SPACE TRIPS SUMMER SCHOOL

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Redressed flows in thermoacoustics

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- **Introduction**

- Some bases

- Turbulence

- Edge effects

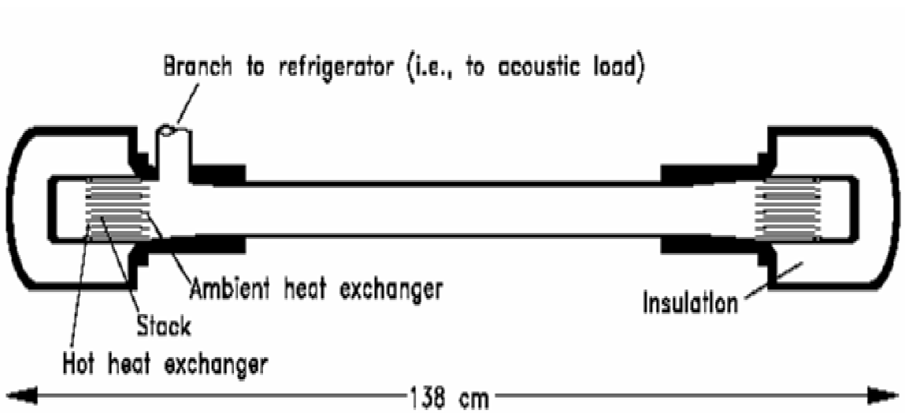
- Acoustic streaming

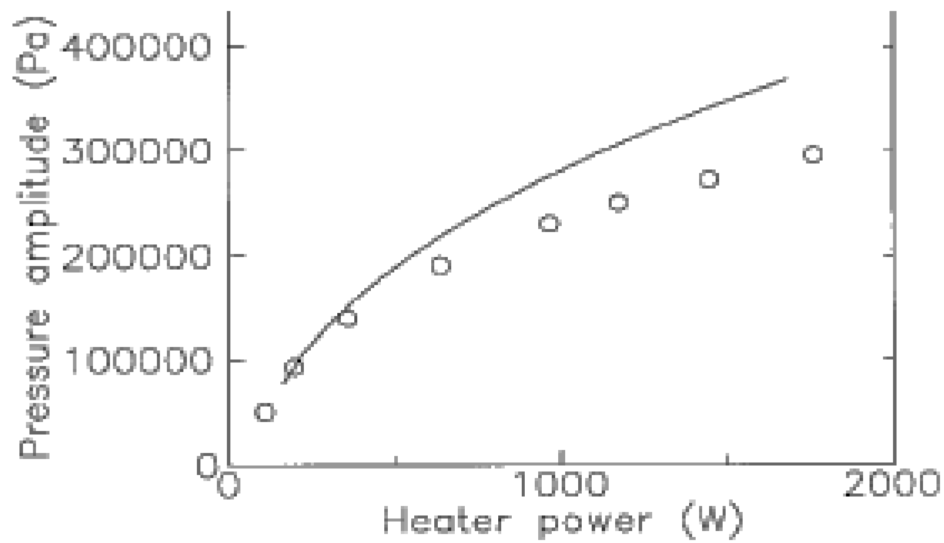
- Conclusion

Thermoacoustics: A unifying perspective
for some engines and refrigerators

Greg Swift
Condensed Matter and Thermal Physics Group
Los Alamos National Laboratory

Fifth draft
29 May 2001
LA-UR 99-895





- Results of measurements

o Results of DELTAE-C calculation

Linear regime
1-D model
Valid for steady state

« Thermoacoustic engines and refrigerators with practical levels of power per unit volume and per unit mass must operate at high amplitudes such as these, where actual behavior deviates significantly from Rott’s acoustic approximation.

In the early days of thermoacoustics research, we were impressed that Rott’s acoustic approximation came so close to the truth at high amplitudes, but our standards are more demanding now: We hope to understand such deviations quantitatively. Those of us who come from an acoustics background must go well beyond our acoustic-based knowledge and intuition;

for example, we must learn about the High-Reynolds-number phenomena encountered in other branches of hydrodynamics, such as aerodynamics and pipeline hydraulics. »

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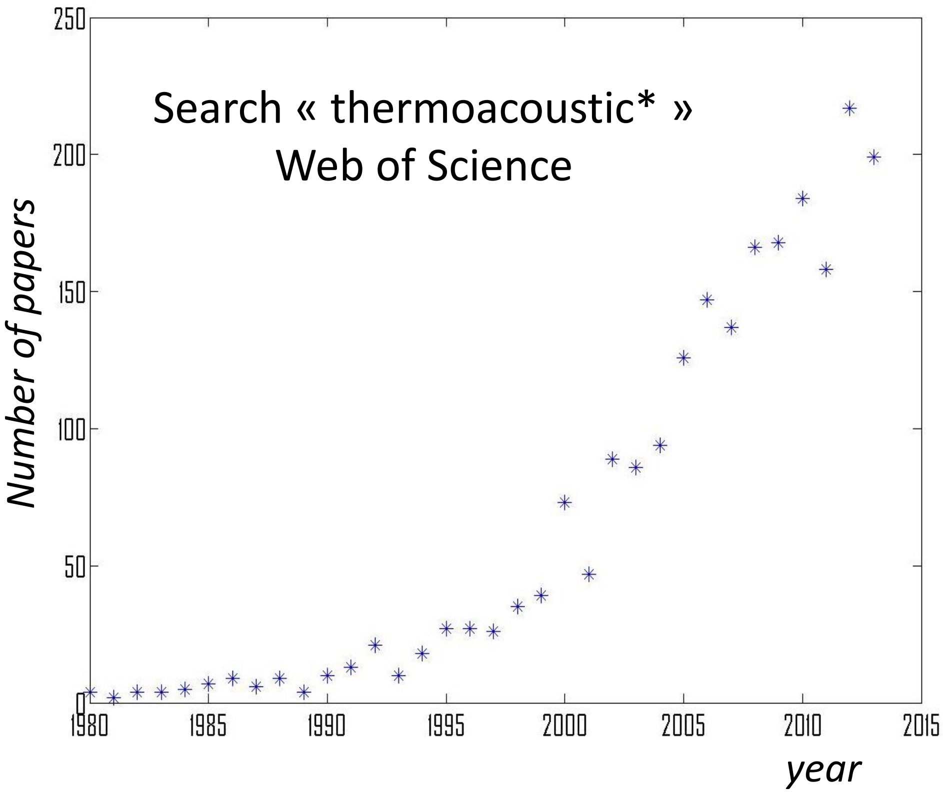

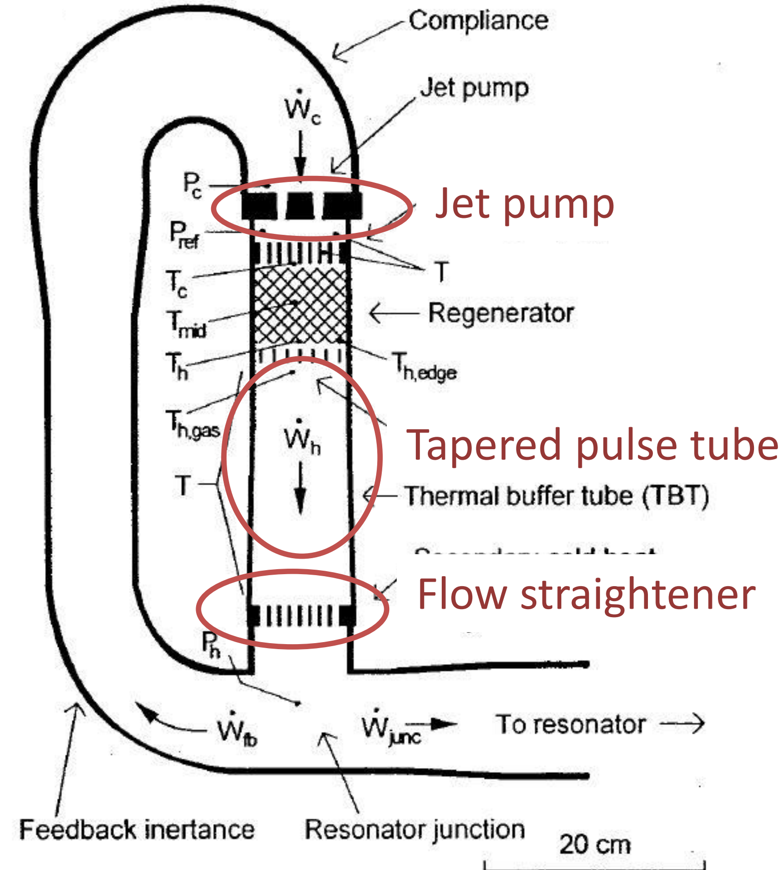

Backhaus and Swift, Nature, 1999

There exist some Academic studies

But still far away from practical devices
and still lot to understand

There exist some Empirical solutions

But work only under certain conditions, not transposable
and still lot to understand

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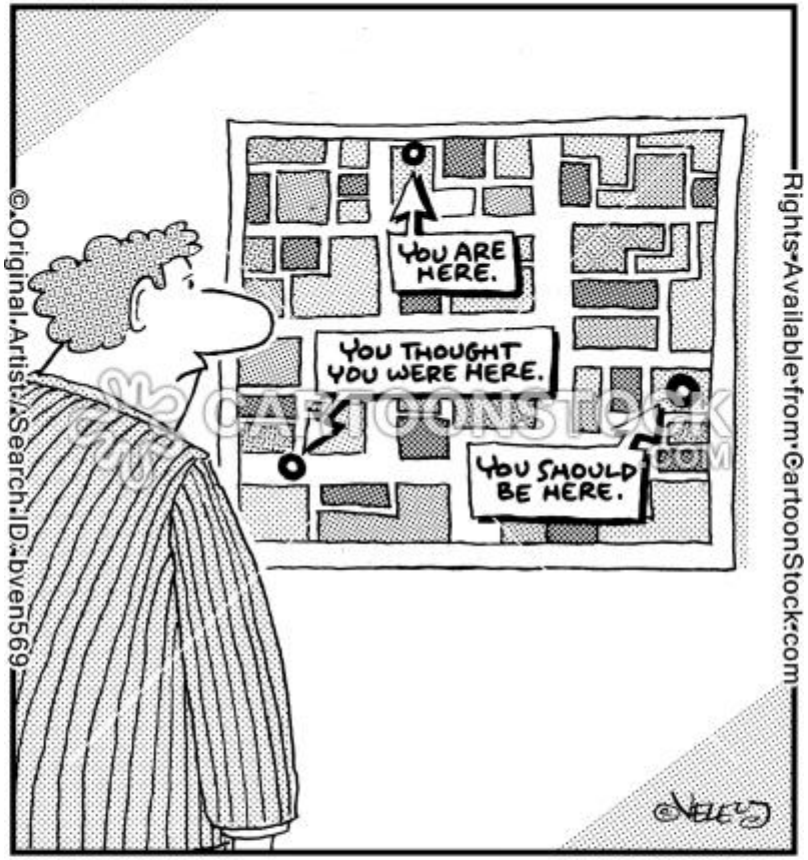
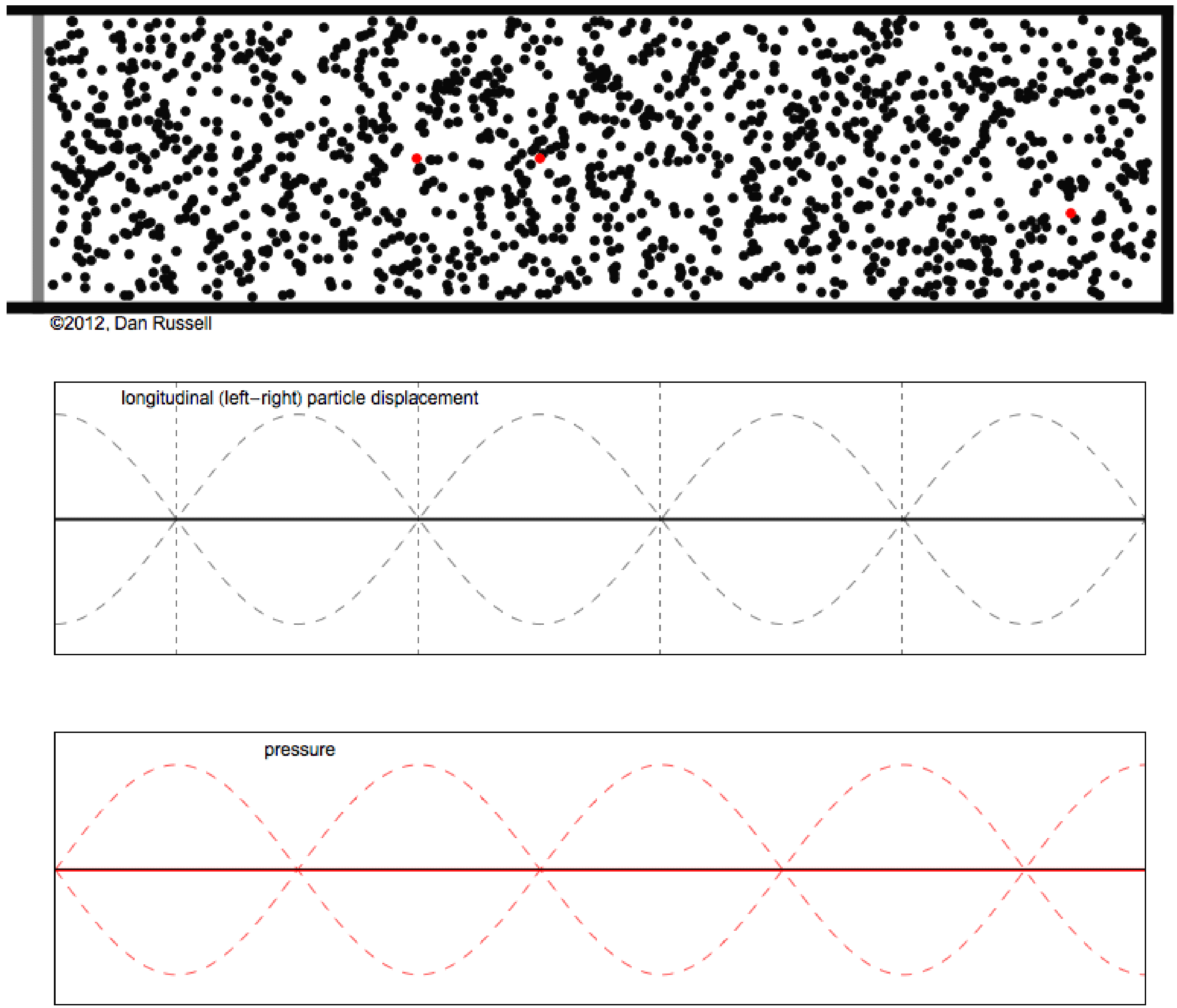
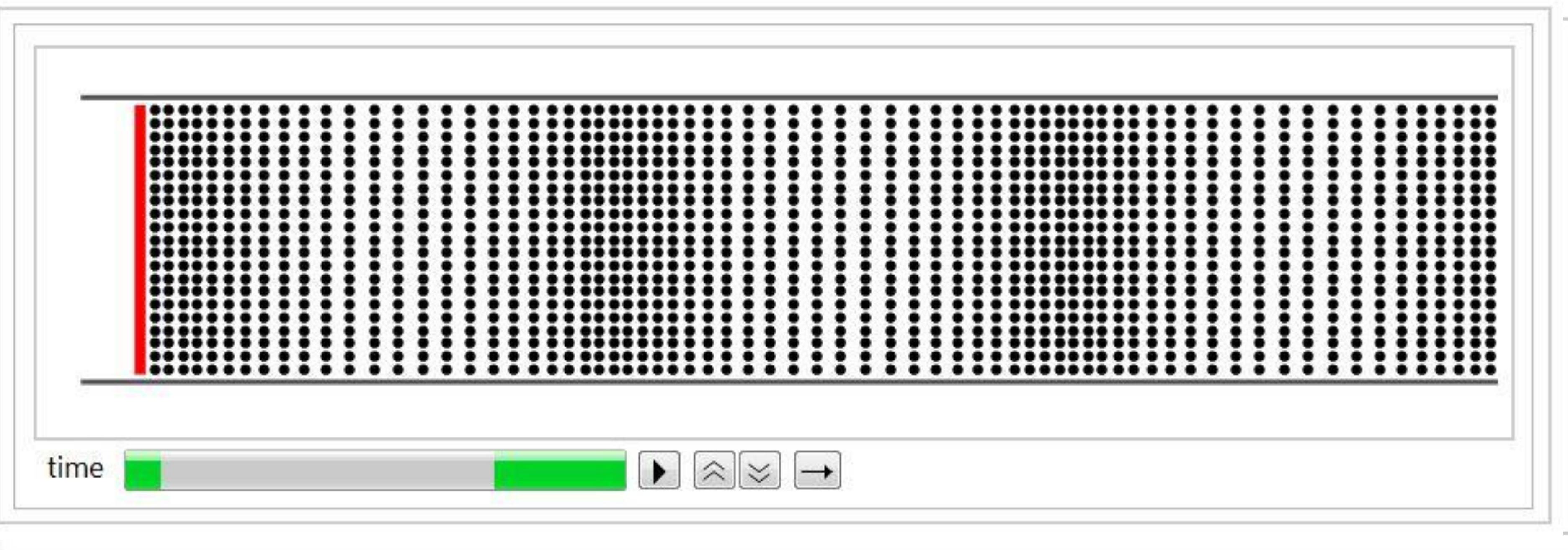
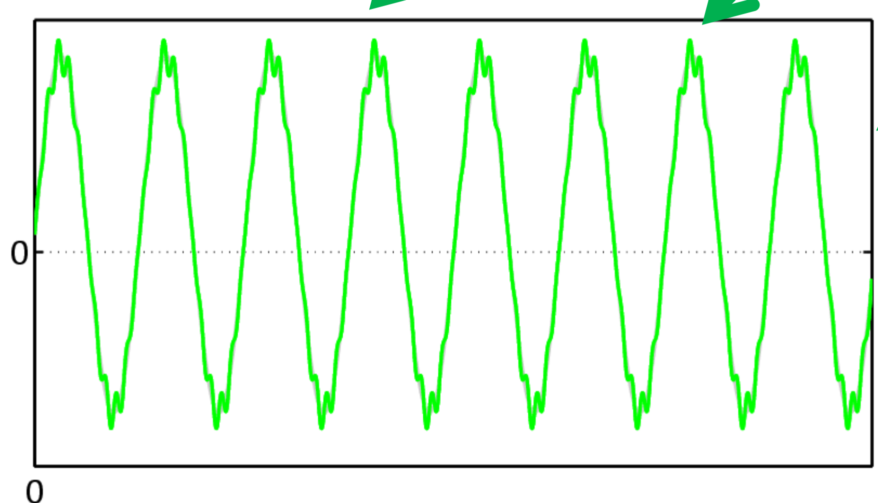




Animation courtesy of Dr. Dan Russell, Grad. Prog. Acoustics, Penn State



$$\underbrace{u(x,r,t)}_{\textit{particle velocity}} = \underbrace{u_0(x,r)}_{\textit{mean velocity } (u_0=0)} + \underbrace{u_{ac}(x,r,t)}_{\textit{acoustic velocity}} + \underbrace{u_2(x,r)}_{\textit{streaming velocity}} + \underbrace{u'(x,r,t)}_{\textit{turbulent component}}$$

