoustic equations for the MHD converter

- Lorentz forces acting on the sodium;
- Navier-Stokes equations for incompressible fluid;
  - Assume relatively small velocities

$$\vec{f} = \vec{j}_1 \times \vec{B}_0$$



$$i\omega\rho\vec{v} = -\nabla p_1 + \eta\Delta\vec{v} + \vec{f}$$

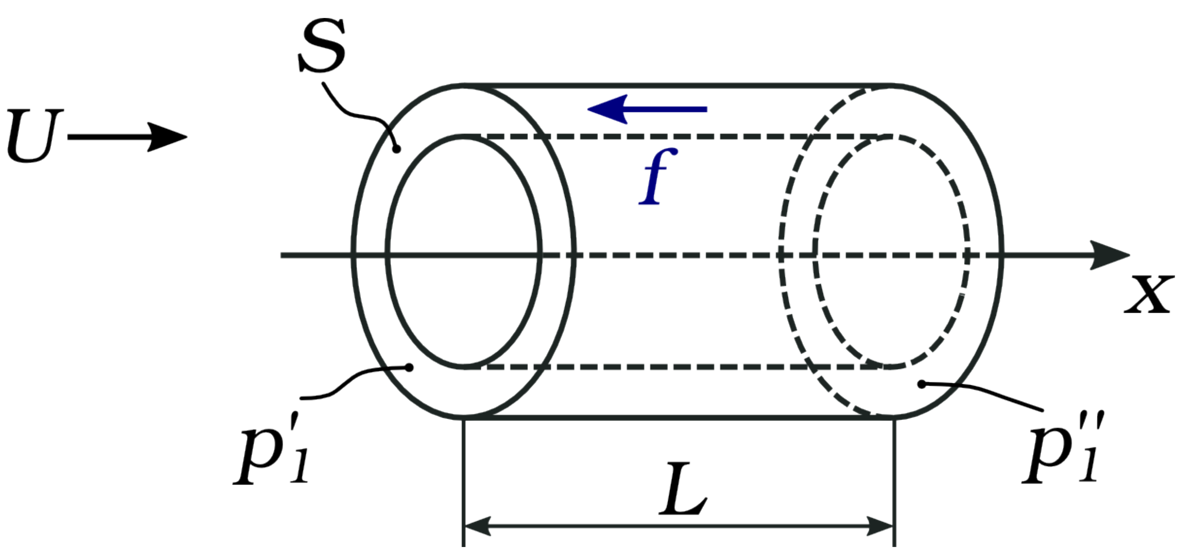
Integration of the acoustic equations (idealized main channel)

$$i\omega\rho v = -\frac{\partial p_1}{\partial x} + f_x$$

$$p_1' - p_1'' = \frac{i\omega\rho UL - F}{S}$$

$$U = Sv$$

$$F = SLf$$

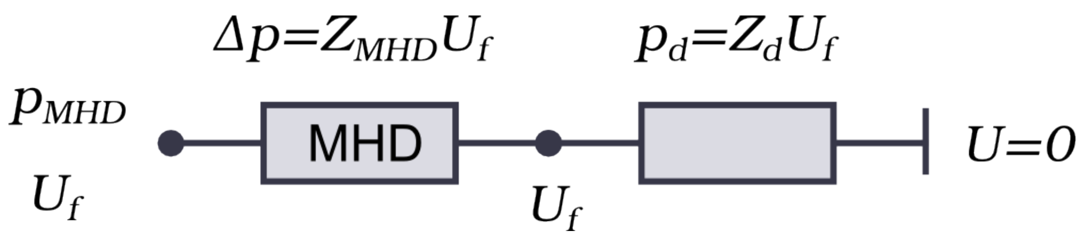


Pressure drop in the MHD engine is a linear function of volume flow rate U. Definition of acoustic impedance:

$$Z(\omega) = \frac{p_1' - p_1''}{U}$$

Acoustic properties of the MHD converter

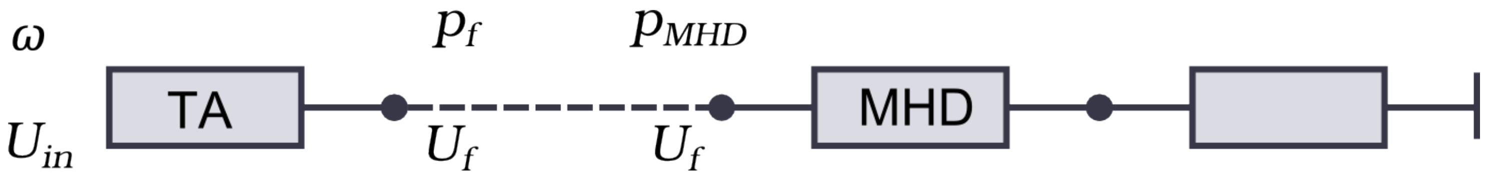
- Consider an MHD converter with a closed duct on its end;
- For given frequency and input volume flow rate, the system will produce a certain pressure amplitude



$$p_{MHD} = (Z_{MHD} + Z_d)U_f$$

Coupling TA and MHD

- TA and MHD acoustic solutions are not compatible, in general:



$$p_f \neq p_{MHD}$$

- Find TA and MHD solution with matching pressure amplitudes by changing frequency and initial volume flow rate:

$$\Delta p = p_f - p_{MHD}$$

$$\Delta p_{Re} = \Delta p_{Re}(\omega, U_{in})$$

$$\Delta p_{Im} = \Delta p_{Im}(\omega, U_{in})$$

Conclusions

Thanks!