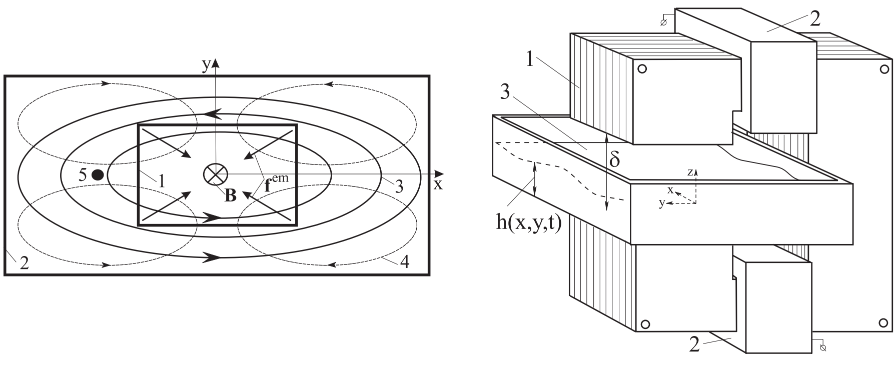
# VORTEX FLOWS GENERATED BY A VARYING MAGNETIC FIELD IN A CONDUCTING FLUID LAYER

*S. Khripchenko, R. Khalilov, I. Kolesnichenko*

*Institute of Continuous Media Mechanics,*
*1 Korolyov str., 614013 Perm, Russia (klsnchnk@icmm.ru)*

**Introduction.** This study deals with vortex flows in the shallow liquid metal layers having free or solid top surface. The vortex flows are generated by a vortex electromagnetic (EM) force. One way to initiate the em-force in a rectangular layer is to pass through the layer a uniform electric current from an external source and to apply ferromagnetic yokes to a local region [1]. This em-force excites the so-called "electro-vortex flow" (EVF) due to initial action of the electric current. In the present study, we consider another way of em-force generation by a local transverse alternating magnetic filed from an external source. Penetrating the layer, this field induces vortex electric current flows in the layer plane around the induction field lines. Their interaction generates a rotational em-force, and, consequently, a rotational flow, which by analogy with the EVM is called a "magneto-vortex flow" (MVF) because it originates from the initial action of the magnetic field. With the magnetic field located at the center of the layer plane, one can excite the MVF as shown in Fig. 1*a* [2]. The MVF can be exploited in metallurgy for stirring and pumping molten metals, therefore, the results of the study are useful in designing devices for these technological processes. Since this flow is unstable [3] for parameters exceeding the critical values, we investigated the stability of the MVF both by experimental and numerical methods.

**1. Experimental setup.** The MVF studies were performed on the experimental setup consisting of the following elements (Fig. 1*b*): 1 – C-shape core 1 made from electrotechnical steel sheets, copper electrical coils 2 and a cell 3 with a layer of liquid metal. The C-core 1 and coils 2 generated an alternating




(*a*) (*b*)

*Fig. 1.* (*a*) Scheme describing MVF generation in the layer (top view): 1 – domain of C-core, 2 – layer domain, 3, 4 – streamlines of the electric current and velocity, 5 – a point at which the velocity was measured in experiment. (*b*) Sketch of setup: 1 – ferromagnetic C-core, 2 – coils, 3 – cell with a liquid metal layer.