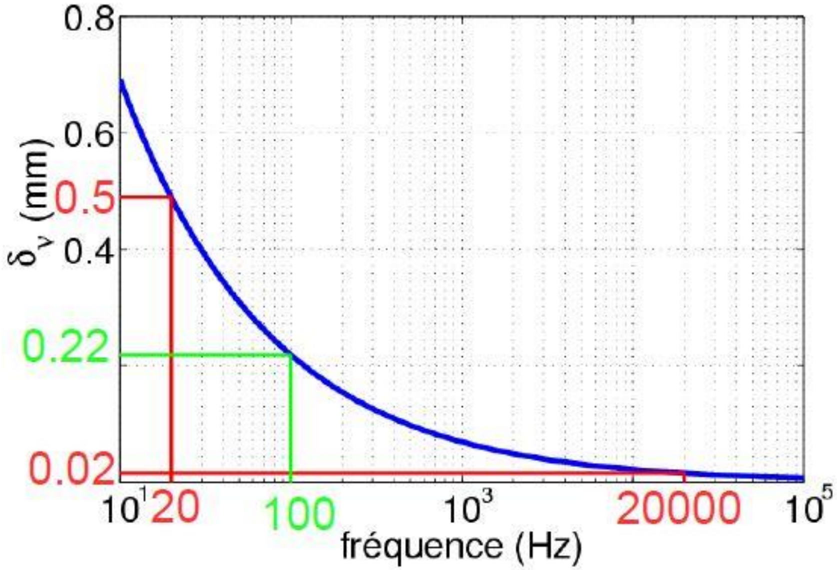


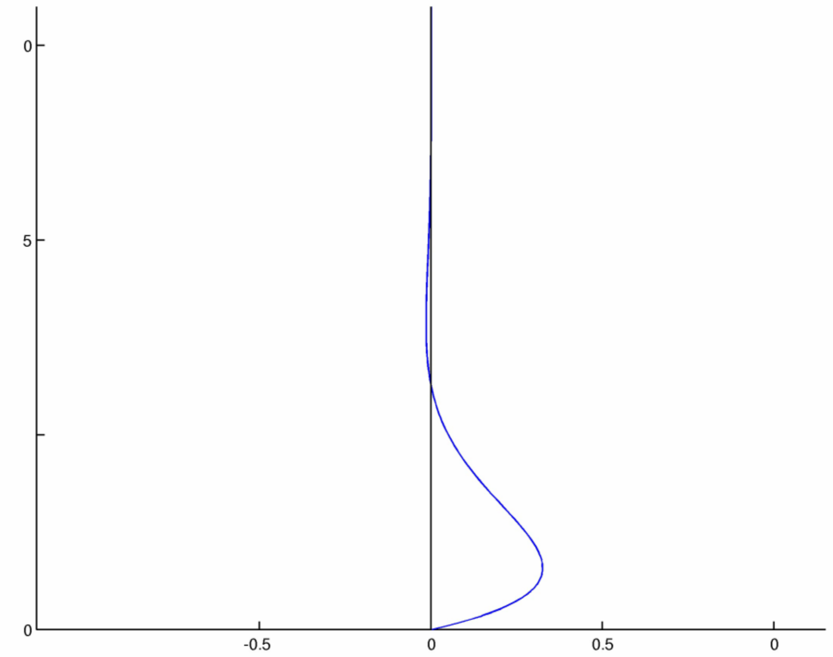
Non linear effects induced by shear phenomena inside a viscous boundary layer associated with high amplitudes

Oscillatory or acoustic boundary layer with a thickness

$$\delta_\nu = \sqrt{\frac{2\nu}{\omega}}$$

ν kinematic viscosity
ω acoustic pulsation





Relevant dimensionless numbers

Consider acoustic propagation along the z-axis of an axisymmetric wave guide for a perfect gaz. Consider that acoustic oscillation corresponds to a *M* order variation of acoustic quantities around 0-state:

$$\frac{P-P_0}{P_0}, \frac{\rho-\rho_0}{\rho_0}, \frac{T-T_0}{T_0}, \frac{S-S_0}{S_0}, \frac{u}{c_0} = 0(M)$$

O-order: These are constant compared to acoustic time scale

We can write

$$\rho = \rho_0(1+M\hat{\rho})$$
$$P = P_0(1+M\hat{P})$$
$$T = T_0(1+M\hat{T})$$
$$S = S_0 + MR_g\hat{S}$$
$$u = c_0 M\hat{u}$$

Dimensional quantities

Dimensionless quantities

$$\hat{t} = \omega t \qquad \frac{\partial}{\partial t} = \omega\frac{\partial}{\partial \hat{t}} \qquad \hat{z} = \frac{2\pi}{\lambda}z$$

$$\hat{r} = \frac{r}{R} \qquad \frac{\partial}{\partial z} = \frac{\omega}{c_0}\frac{\partial}{\partial \hat{z}}$$

Note: $\partial/\partial r$ becomes $\frac{1}{R}\frac{\partial}{\partial \hat{r}_1}$ for *v* and

$\frac{1}{Re}\frac{1}{\delta_\nu}\frac{\partial}{\partial \hat{r}_2}$ for other quantities

z –projection of Navier-Stokes

$$\rho\left(\frac{\partial\vec{u}}{\partial t}+(\vec{u}.\vec{\nabla})\vec{u}\right)=-\vec{\nabla}P+\mu\nabla^2\vec{u}+\left(\eta+\frac{\mu}{3}\right)\vec{\nabla}(\vec{\nabla}.\vec{u})$$

Instationarity

Convective effects

Viscosity effects

$$\rho\left(\frac{\partial u}{\partial t}+u\frac{\partial}{\partial z}u+v\frac{\partial}{\partial r}u\right)=-\frac{\partial P}{\partial z}+\mu\left[\frac{\partial^2}{\partial z^2}+\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}\right)\right]u+(\eta+\frac{\mu}{3})\frac{\partial}{\partial z}\left[\frac{\partial u}{\partial z}+\frac{1}{r}\frac{\partial}{\partial r}(rv)\right]$$

That gives

$$(1+M\hat{\rho})\frac{\partial\hat{u}}{\partial\hat{t}}+(1+M\hat{\rho})\boxed{M}\left(\hat{u}\frac{\partial}{\partial\hat{z}}\hat{u}+\left(\frac{1}{Re}\right)\hat{v}\frac{\partial\hat{u}}{\partial\hat{r_2}}\right)$$

$$=-\frac{1}{\gamma}\frac{\partial\hat{P}}{\partial\hat{z}}+\boxed{\frac{1}{Re}}\boxed{\left[\frac{\eta}{\mu}+\frac{4}{3}\right]\frac{\partial^2\hat{u}}{\partial\hat{z}^2}}+\boxed{\left(\frac{1}{Re}\right)}^2\boxed{\frac{\partial^2\hat{u}}{\partial\hat{r_2}^2}}+\boxed{\left(\frac{Sh}{Re}\right)}\boxed{\frac{1}{\hat{r_1}}\frac{\partial\hat{u}}{\partial\hat{r_2}}}$$

$$+\boxed{\left(\frac{Sh}{Re}\right)}\boxed{\left(\frac{\eta}{\mu}+\frac{1}{3}\right)\frac{1}{\hat{r_1}}\frac{\partial}{\partial\hat{r_1}}\left[\hat{r_1}\frac{\partial\hat{v}}{\partial\hat{z}}\right]}$$

$M=u/c_0$ Mach number

Shear number $Sh=\dfrac{\delta_\nu}{R}$

Acoustic Reynolds number $Re=\dfrac{\rho_0c_0^2}{\mu\omega}=2\pi^2\left(\dfrac{\lambda}{\delta_\nu}\right)^2$

$M << 1$ *Weakly non linear acoustics*

$\frac{1}{Re} << 1$ *Low viscosity*

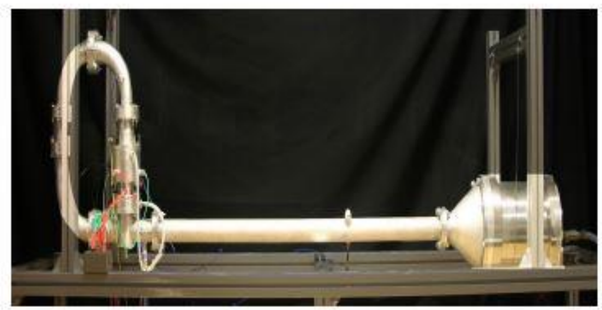
$Sh << 1$ *Wide guide. Viscosity only in a thin layer of fluid*

$\frac{1}{ReSh} << 1$ *Bulk viscosity negligible*

	Prototype 1	Prototype 2
Fluid	Air	Hélium
P0 [Pa]	6,01E+005	2,20E+006
Tc and Th [K]	300 / 577	300/907
sound velocity at Tc/Th [m/s]	347/481	1019/1772
ρ0 at Tc/Th [kg/m3]	6.98/3.66	3.53/1.17
F [Hz]	40,6	79,7
Drive ratio [%]	3	5
U in regenerator (in/out) [m3/s]	1.41e-3/2.86e-3	4.37e-3/1.38e-2
Hydraulic radius regenerator [m]	3,06E-005	4,06E-005
Porosity	6,90E-001	0,72
Regenerator radius [m]	0,02815	0,02815
U in main guide [m3/s]	1,40E-002	3,70E-002
main guide radius [m]	2,22E-002	6,50E-002

Writing other fundamental equations only adds the Prandtl number

$$Pr = \frac{\mu_0 c_p}{k_0}$$



Mach number in the guide	**0,03**	**0,01**
1 / Acoustic Reynolds number	**5,16E-009**	**2,72E-009**
Viscous penetration depth (m)	**1,44E-004**	**1,47E-004**
Shear number	**0,01**	**2,26E-003**
Prandtl number	**0,7**	**0,2**
1/(ReSh)	**7,94E-007**	**1,20E-006**

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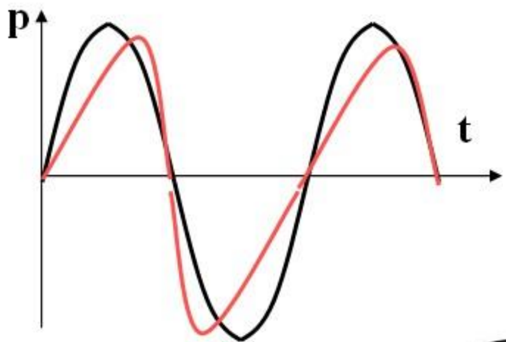
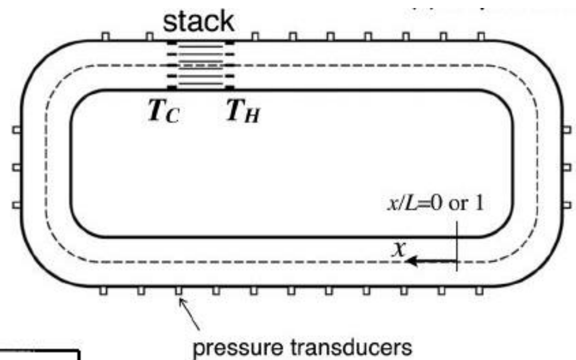
Non linear propagation





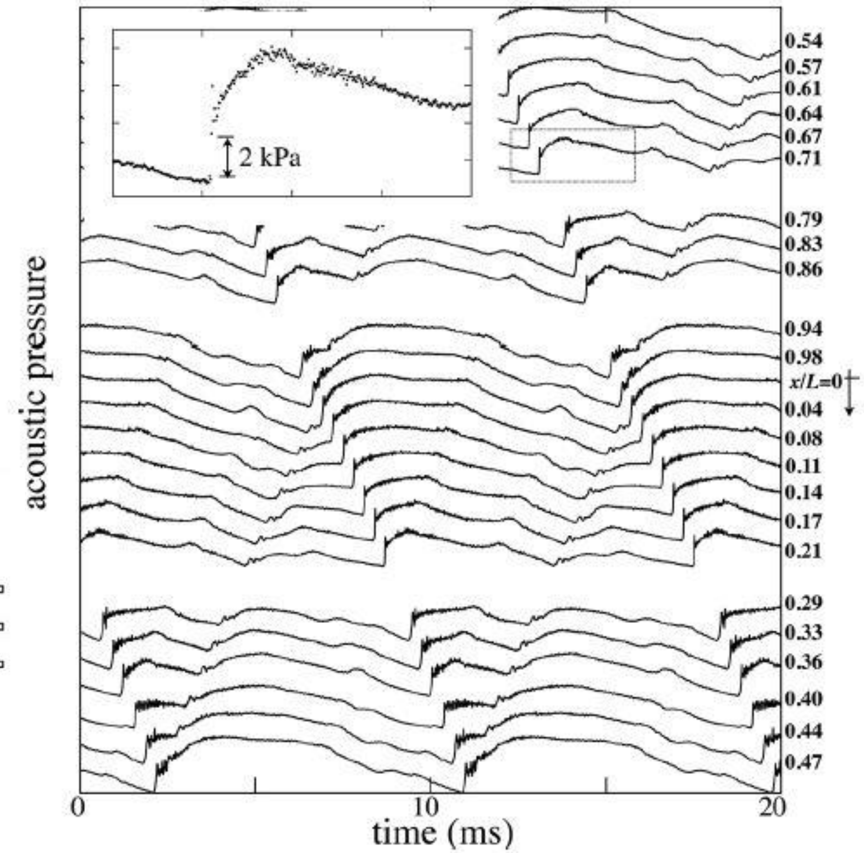
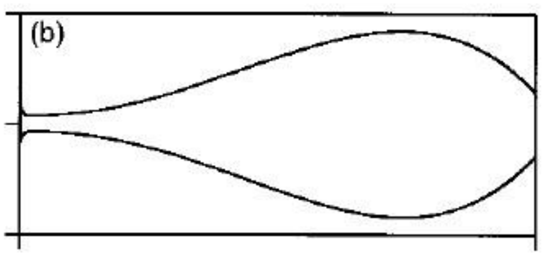


FIG. 2. Spatial and temporal evolution of thermoacoustic shock waves. Acoustic pressures are shown with vertical offsets according to their axial positions. The inset shows a magnified view of the data ($x/L = 0.71$) in the region enclosed by the dotted square.



Ilinski et al JASA 2001

Taking into account non linear propagation allows to account for 10 to 20% of non linear losses only

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Observation of traveling thermoacoustic shock waves (L)

Tetsushi Biwa[a)] and Takuma Takahashi

Department of Mechanical Systems and Design, Tohoku University, Sendai, 980-8579, Japan

Taichi Yazaki

Department of Physics, Aichi University of Education, Kariya, 448-8542, Japan

(other non linear effects)

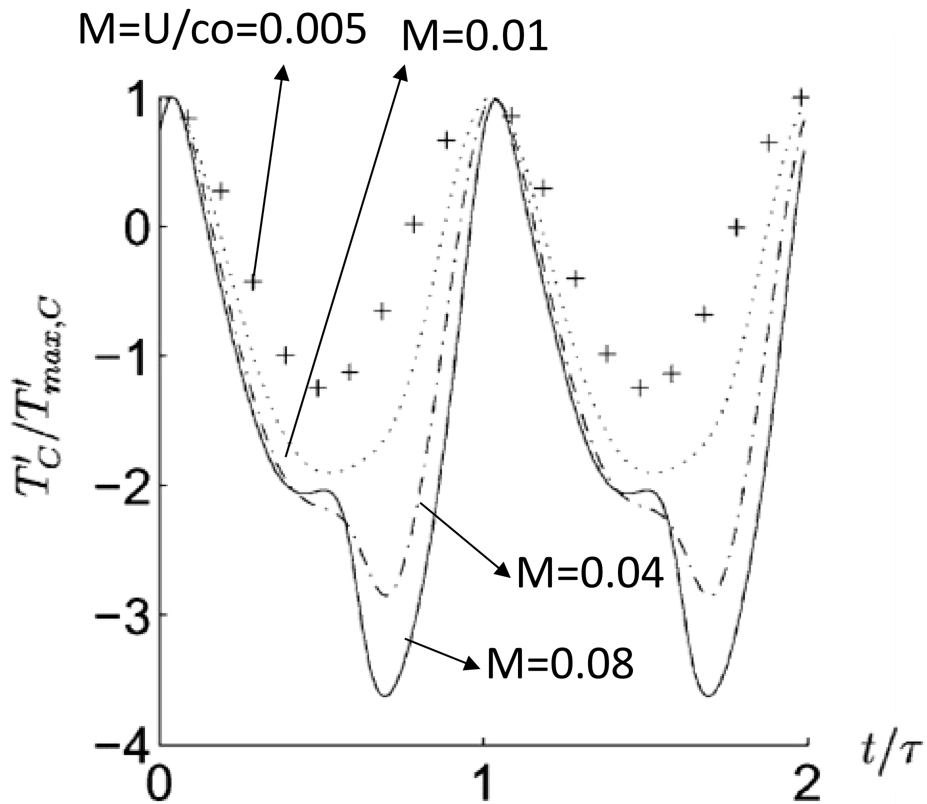
The end of stack region is associated with a strong singularity in terms of heat transfer.
There exist higher harmonics for the oscillating temperature even for sinusoïdal acoustic pressure (*Gusev JASA 2001)*

mesh grids

RVC foam

The linear 1D models cannot describe sound propagation in complex shaped stack or regenerators

Methods have been developped to access the thermal and viscous functions of such « stack » i.e. *Petculescu JASA 2001, Guédra JASA 2011*



Oscillating temperature in the end of stack region (Marx & B-Benon JASA 2005)

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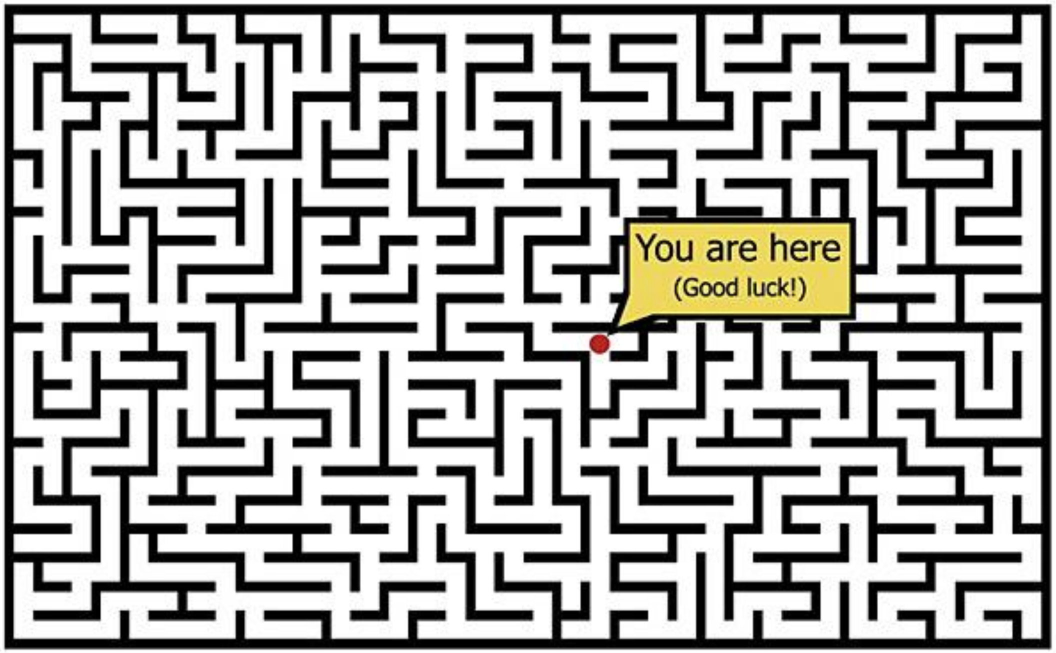
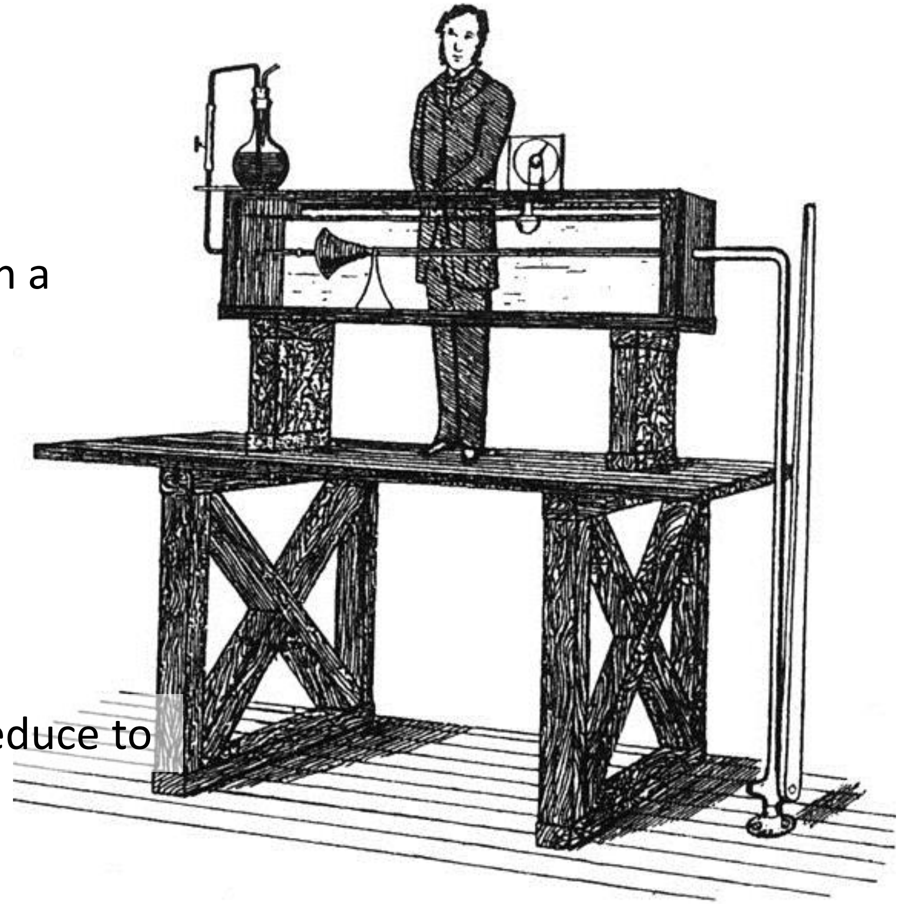


Back to fluid mechanics :
Osborne Reynolds demonstrated the transition to turbulent flow in a classic experiment in which he examined an outlet from a large water tank through a small tube. Reynolds identified the governing parameter, the dimensionless Reynolds number.



$$\Re e = \frac{UD}{\nu} = \frac{T_{diff}}{T_{ref}}$$

$$T_{diff} = \frac{D^2}{\nu} \quad , \quad T_{ref} = \frac{D}{U}$$

For incompressible fluid Navier-Stokes equations reduce to

$$\rho\left(\frac{\partial \vec{u}}{\partial t} + (\vec{u}.\vec{\nabla})\vec{u}\right) = -\vec{\nabla}P + \mu\nabla^2\vec{u}$$

That is

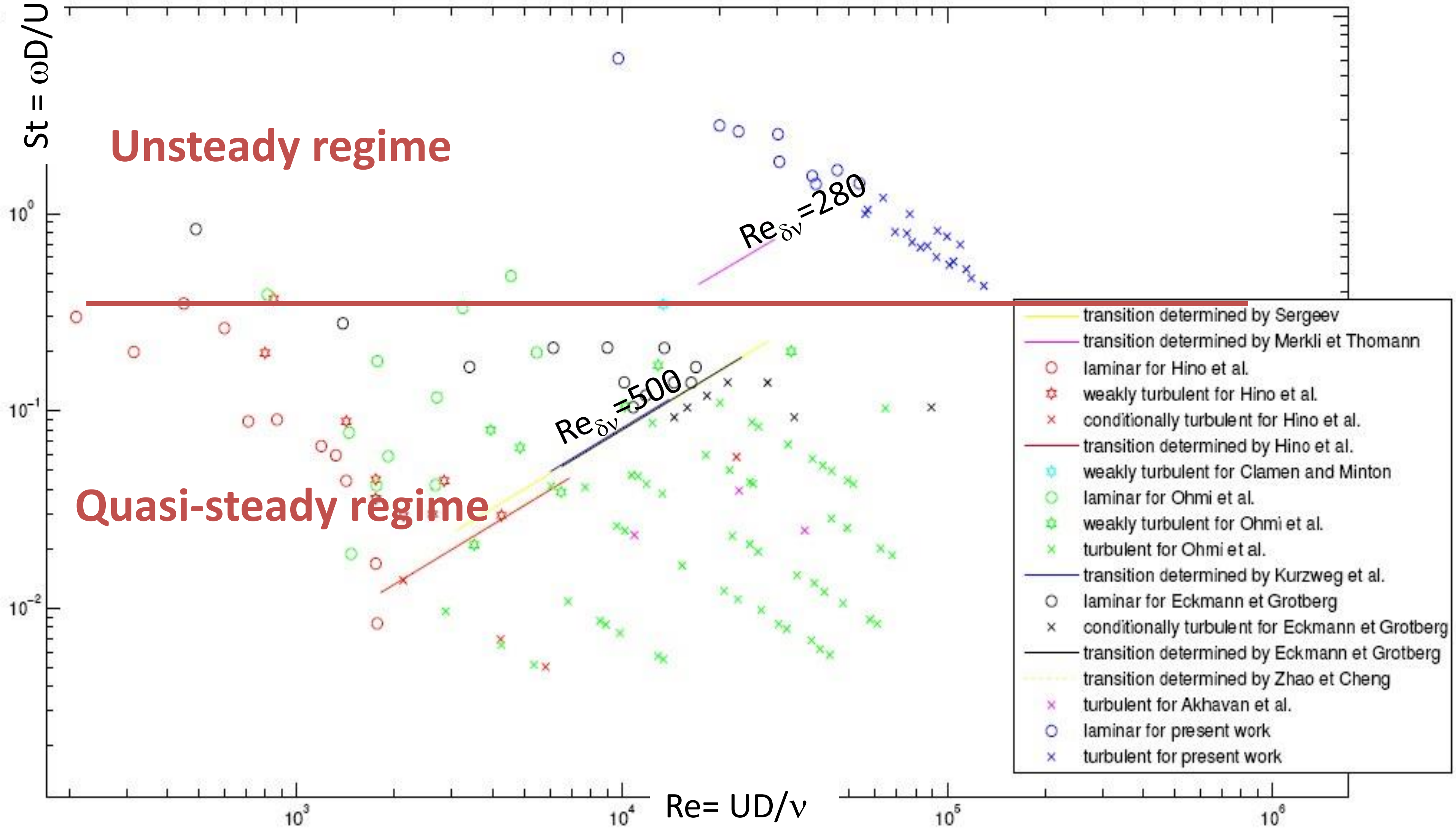
$$\underbrace{\boxed{\frac{\omega D}{U}}}_{St}\frac{\partial \hat{\vec{u}}}{\partial \hat{t}} + (\hat{\vec{u}}.\hat{\vec{\nabla}})\hat{\vec{u}} = -\frac{1}{\rho U^2}\hat{\vec{\nabla}}P + \underbrace{\boxed{\frac{\nu}{DU}}}_{\Re e}\hat{\nabla}^2\hat{\vec{u}}$$

$$\hat{t} = \omega t \quad , \quad \vec{u} = U\hat{\vec{u}} \quad , \quad \nabla = \frac{1}{D}\hat{\nabla}$$

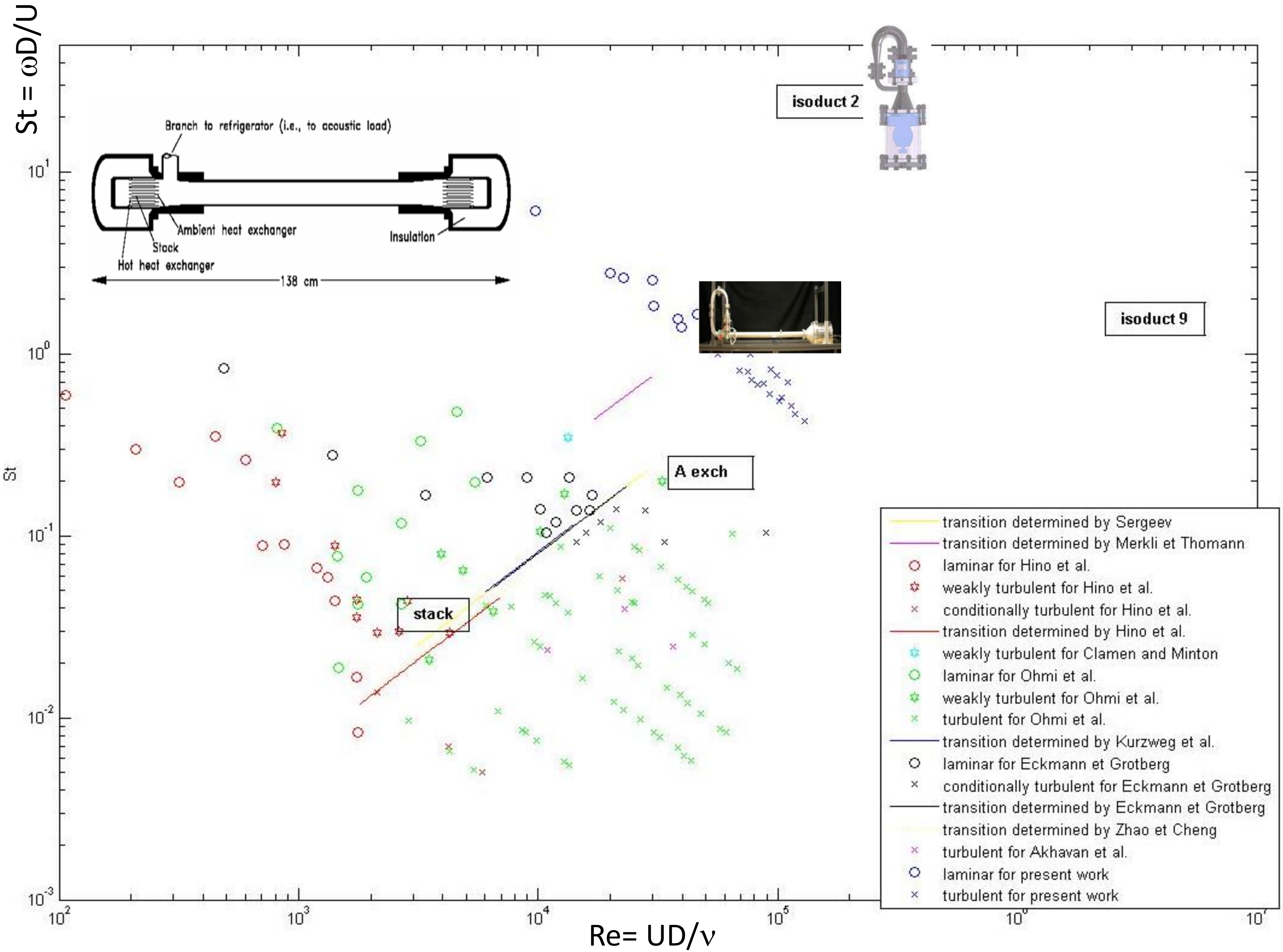
Transition to turbulence far more studied for cases where instationarity can be neglected :

$T_\omega >> T_m >> T_\nu$ with $T_\omega = \frac{1}{\omega}$ so $\Re e = \frac{UD}{\nu} >> 1$ and $St = \frac{\omega D}{U} << 1$ than in acoustic wave guides

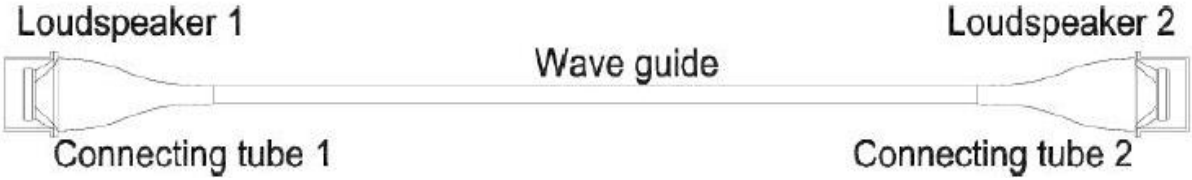
Stability diagram St=f(Re)

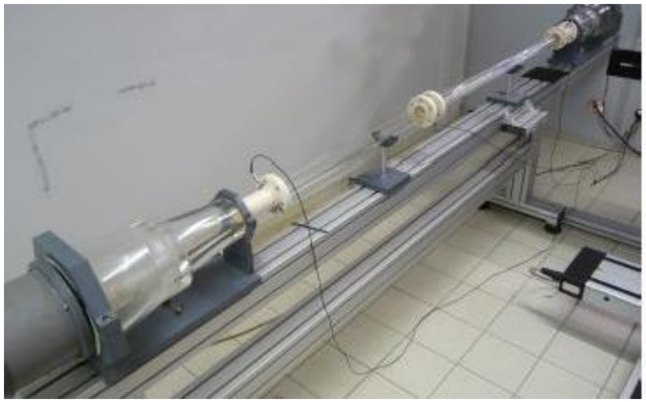


Transition Re$_{\delta\nu}$=500 determined by several authors can be suspected to be valid for quasi-steady regimes only.

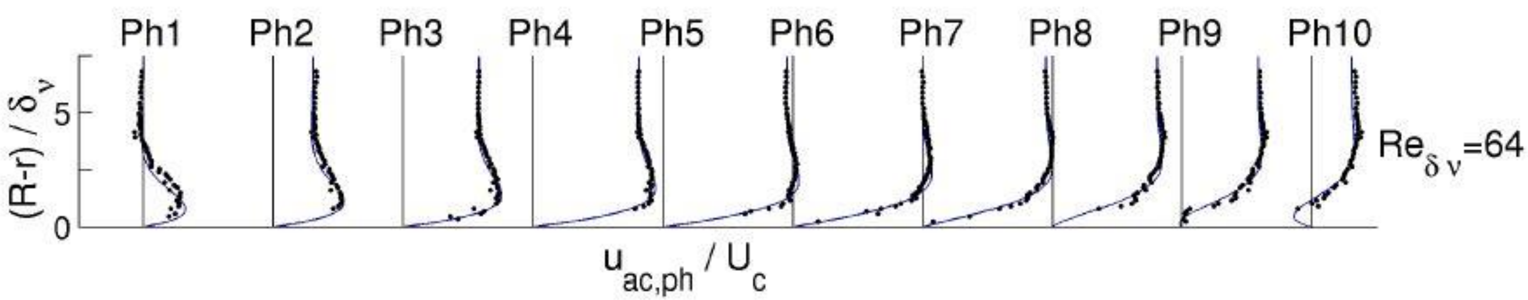


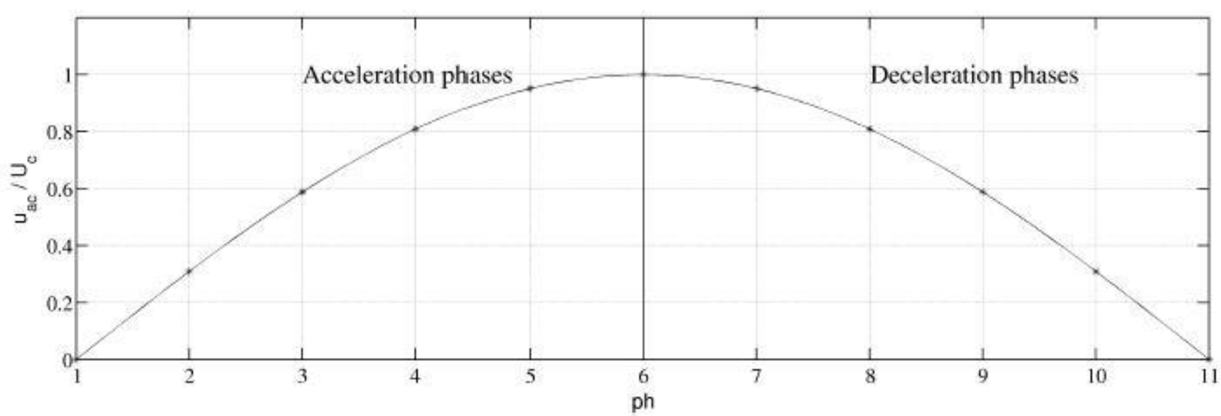
St = ωD/U
St
Re= UD/ν
Branch to refrigerator (i.e., to acoustic load)
Ambient heat exchanger
Stack
Hot heat exchanger
Insulation
138 cm
isoduct 2
isoduct 9
A exch
stack
transition determined by Sergeev
transition determined by Merkli et Thomann
laminar for Hino et al.
weakly turbulent for Hino et al.
conditionally turbulent for Hino et al.
transition determined by Hino et al.
weakly turbulent for Clamen and Minton
laminar for Ohmi et al.
weakly turbulent for Ohmi et al.
turbulent for Ohmi et al.
transition determined by Kurzweg et al.
laminar for Eckmann et Grotberg
conditionally turbulent for Eckmann et Grotberg
transition determined by Eckmann et Grotberg
transition determined by Zhao et Cheng
turbulent for Akhavan et al.
laminar for present work
turbulent for present work

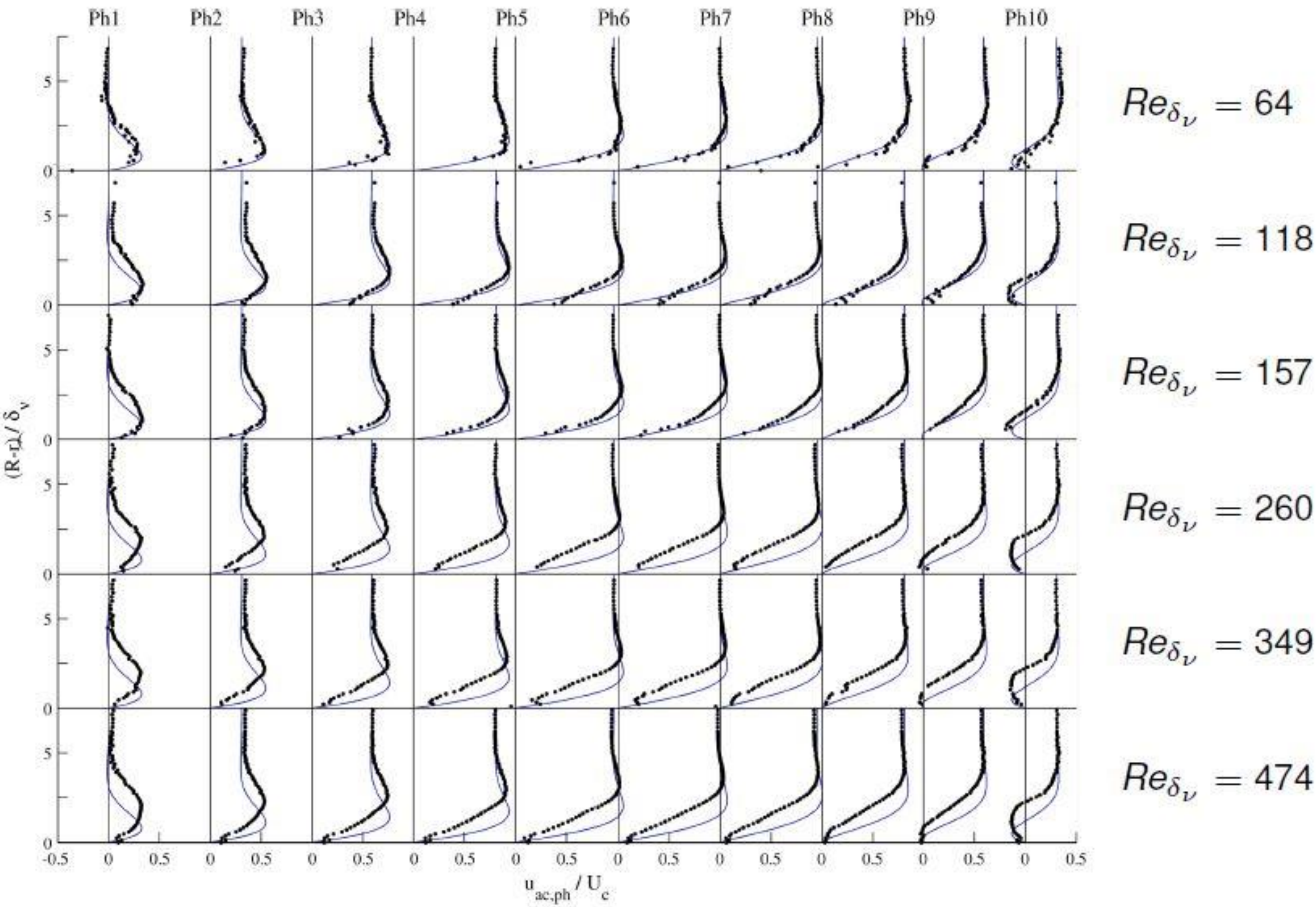




- LDV measurements
- Axial velocity along the radius
- $f = 25\ Hz$
- $Re_{\delta_\nu} = 64$ to 474
- $\delta_\nu = 0.435\ mm$
  $\Rightarrow$ several measurement points inside the boundary layer

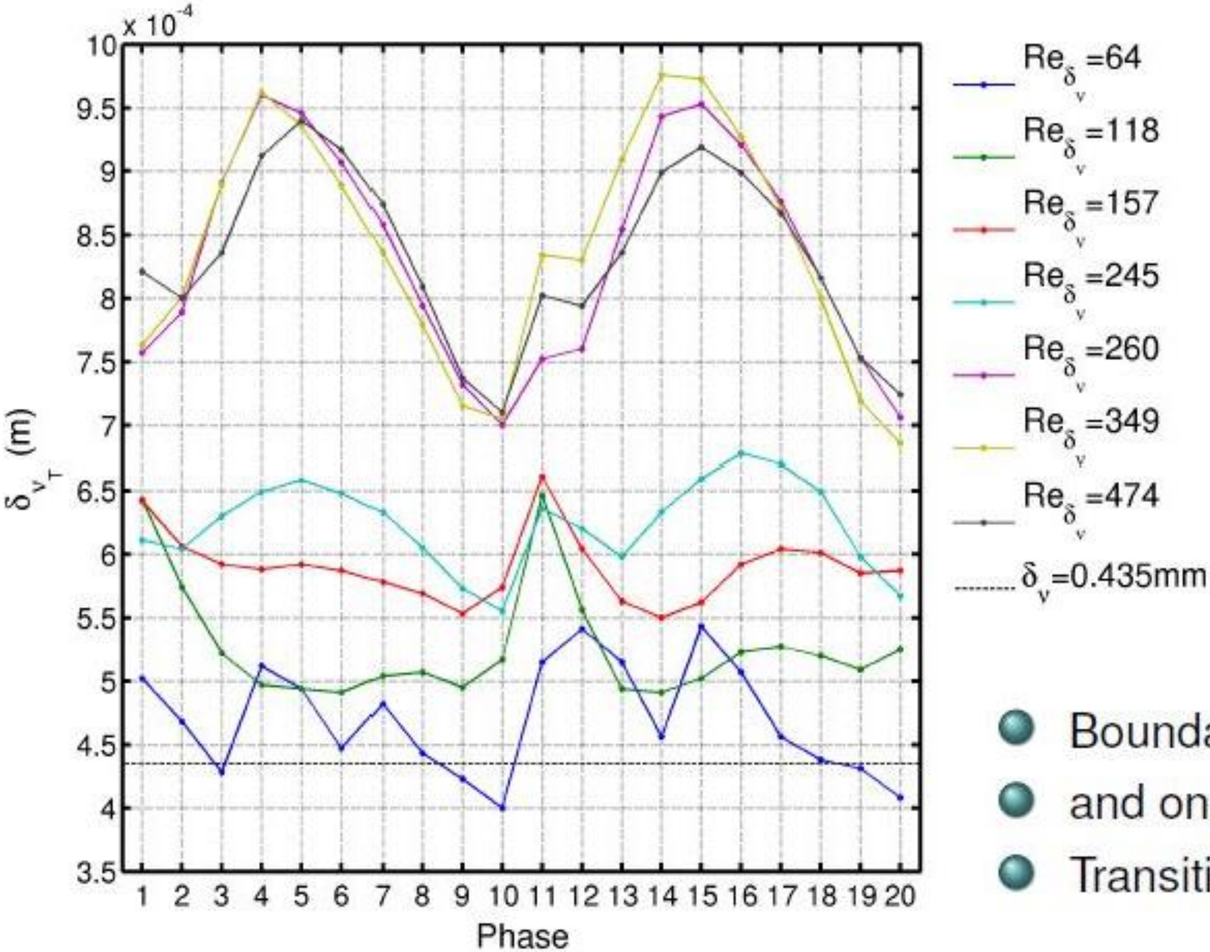





Ph1
Ph2
Ph3
Ph4
Ph5
Ph6
Ph7
Ph8
Ph9
Ph10
$Re_{\delta_\nu} = 64$
$Re_{\delta_\nu} = 118$
$Re_{\delta_\nu} = 157$
$Re_{\delta_\nu} = 260$
$Re_{\delta_\nu} = 349$
$Re_{\delta_\nu} = 474$
$(R-r)/\delta_\nu$
$u_{ac,ph}/U_c$
-0.5
0
0.5
5

As Re_{δ_ν} increases, the effective boundary layer thickness increases.