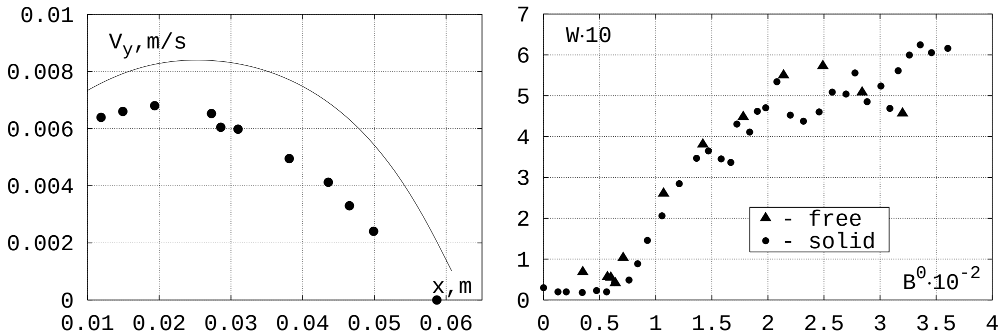



*Fig. 2.* ($a$) Velocity profile for $y = 0$, surface is free (hereinafter: dots – experiment, solid lines – calculations). ($b$) Oscillation energy versus the parameter of MHD-interaction ($\mathrm{G} = 10^8$) used to define the bifurcation point.

magnetic field of frequency $\omega = 50\,\mathrm{s}^{-1}$ and the effective value $B_0$ in the case of empty gap $\delta = 0.04\,\mathrm{m}$ of the core (the maximal value of $B_{\max} = 0.05\,\mathrm{T}$). The magnetic field penetrated the cell 3 in a transverse direction. The cell encloses the layer of liquid gallium alloy (2%Zn+87.5%Ga+10.5%Sn) with thickness $d_0 = \{8 \div 14\} \cdot 10^{-3}\,\mathrm{m}$ and sizes in plane $a_0 = 0.1\,\mathrm{m}$, $b_0 = 0.2\,\mathrm{m}$. The gallium alloy had the following properties: $\rho = 6256\,\mathrm{kg/m^3}$ – density, $\nu = 3 \cdot 10^{-7}\,\mathrm{m^2/s}$ – kinematic viscosity, $\sigma = 3.56 \cdot 10^6\,\mathrm{S/m}$ – electric conductivity. We studied the layers having both free and solid top boundaries. When the surface was free, we applied a thin layer of water solution of hydrochloric acid over the surface of the liquid metal. Small bubbles of hydrogen generated on the surface in the course of chemical reactions served as tracers allowing to detect the appearance of any flow modes. Additionally, the acid removed gallium oxide from the layer surface enabling the fluid near the surface to move freely. Using this method, we studied the most stable two-eddies MVF and compared the obtained results with numerical simulations (Fig. 2$a$). For both free and solid top boundaries, we also used a conductive anemometer with a permanent samarium-cobalt magnet and couple of electrodes. The probe was placed at point 5 (Fig. 1$a$), where the MVF velocity was high enough to provide good level of the signal in studying the MVF oscillations. Working with this setup, we can obtain maximal values for the following parameters: $V_{\max} = 0.1\,\mathrm{m/s}$ for velocity, $\mathrm{Re} = V_{\max} d_0/\nu = 4322$ for the hydrodynamic Reynolds number, $\mathrm{Rm} = V_{\max} d_0 \sigma \mu_0 = 8.18 \cdot 10^{-3}$ for the magnetic Reynolds number ($\mu_0 = 4\pi \cdot 10^{-6}\,\mathrm{Hn/m}$), and $\mathrm{Ha} = B_{\max}\delta\sqrt{\sigma/\rho\nu} = 85$ for the Hartmann number.

**2. Mathematical model.** The model is formulated in terms of a reduced system of MHD-equations. Here we use the inductive-free approach, which is valid for small magnetic Reynolds numbers ($\mathrm{Rm} \ll 1$) and the shallow-water approach commonly applied to layers whose thickness is much less than the planar dimensions. The former approach allows us to get the hydrodynamic and electrodynamic fields separately because the transition of the magnetic field by the flux is negligible. The use of the latter approach leads to 2D-equations after integrating the equations across the layer from the bottom to the solid top or free surface described by the function $h(x, y, t)$. In this case, the 3D velocity components are approximated using the velocity profile and 2D components $V_i^{3D}(x, y, z, t) = V_i(x, y, t) f_V(z), i = 1, 2$ (the notation: $V_1 = V_x, x_1 = x, ...$). The solid boundaries are under the constraint that $V_i^{3D} = 0$ ($i = \{1, 2, 3\}$), $\partial_z V_i^{3D}|_{z=0} = \kappa V_i$