Estimation of the modified boundary layer thickness as a function of the phase and Reynolds number.



- Boundary layer depending on the phase,
- and on the acoustic level.
- Transition behaviour around $Re_{\delta_\nu} = 250$.

Back to real devices and their modelling

- *Ilinskii JASA2001:* For high amplitude wave excitation in a resonator with a shape that suppresses harmonic generation, significant excess losses are mostly not associated with harmonic generation but with increasing effective viscosity due to turbulence

$$\eta = \eta_0 + \eta_e$$

$$Re = \sqrt{2} Re_\delta$$

$$Re_0 = 400$$

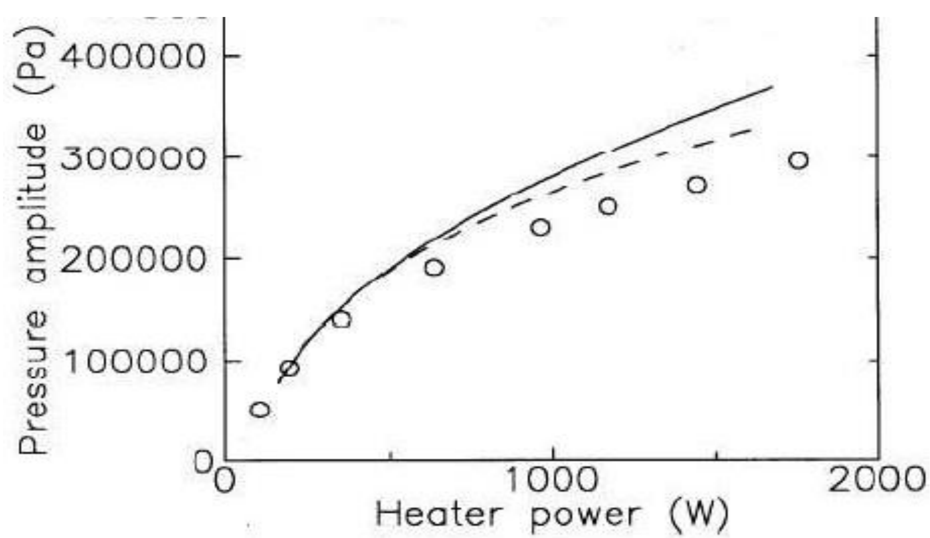
Eddy viscosity $$\eta_e = \psi \eta_0 \left[\sqrt{1 + \{\frac{Re}{Re_0}\}^2} - 1 \right]$$

Ψ is determined by matching calculated and measured dissipation

- *Swift2001* : A time dependent friction factor is estimated based on steady flow theory and a Taylor-series expansion around the peak Reynolds number

$$f_M(t) = f_M + \frac{df_M}{dRe} Re \left[\frac{|\Re\left[U e^{i\omega t}\right]|}{|U|} - 1 \right]$$

$$\Delta p = f_M \frac{L}{D} \frac{1}{2} \rho \langle u \rangle^2 .$$



Approach considered as unsatisfactory by the authors themselves

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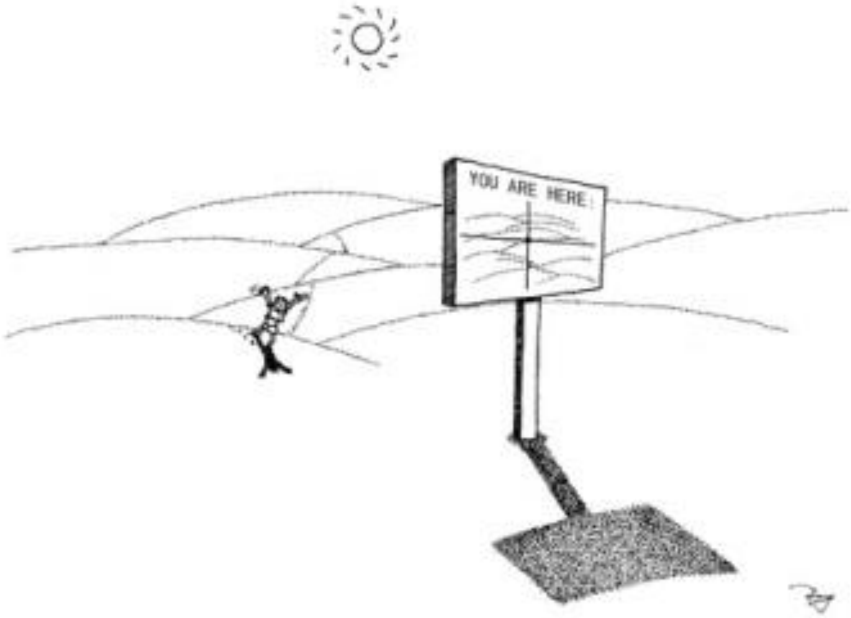

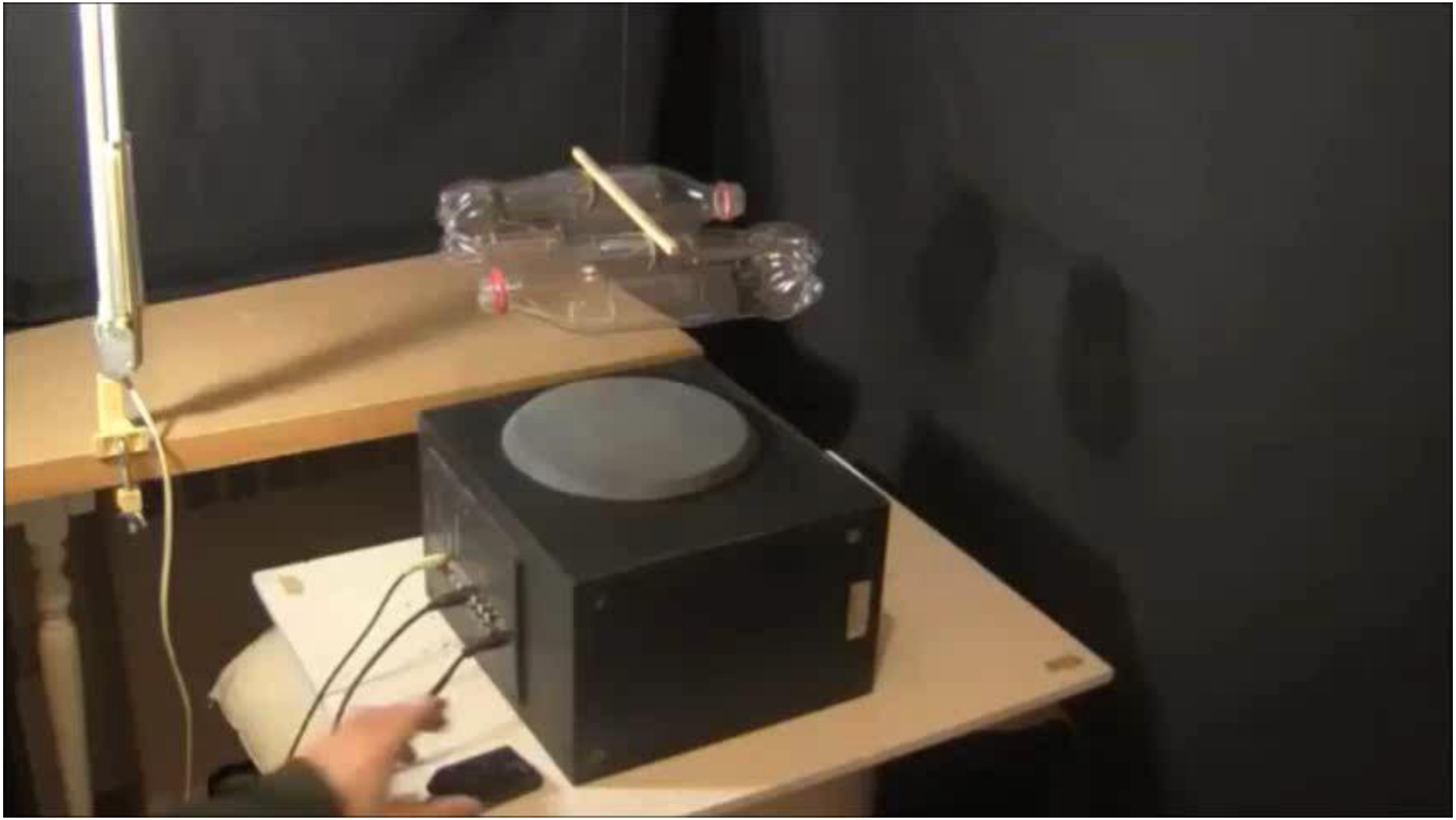

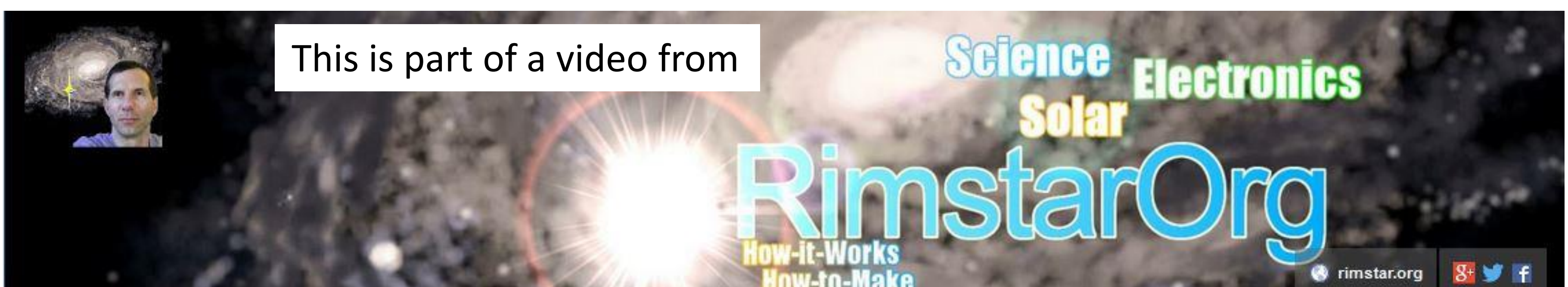
This is part of a video from
Science
Electronics
Solar
RimstarOrg
How-it-Works
How-to-Make
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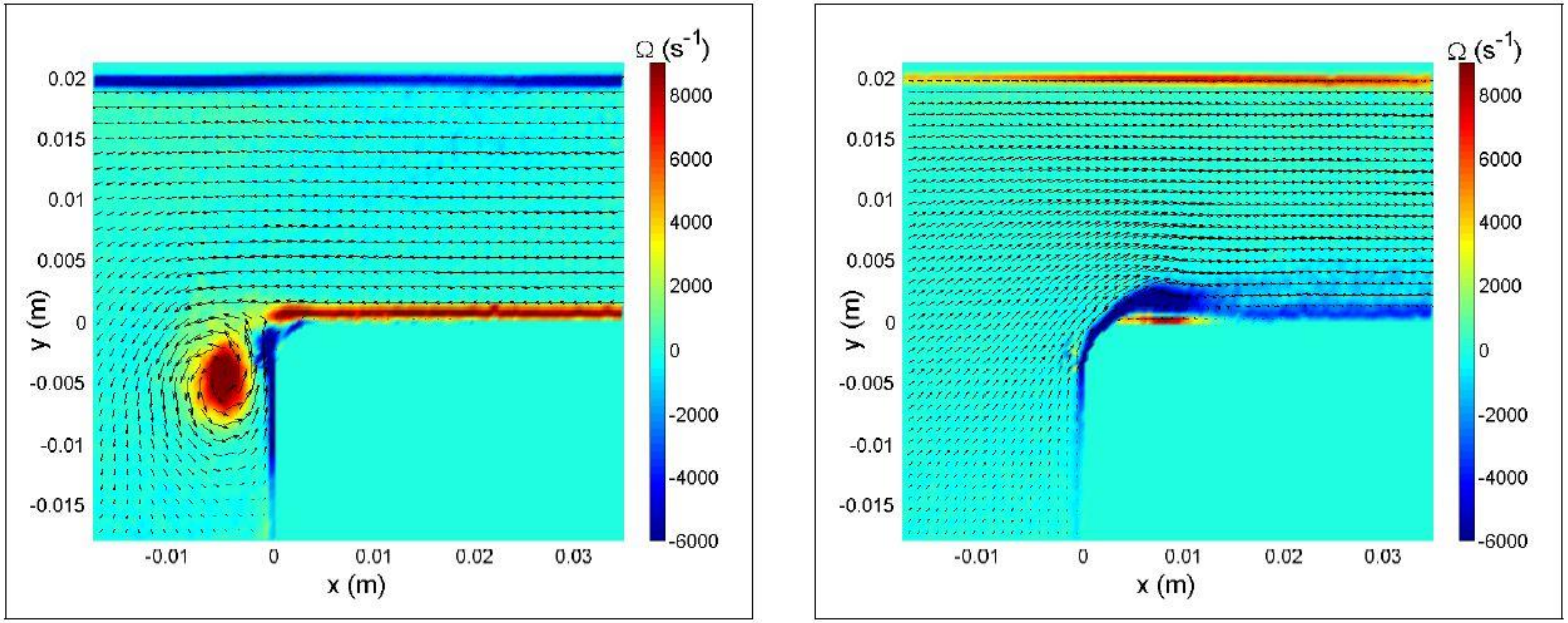

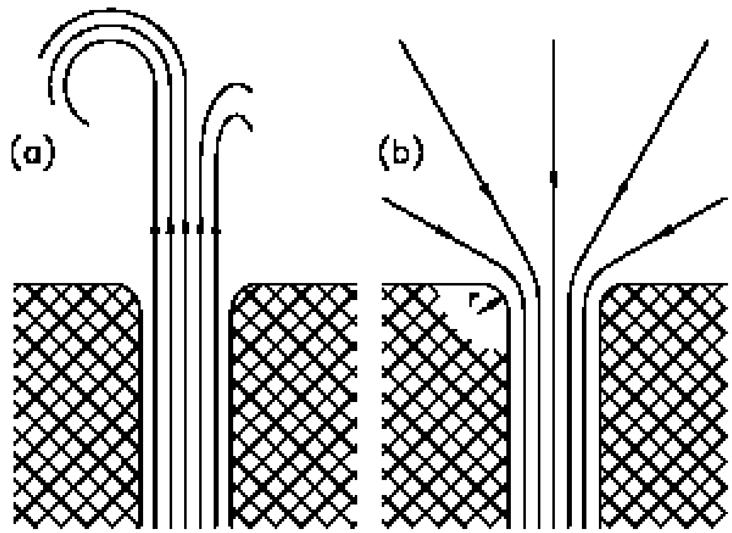

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Figure 7.14: Asymmetry of high-Reynolds-number flow at a transistion between a small tube and wide-open space. (a) For outflow, a jet extends far into the open space, and downstream turbulence dissipates kinetic energy. (b) For inflow with a well rounded entrance lip, there is little dissipation.

In an oscillating flow, a net pressure difference is established with the pressure in the smaller channel being greater than that in the larger one

↓

Minor losses

Minor losses in oscillating flow with quasi-steady hypothesis:

Mean pressure drop

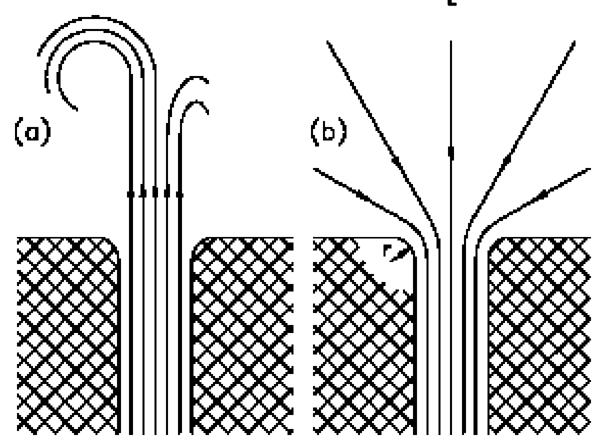
$$\Delta p(t) = -\frac{1}{2A^2} K(t) \rho(t) \, |U(t)| \; U(t).$$

$$\begin{aligned} \overline{\Delta p_{ml}} &= -\frac{\omega}{2\pi A^2} \left(\int_0^{\pi/\omega} K_{\text{out}} \frac{1}{2} \rho |U_1|^2 \sin^2 \omega t \, dt - \int_{\pi/\omega}^{2\pi/\omega} K_{\text{in}} \frac{1}{2} \rho |U_1|^2 \sin^2 \omega t \, dt \right) \\ &= -\frac{1}{8A^2} \rho |U_1|^2 \left(K_{\text{out}} - K_{\text{in}} \right). \end{aligned}$$

Mean energy loss

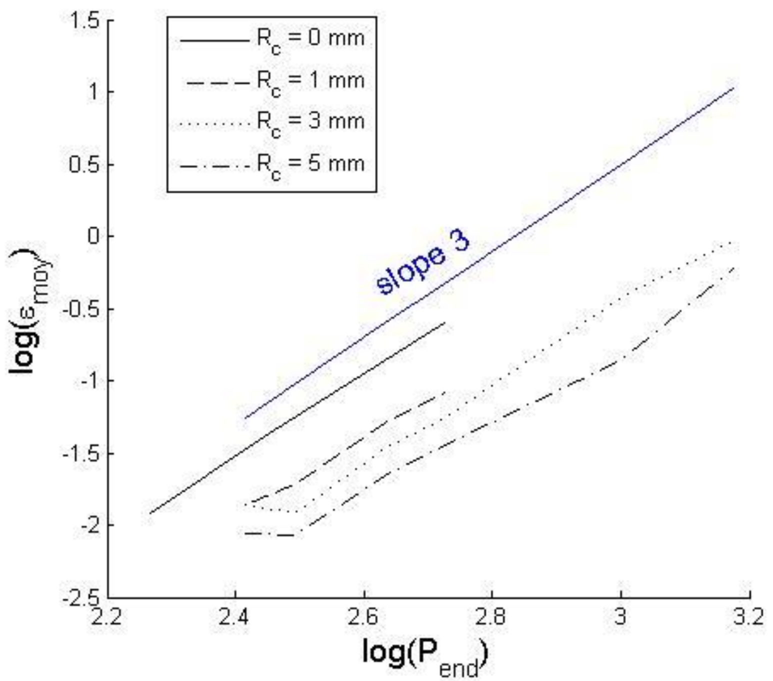
$$\overline{\Delta E} = \overline{u \Delta p} \propto \frac{1}{3\pi} \rho U_{ac}^3 S_1 \left(K_{ejec} + K_{inj} \right) \approx U_{ac}^3$$

$K_{out} = 1$ $\quad$ $K_{in} \in [0.04\text{-}0.5]$



Mean dissipation

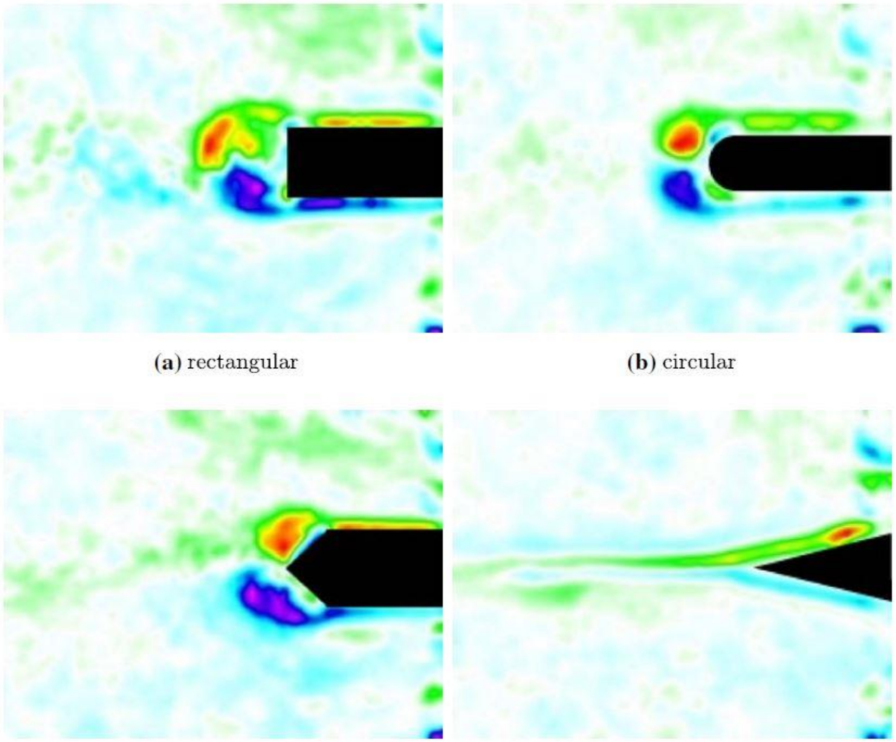
Marx Acta Acustica 2008



$\Rightarrow \varepsilon_{moy} \approx P_{end}^3 \approx U_{ac}^3$ (as expected)

Time-Space averaged turbulent dissipation

Morris Acta Acustica 2004 found that the calculated level of the pressure difference is greater by a factor of three than that estimated using a quasi-steady hypothesis



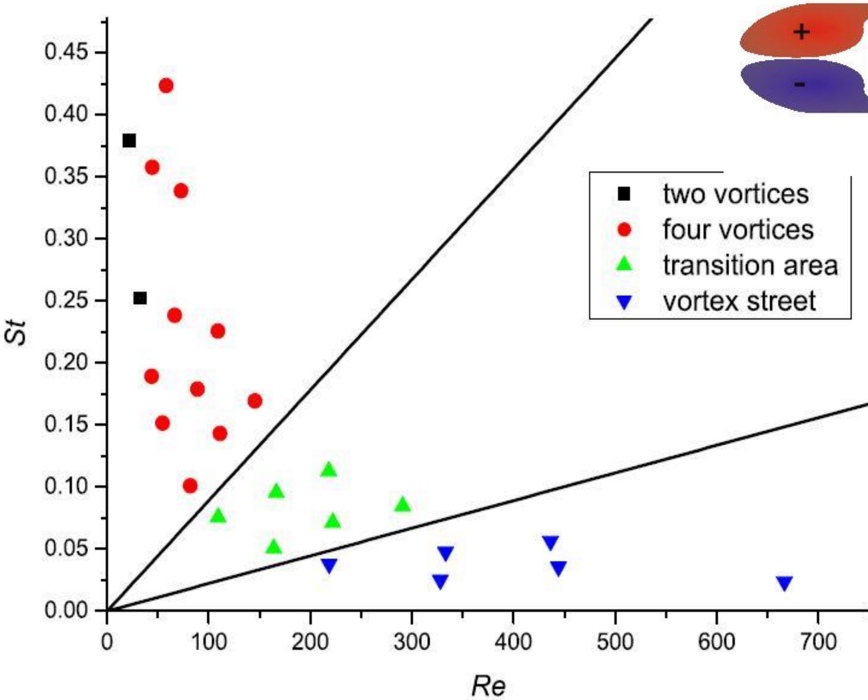
(a) rectangular

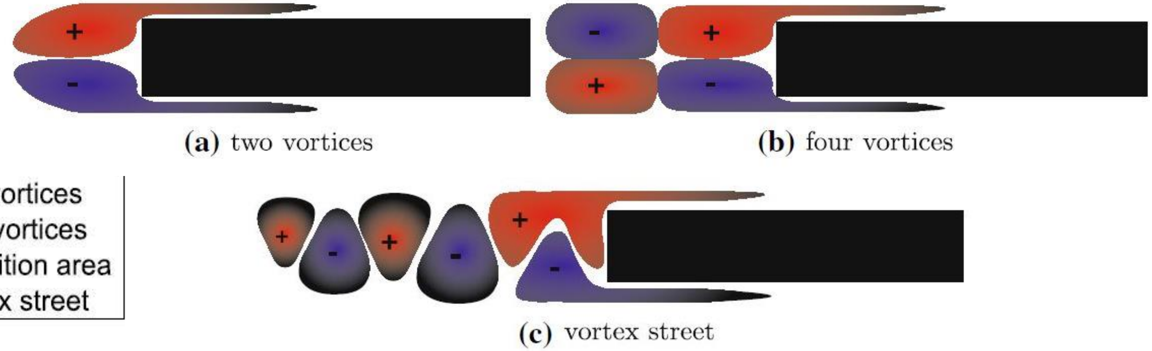
(b) circular

(c) triangular

(d) sharp triangular

Abem Exp Fluids 2009

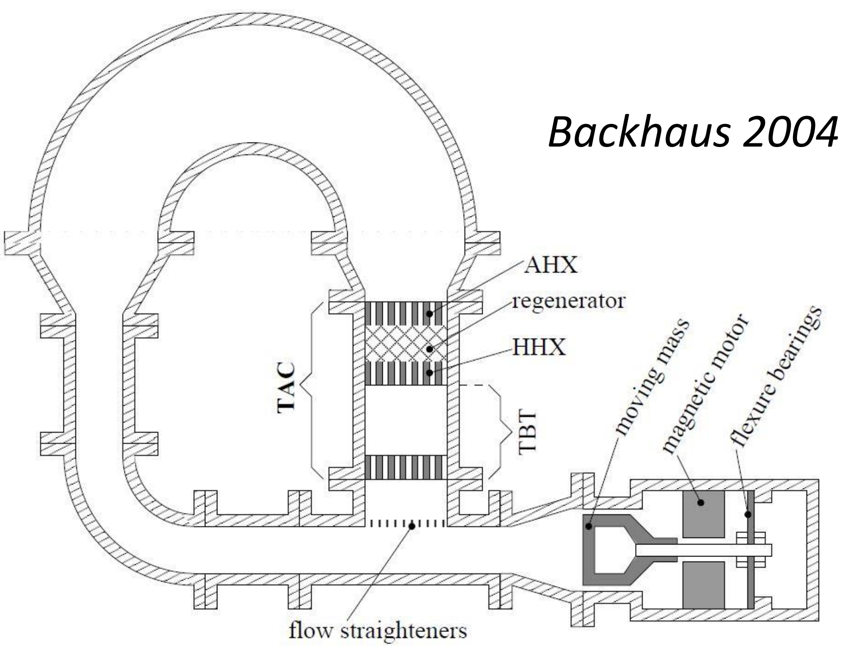


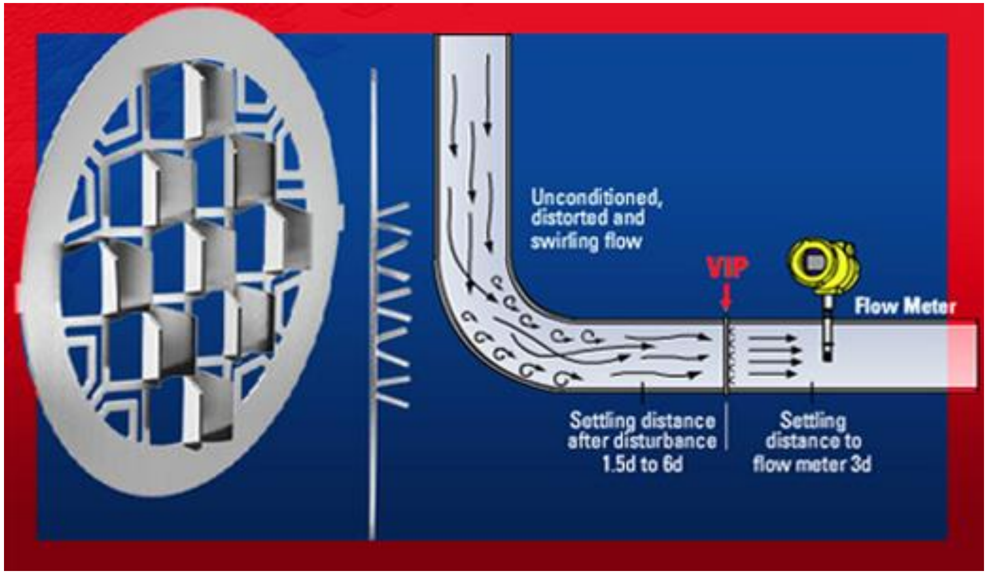


$$Re = \frac{Vd}{\nu}, St = \frac{fd}{V},$$

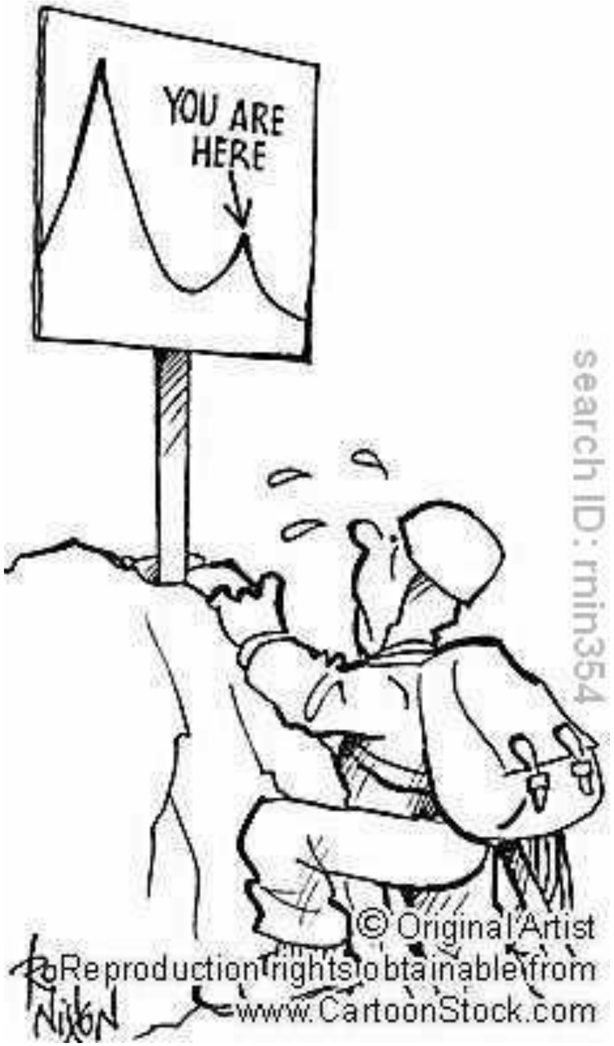
Abem Exp Fluids 2009

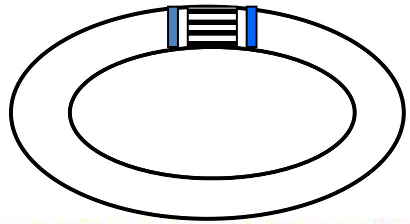
Backhaus 2004

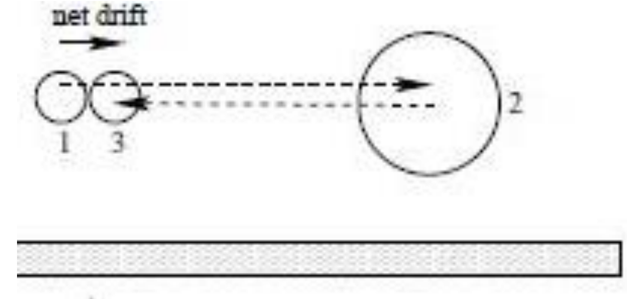




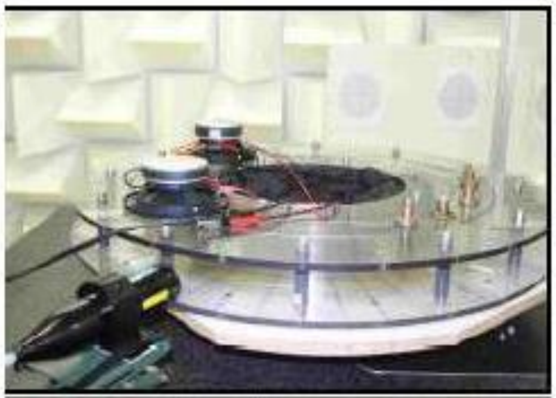
- Introduction
- Some bases
- Turbulence
- Edge effects
- **Acoustic streaming**
- Conclusion

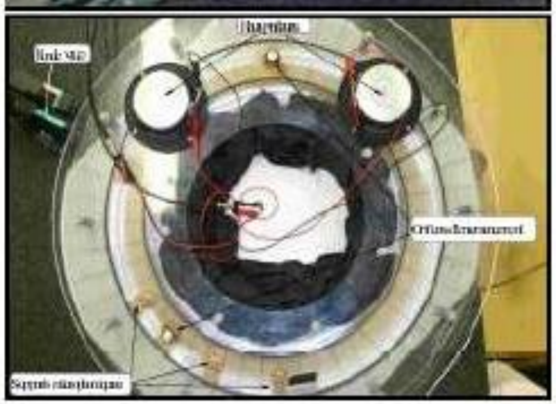


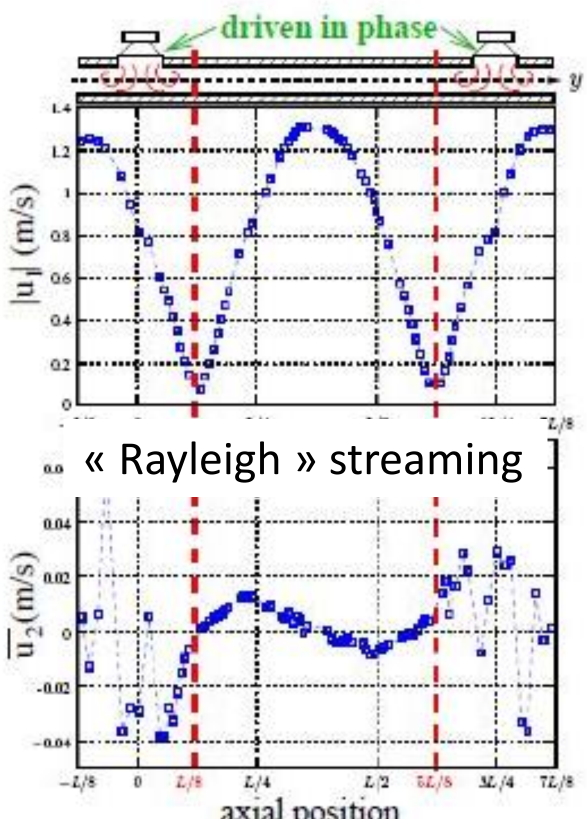


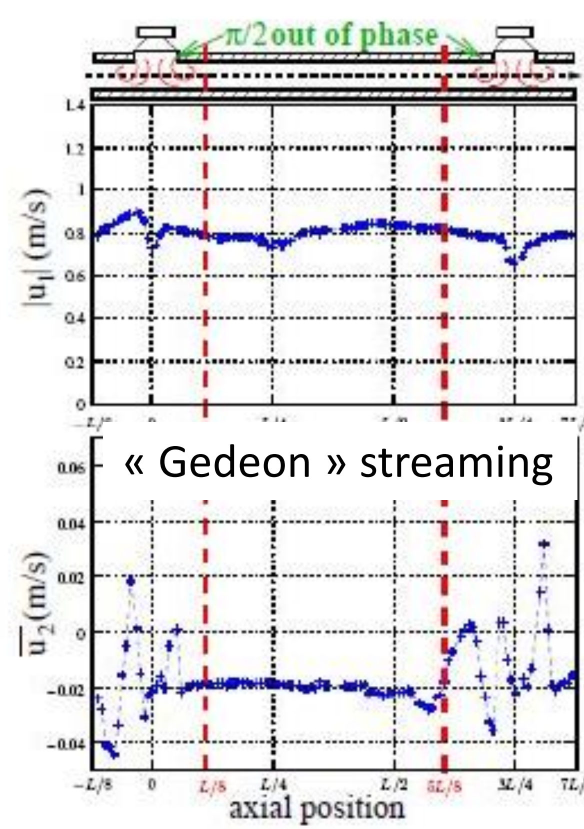


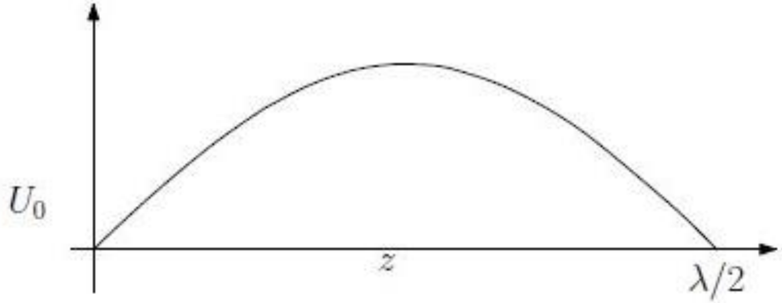
Acoustic streaming in a closed-loop resonator (Desjouy et al.,JASA,2009)

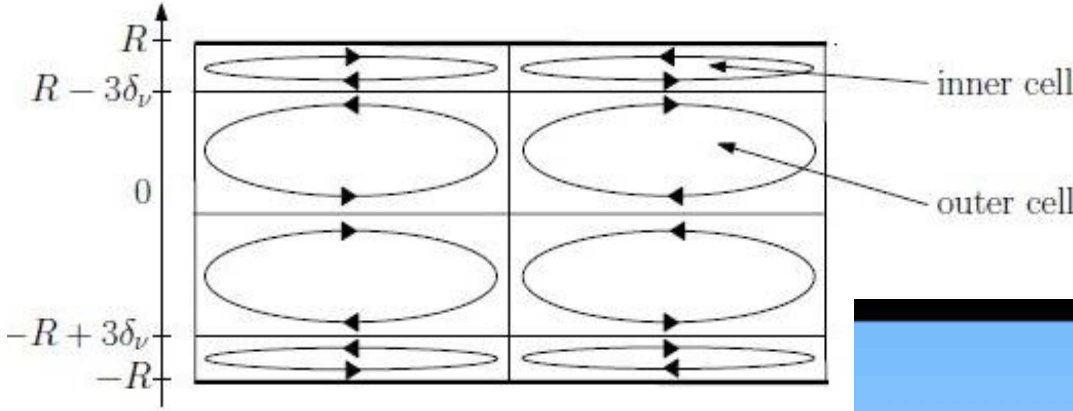






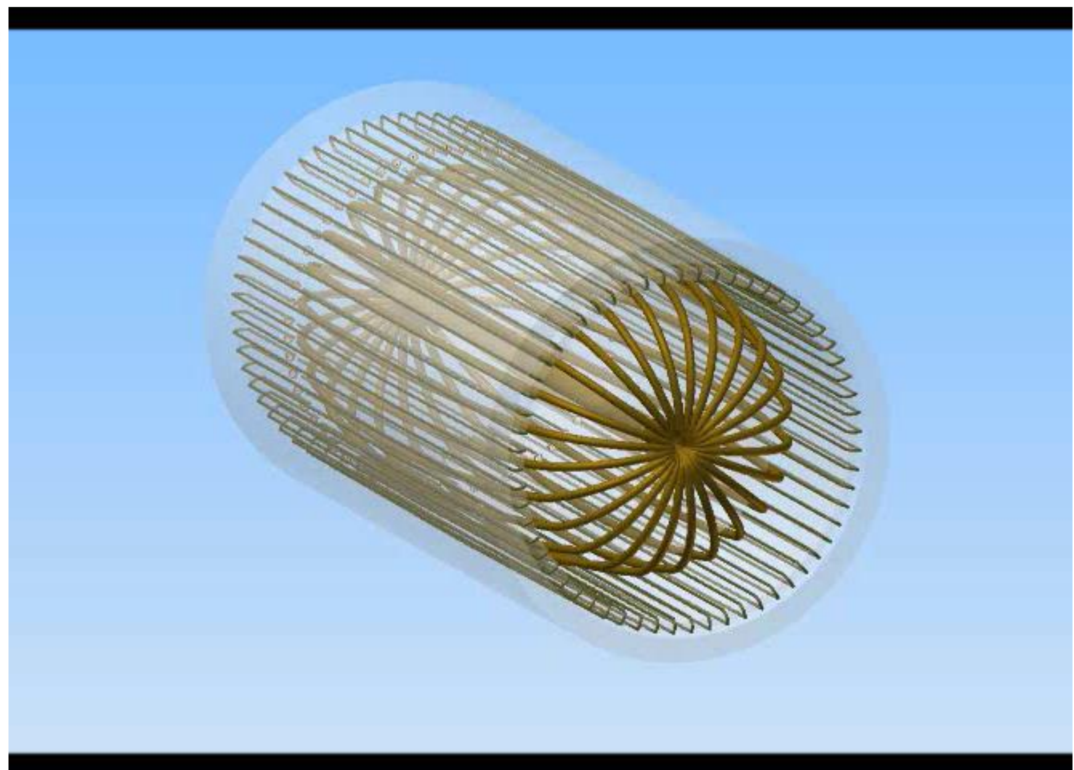


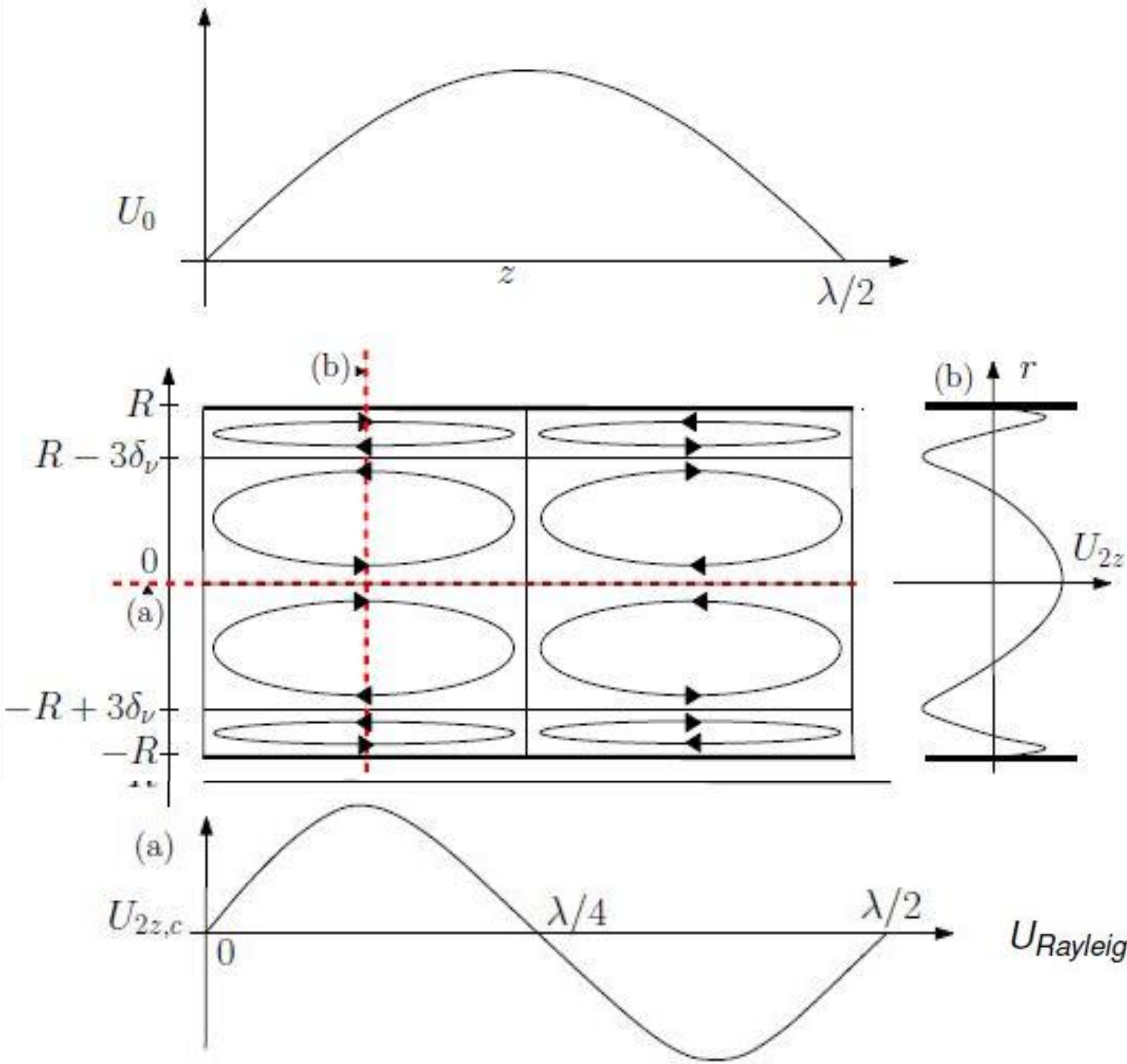




·oïdal vortices with $\lambda/2$ periodicity

re $R/\delta\nu = 10$





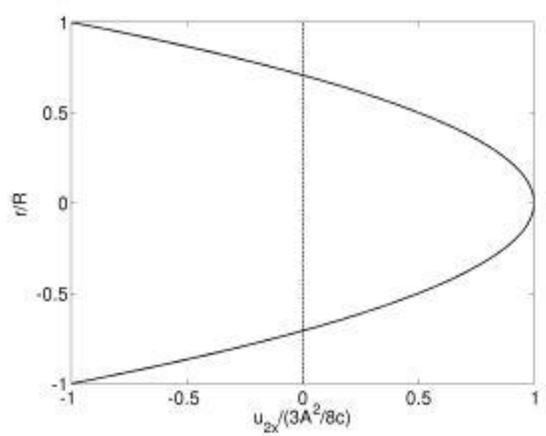
$$U_{Rayleigh} = \frac{3U_0^2}{8c_0}\sin\left(\frac{4\pi z}{\lambda}\right)$$

The theorical developments available in the litterature for 2D channels : 2 infinite plates or in a cylindrical wave guide.

For a velocity field outside the boundary layer : $U_{ac} = A\sin(kx)\cos(\omega t)$

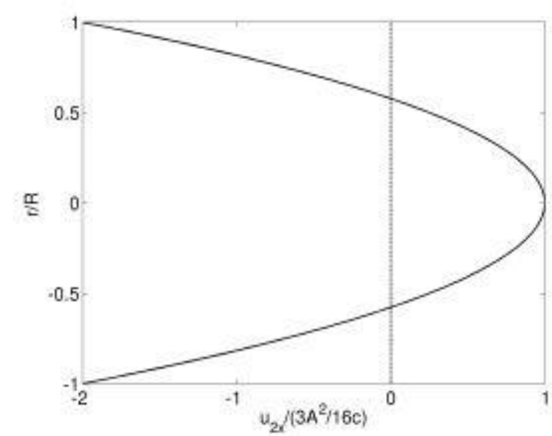
expression of outer streaming in a cylindrical guide

$$u_{2x}(x,r) = \frac{3A^2}{8c}(1-2\frac{r^2}{R^2})\sin(2kx)$$

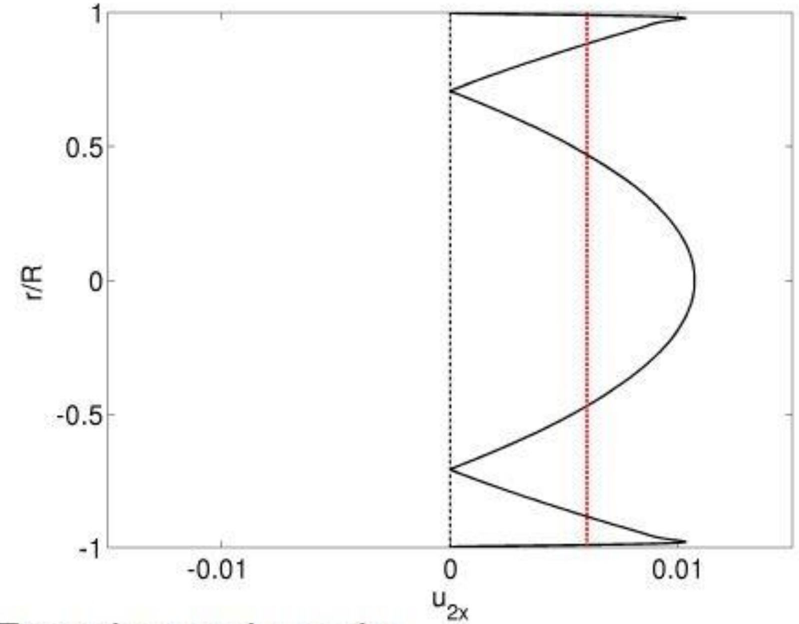


expression of outer streaming between 2 plates

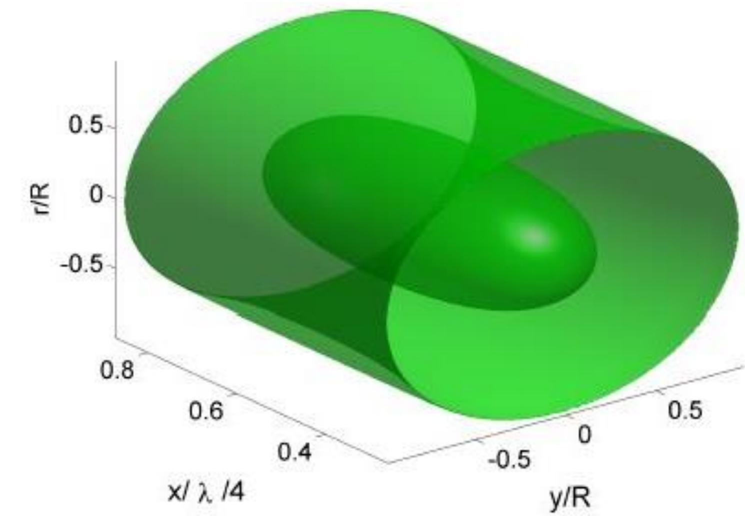
$$u_{2x}(x,r) = \frac{3A^2}{16c}(1-3\frac{r^2}{R^2})\sin(2kx)$$



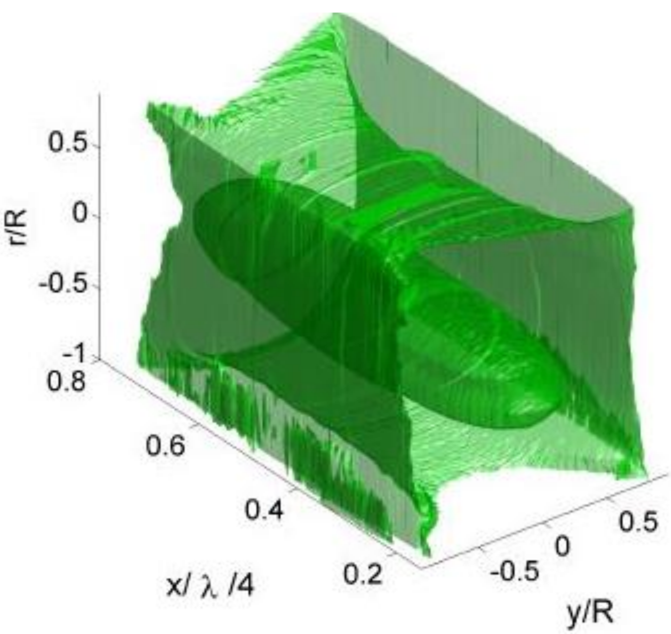
Radial profile of axial streaming velocity

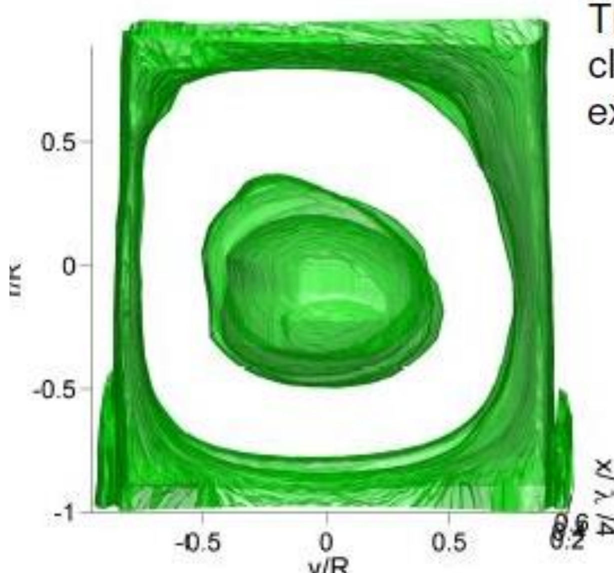


isosurface for streaming
velocity magnitudes equal to $0.006 m/s$



Experimental results





The experimental results are very close to the cylindrical shape except in the near wall region.

$$M \ll 1, \qquad Re_a \gg 1, \qquad Sh \ll 1$$

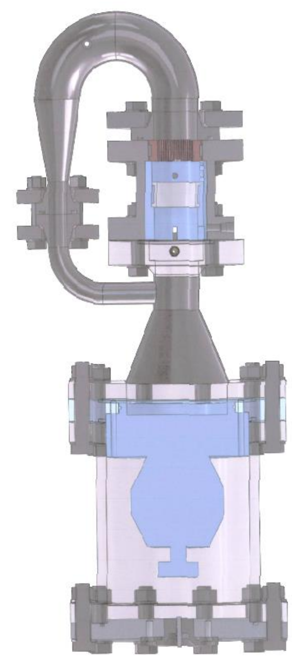
Fundamental equations for acoustic streaming *(Menguy & Gilbert 2000)*

$$\begin{cases} \dfrac{\partial u_{2s}}{\partial z} + \dfrac{1}{r}\dfrac{\partial}{\partial r}(r v_{2s}) = 0 \\ \dfrac{1}{r}\dfrac{\partial}{\partial r}\left(r\dfrac{\partial u_{2s}}{\partial r}\right) = \underbrace{\dfrac{M^2}{Sh^2}}_{Re_{NL}}\left(u_{2s}\dfrac{\partial u_{2s}}{\partial z} + v_{2s}\dfrac{\partial u_{2s}}{\partial r}\right) + f(z) \end{cases}$$

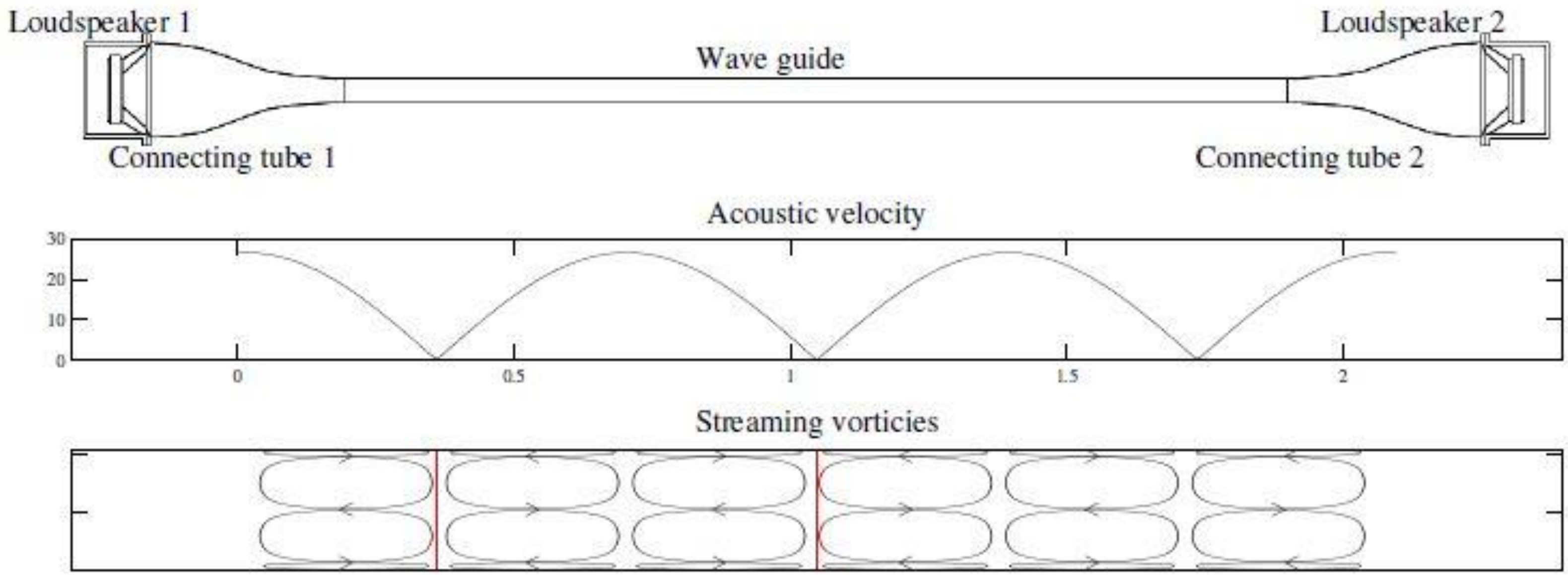
f(z) includes the second order pressure gradient + quadratic terms

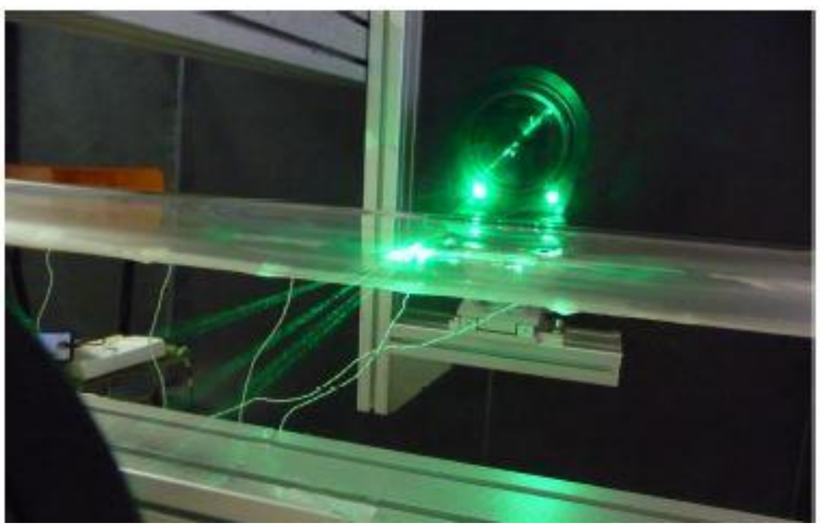
Acoustic streaming well described for $Re_{NL} \ll 1$ (slow streaming) only

$Re_{NL} = 16$



$Re_{NL} = 13$

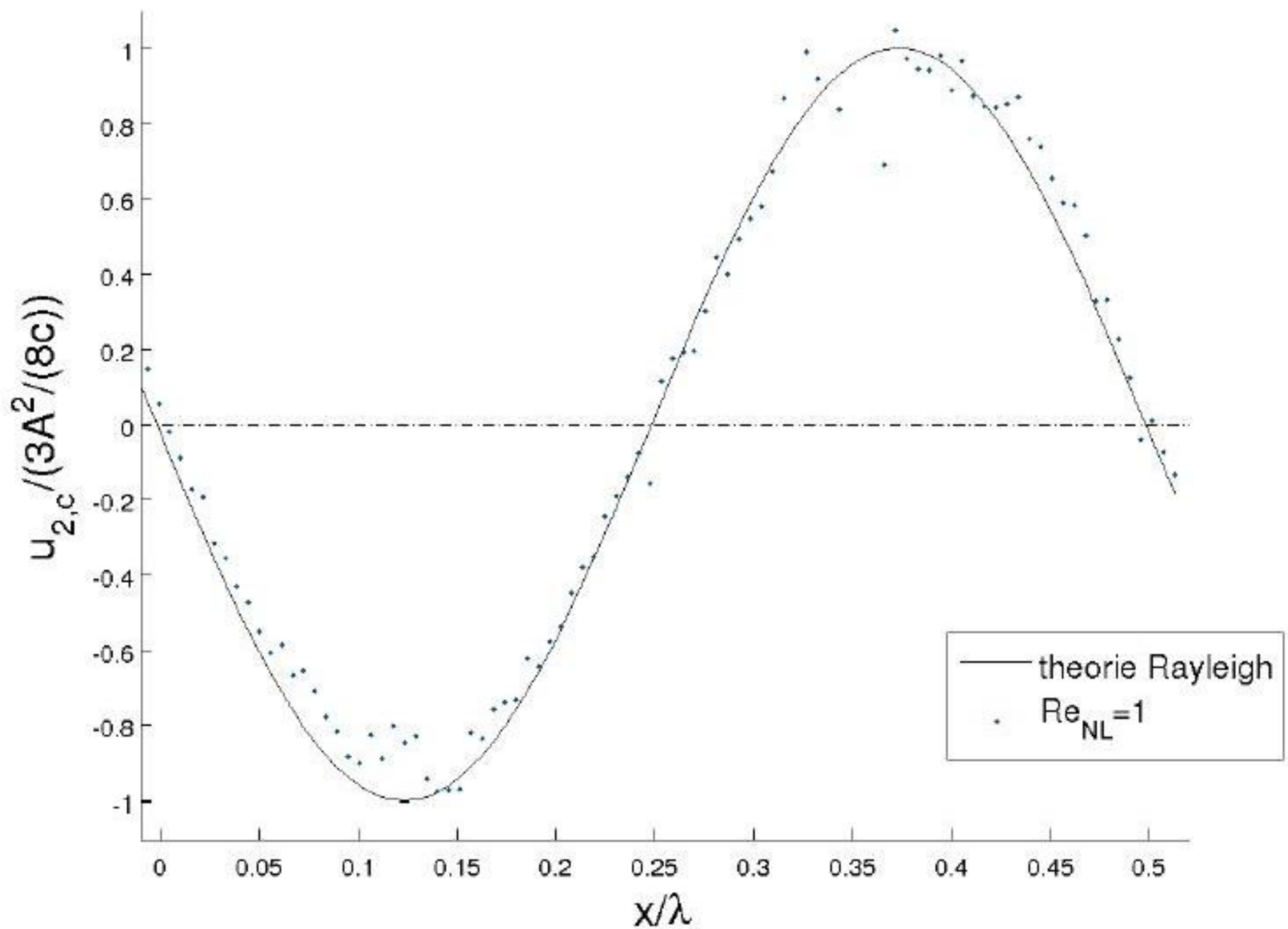




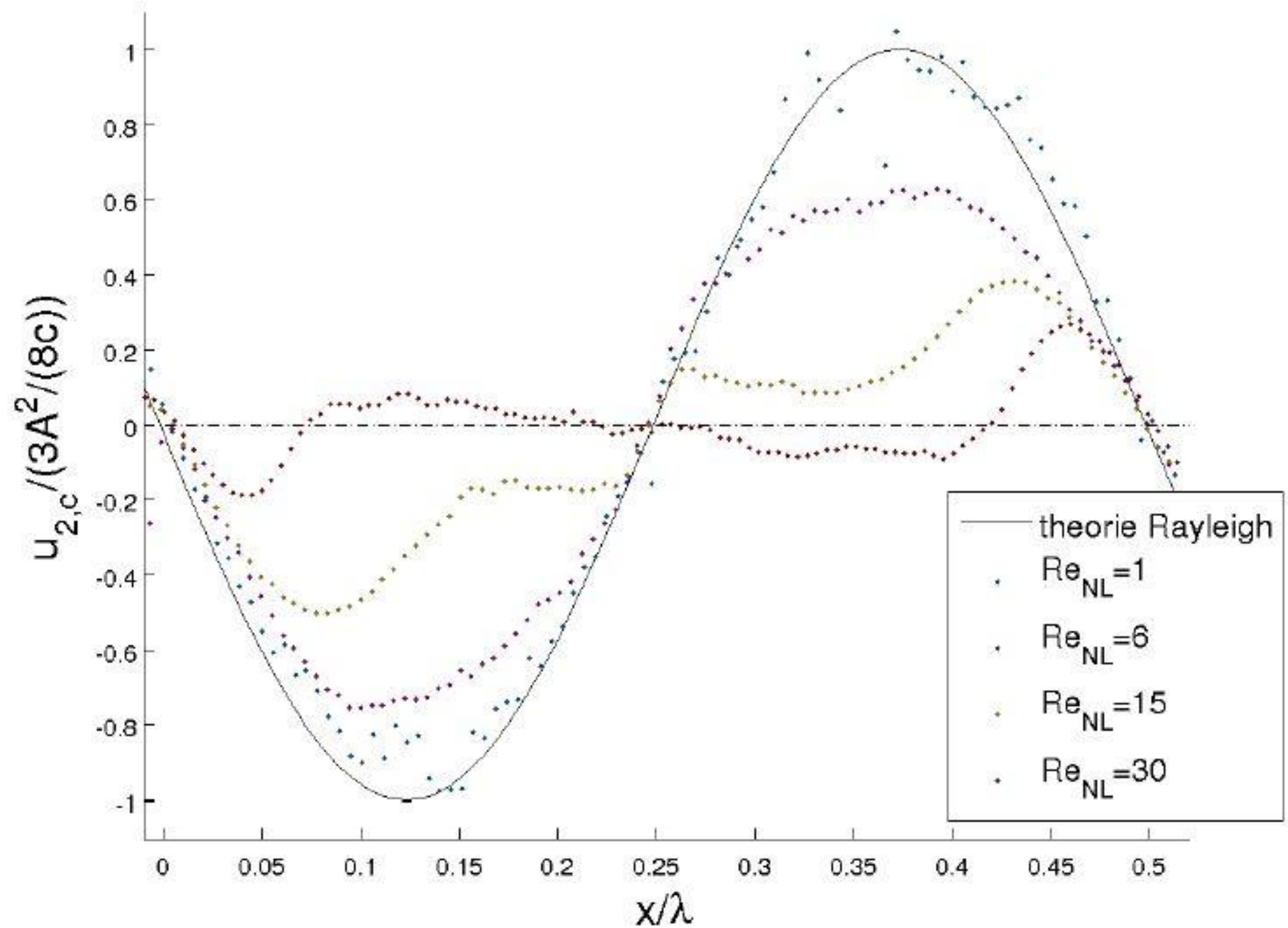
- fréquence : $f_{ac} = 240Hz$
- guide cylindrique
- $R = 19.5mm$
- $L_{exp} = 3\lambda/2 = 2.13m$
- air à pression atmospherique
- paroi quasi-isotherme
- mesure LDV

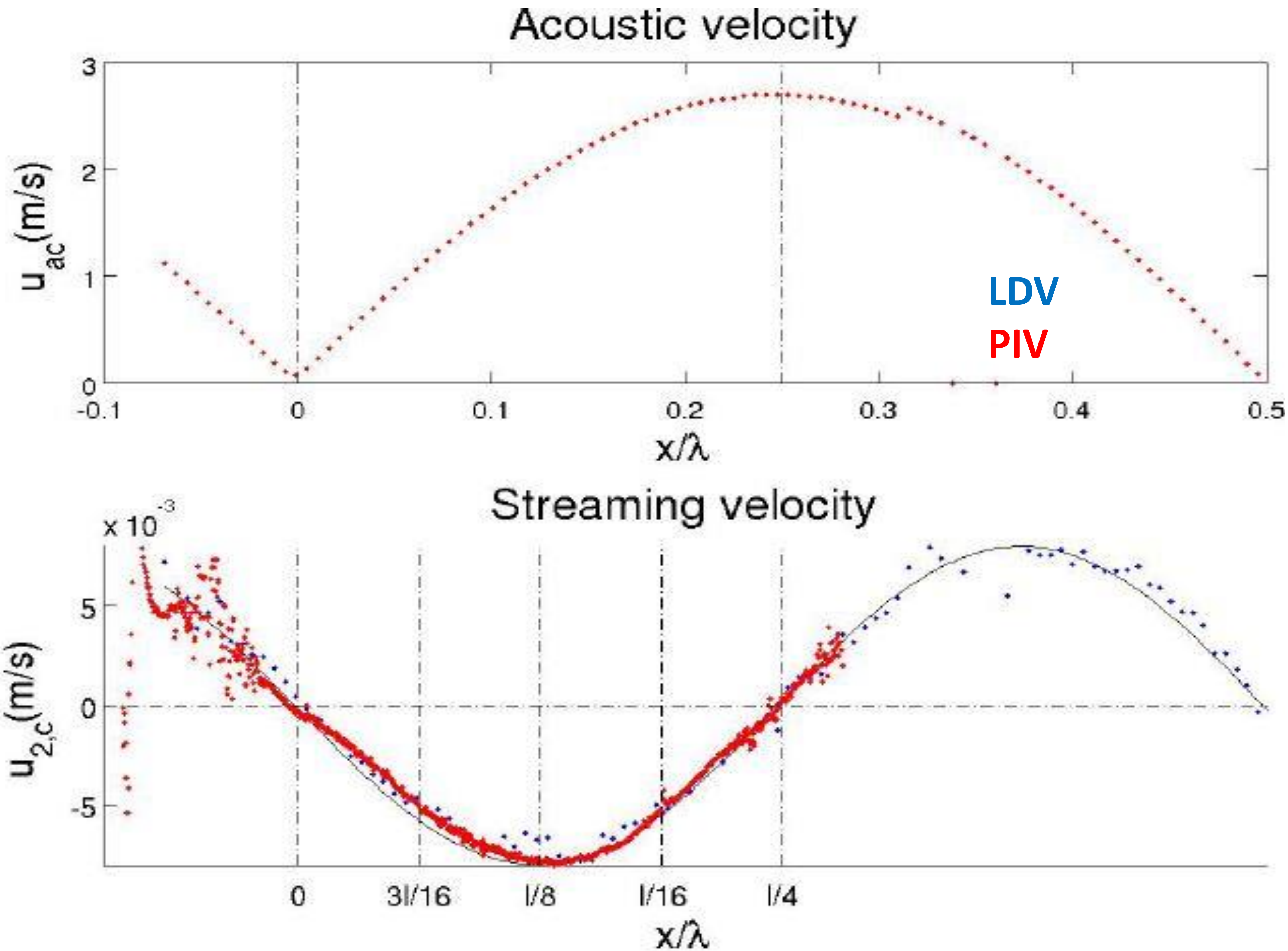
Centerline axial streaming velocity

A : acoustic velocity amplitude

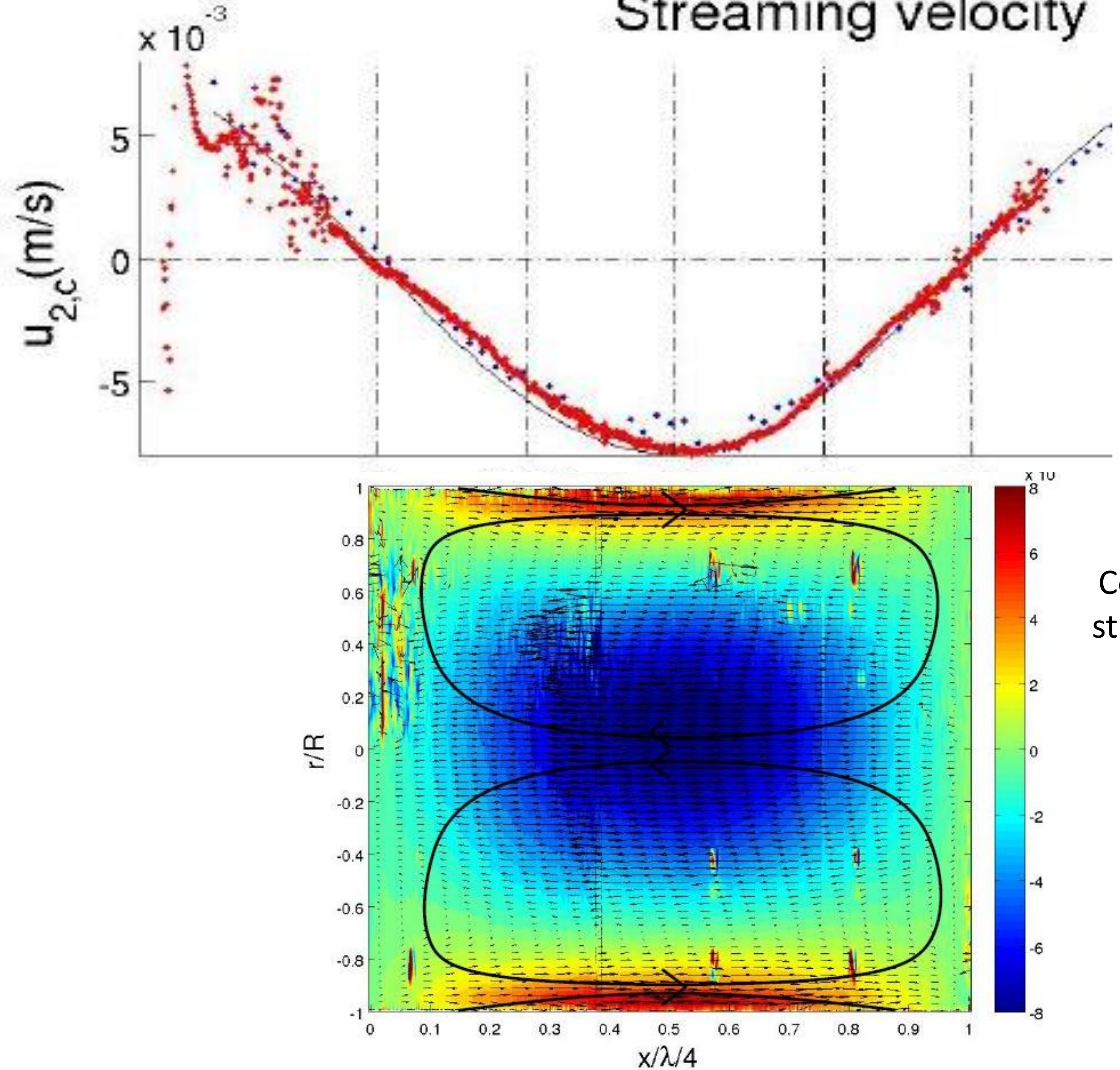


Reyt JASA 2014

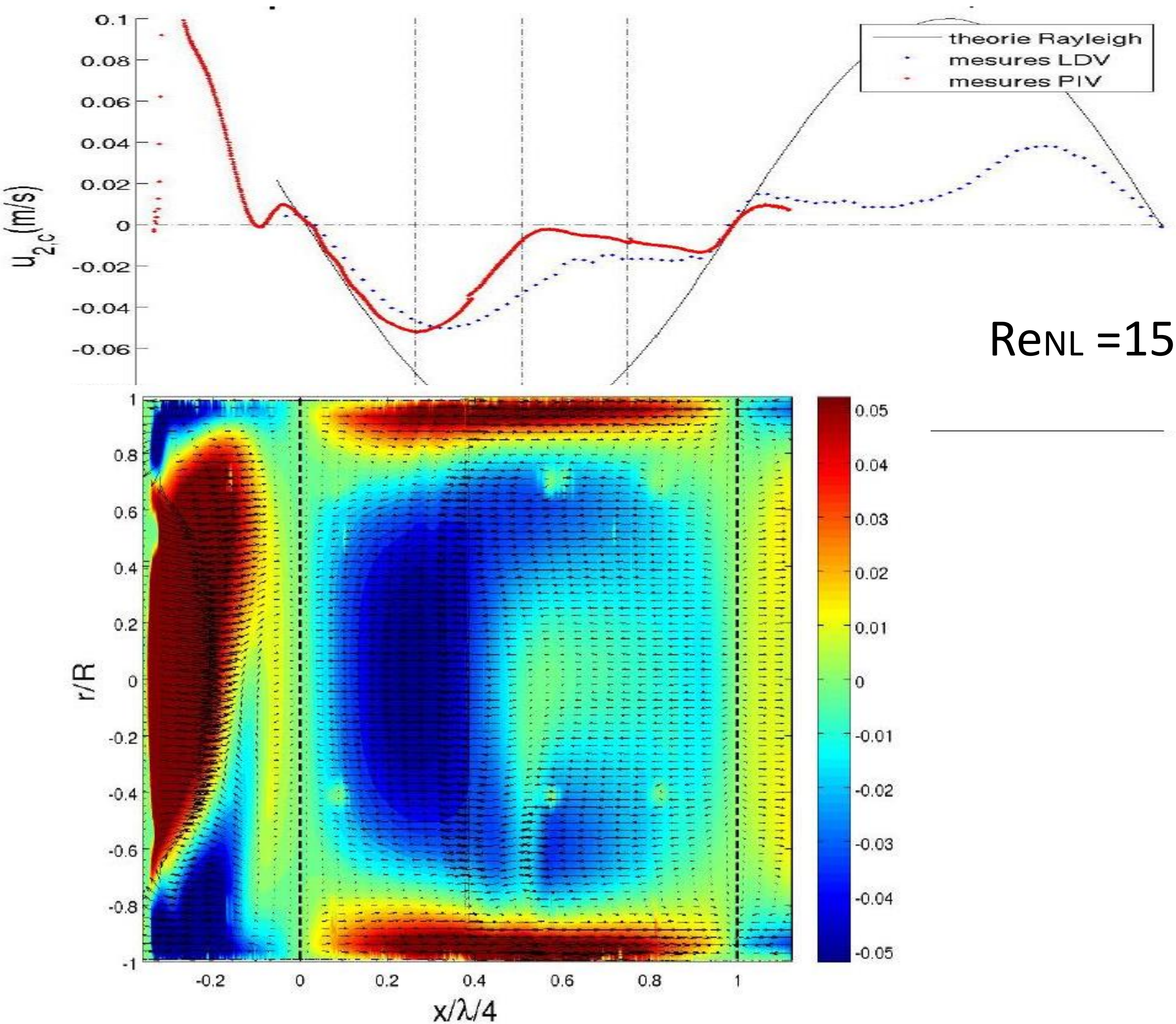



Acoustic velocity
u_{ac}(m/s)
3
2
1
0
-0.1
0
0.1
0.2
0.3
0.4
0.5
x/λ
LDV
PIV
Streaming velocity
x 10^-3
$u_{2,c}$(m/s)
5
0
-5
0
3l/16
l/8
l/16
l/4
x/λ

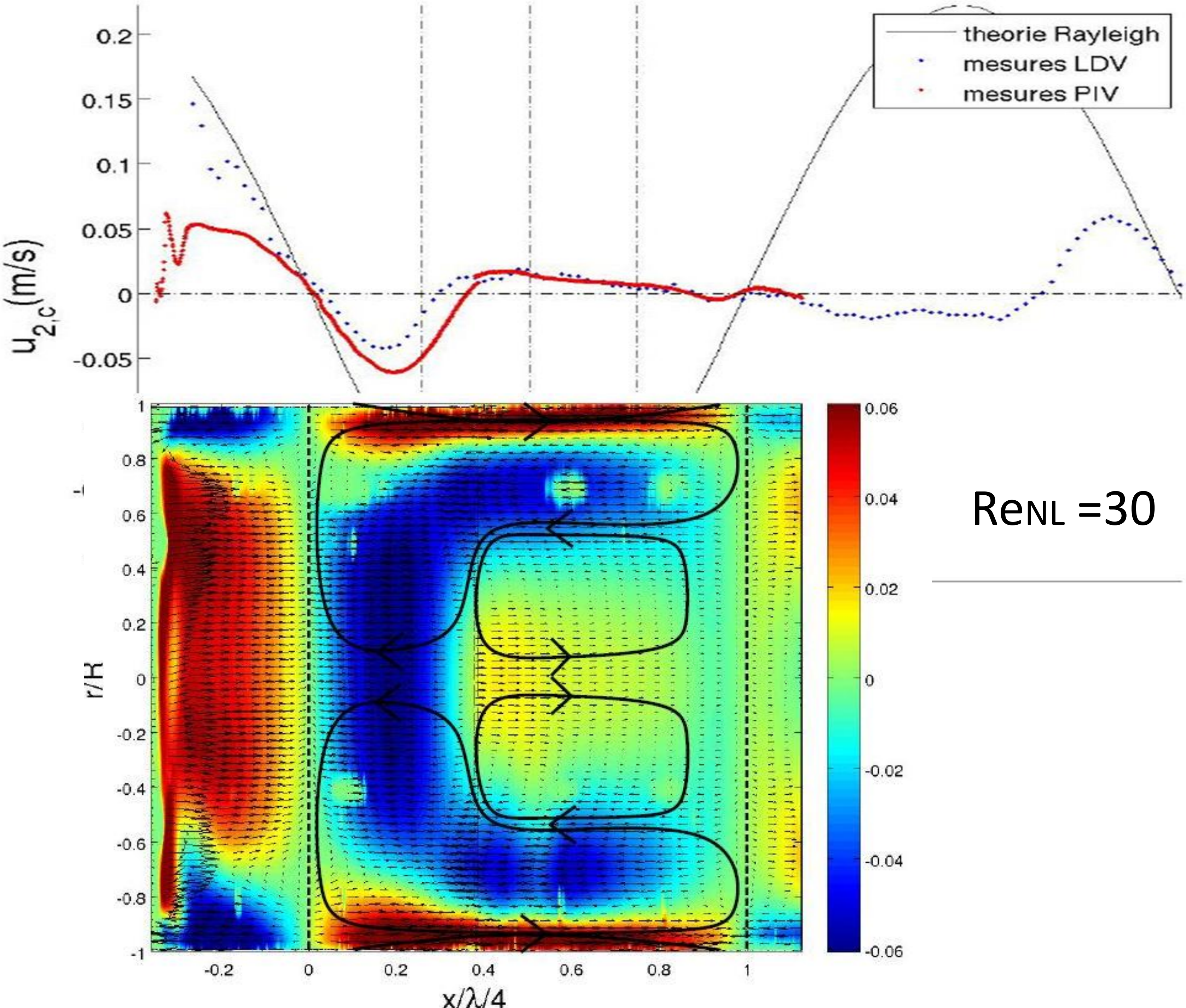
Streaming velocity



Colormap -> axial streaming velocity

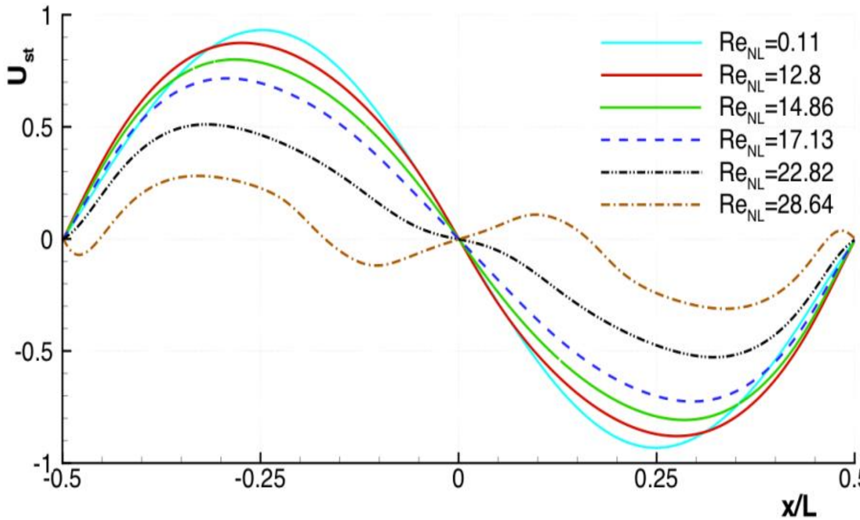


Re_{NL} =15

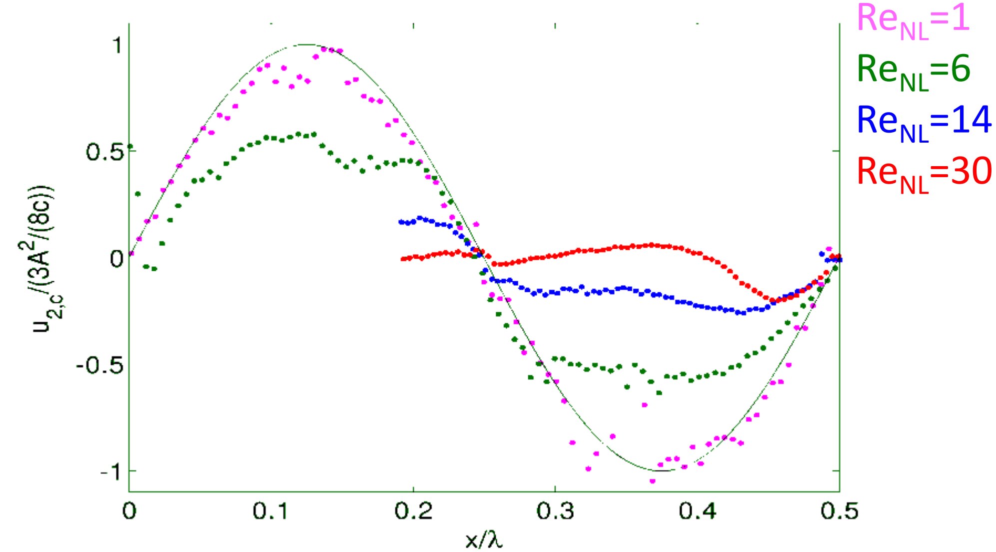


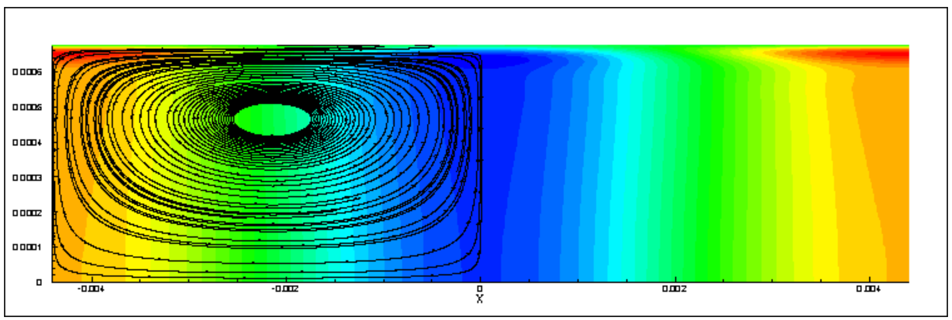
Re$_{NL}$ =30

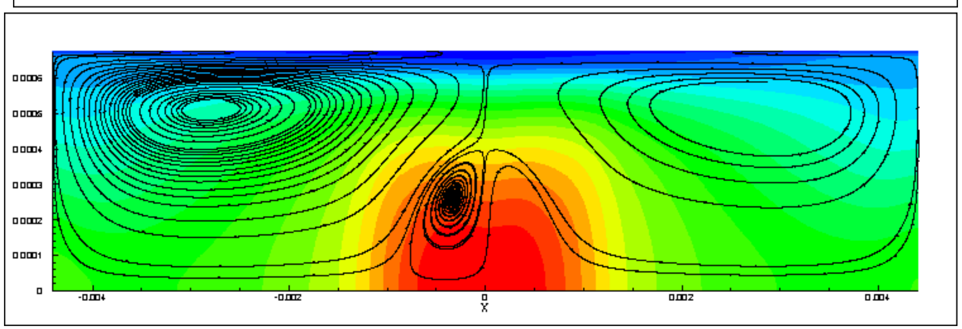
Navier-Stokes simulation

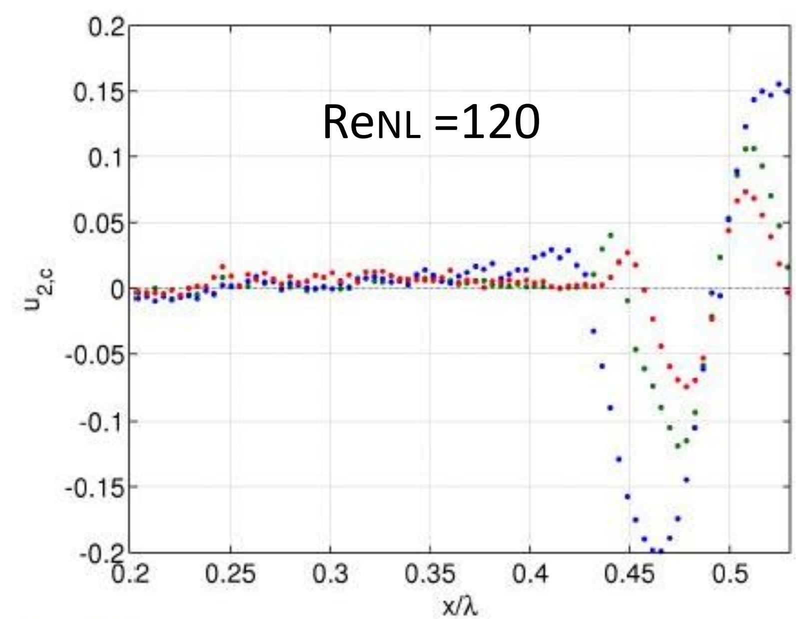


Reyt JASA 2013

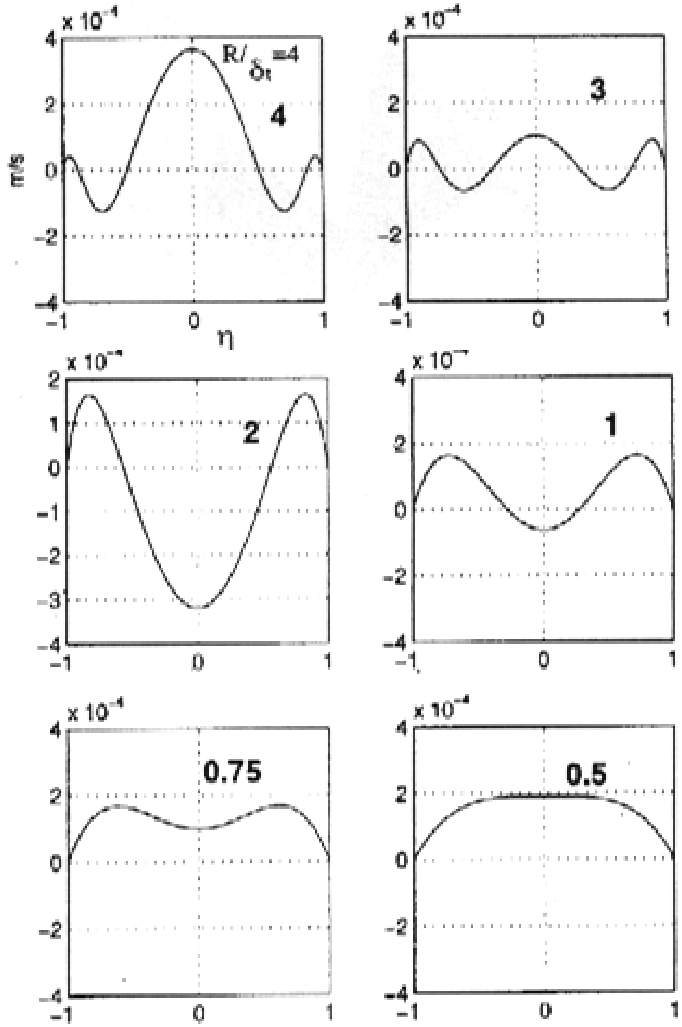




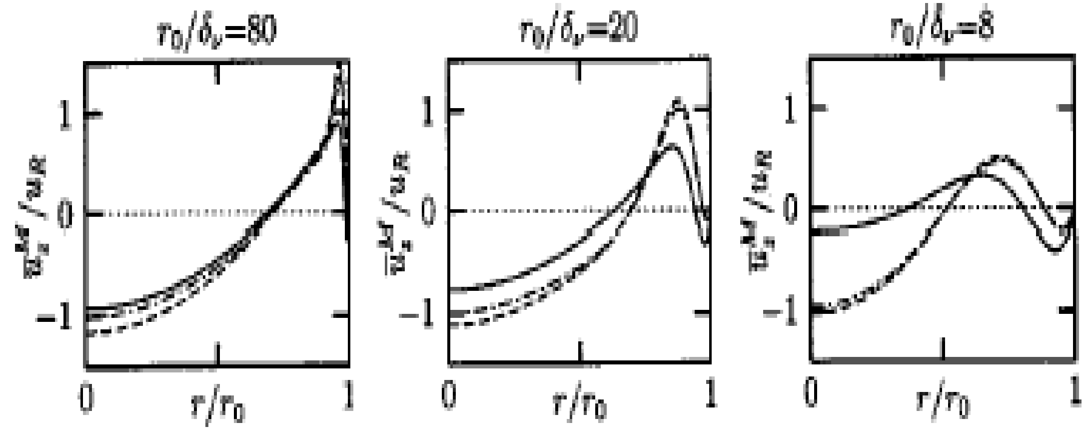




Ventilated : 1°C
Uncontrolled : 6.5°C
Insulated : 10°C

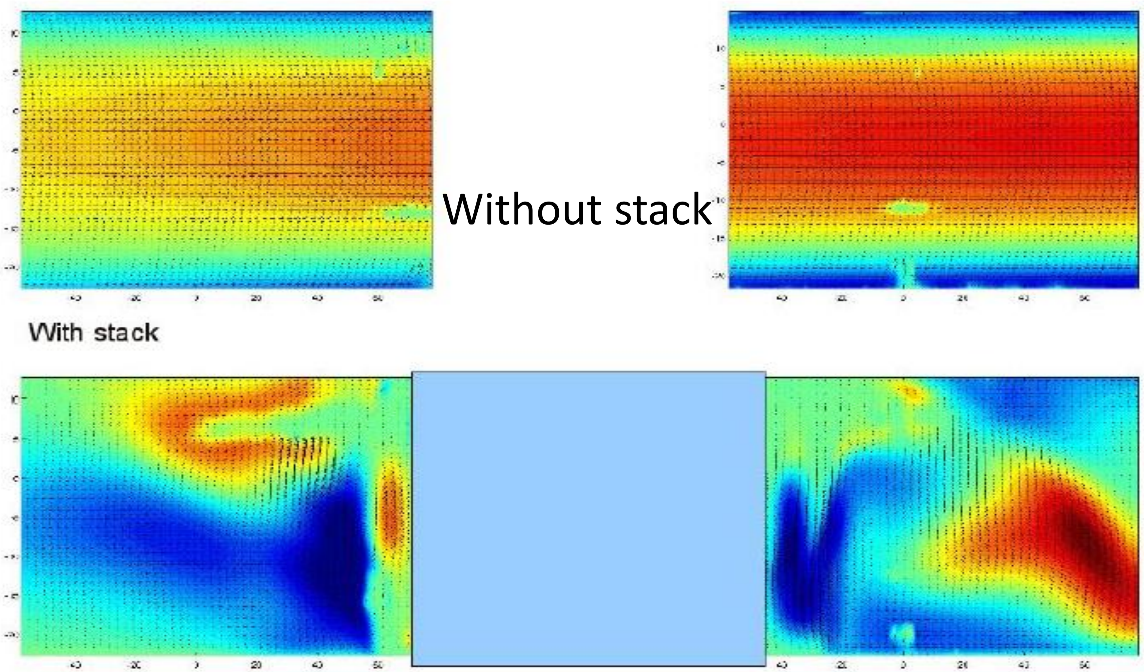


Bailliet JASA 2001



Hamilton JASA 2003

FIG. 6. Acoustic streaming patterns corresponding to Fig. 5, but illustrating influence of viscosity dependence on temperature in cylindrical tubes of different radii. Solid lines are for Pr=∞ and $b=0$, dashed lines are for Pr $=0.2$ and $b=0$, and dot-dash lines are for Pr$=0.2$ and $b=1$.



Moreau JASA 2009

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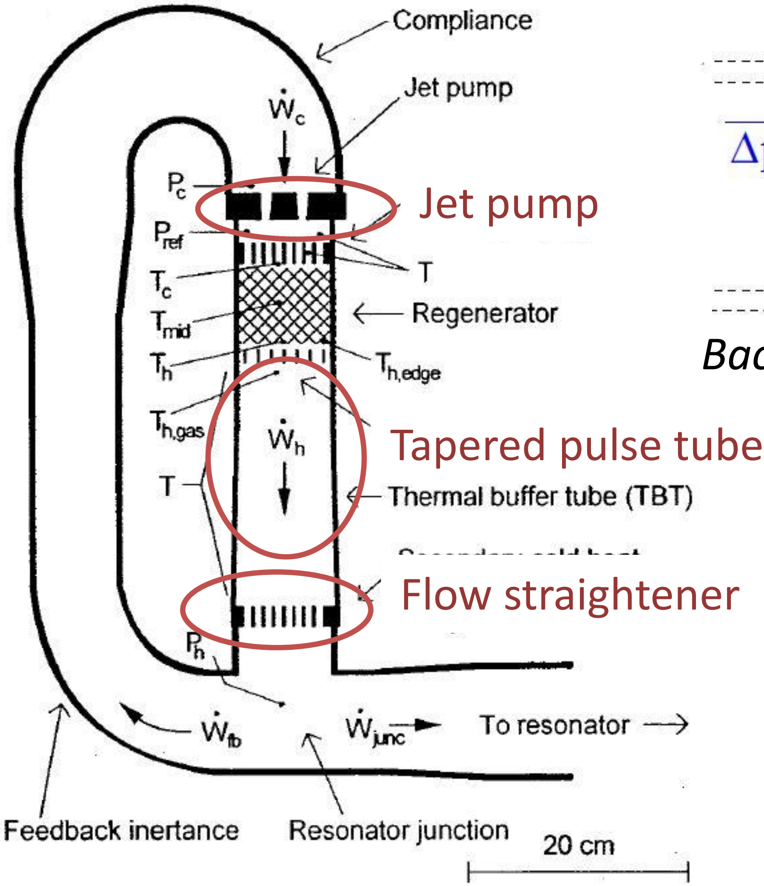
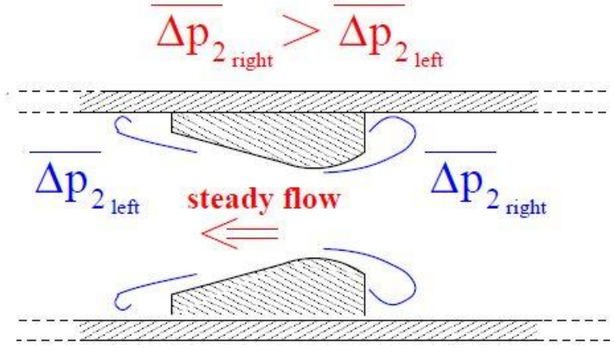
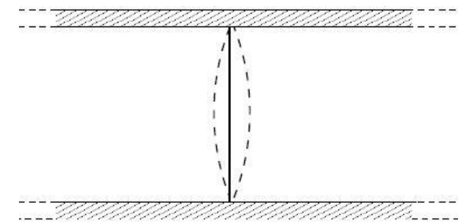
- **Conclusion**



Backhaus and Swift, Nature, 1999



Backhaus and Swift 2003



Tijani Appl. Therm. En. 2013

Olson and Swift 1997

There exist some Empirical solutions

But work only under certain conditions, not transposable and still lot to understand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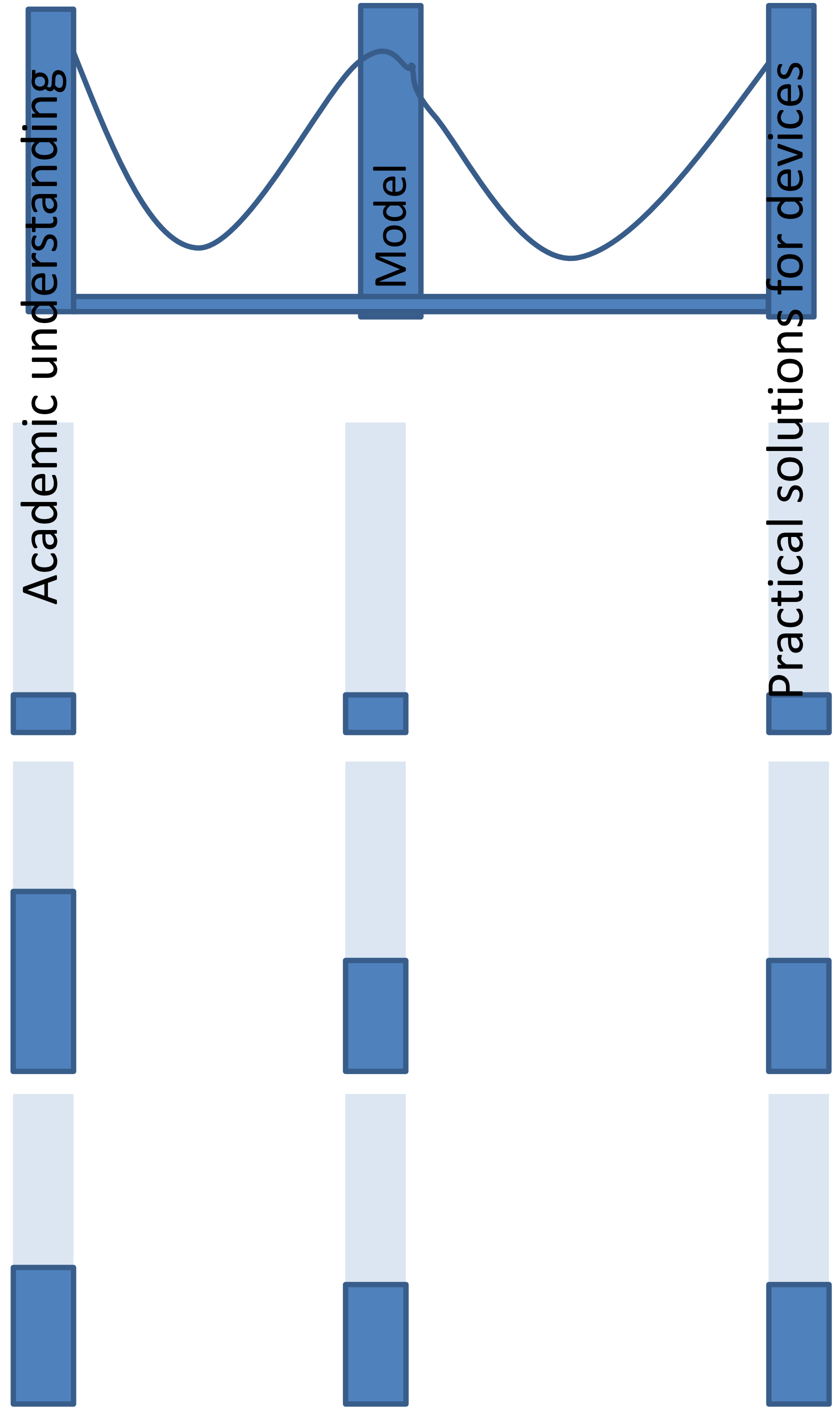
Academic understanding
Model
Practical solutions for devices
Turbulence
Edge effects
Streaming


UNDER
CONSTRUCTION