($i = \{1, 2\}$), and the free boundaries meet the condition $V_z^{3D}(x, y, h, t) = \mathrm{d}h/\mathrm{d}t$. The pressure gradient is $\nabla P = \rho g \nabla h$. The model uses the following scale parameters to reduce the related equations to the non-dimensional form: $d_0$, $\nu/d_0$, $d_0^2\nu$, $\rho\nu^2/d_0^2$, $B^0$ for length, velocity, time, pressure, and magnetic induction, respectively. The final system of equations for the local flux $V_i(x, y, t)$ and the free surface position $h(x, y, t)$ is

$$\frac{\partial V_i}{\partial t} + \frac{\partial(V_i V_j q)}{\partial x_j} + V_i r \frac{\mathrm{d}h}{\mathrm{d}t} = -\mathrm{G}h\frac{\partial h}{\partial x_i} + \frac{\partial^2 V_i}{\partial x_j^2} + \kappa V_i + \mathrm{S}h f_i^{em}; \quad (1)$$

$$\frac{\partial h}{\partial t} + V_j \frac{\mathrm{d}h}{\mathrm{d}x_j} = -\frac{\partial V_j}{\partial x_j}; \qquad i, j = \{1, 2\}. \quad (2)$$

In (1) there are two dimensionless criteria: $\mathrm{G} = g d_0^2/\nu^2$ (Galileo parameter) and $\mathrm{S} = d_0\delta(B^0)^2/\rho\mu_0\nu^2$, and three functions $q(x, y, t)$, $r(x, y, t)$, $\kappa(x, y, t)$, which are defined using the local $Re$ and $Ha$ numbers and position of the free surface [1]. The function $\kappa$ describes the local friction near the horizontal walls taking into account the type of the flow (laminar or turbulent) and the Hartmann effect. The boundary conditions for equations (1) and (2) are $V_i = 0$, $\partial h/\partial x_i(\mathbf{e}_i \cdot \mathbf{n}) = 0$, $i, j = \{1, 2\}$, where $\mathbf{e}_i$ is the unit vector and $\mathbf{n}$ is the normal vector to the boundary. The initial alternating magnetic field $\mathbf{B}^0$ induces the vortex electrical current in the layer $\mathbf{j}$ which, in its turn, induces the vortex magnetic field $\mathbf{B}^{\mathrm{i}}$ in the direction opposite to that of the initial field. The EM force $\mathbf{f}^{\mathrm{em}}$ is caused by the interaction between the resulting magnetic field $\mathbf{B} = \mathbf{B}^0 + \mathbf{B}^{\mathrm{i}}$ (where $\mathbf{B}^0$ is the field in the empty gap) and the current $\mathbf{j}$. The horizontal components of the force vector are much larger than the vertical one, which can be ignored. Using the Maxwell and Ohm equations one can derive the following equations (projected on the vertical axis)

$$\frac{\partial^2\psi}{\partial x_j^2} = -\imath\omega\sigma(B_z^0 + B_z^{\mathrm{i}}) - \sigma\frac{\partial B_z^{\mathrm{i}}}{\partial t}, \qquad B_z^{\mathrm{i}} = \varphi(x, y)\psi\frac{d_0\mu_0}{\delta + l/\mu}. \quad (3)$$

Here $\psi$ is the stream function defined by $j_x = -\partial_y\psi$, $j_y = \partial_x\psi$, $l$ is the effective length of the magnetic field line in the C-core, $\mu$ is the magnetic permeability of the C-core. $\varphi(x, y)$ is the function of the magnetic field dispersion defined from the problem modeling a potential distribution in the space between the C-core poles. The boundary condition for equation (3) is derived from the non-leakage condition for the electric current $(\mathbf{j} \cdot \mathbf{n}) = 0$.

**3. Results.** The theoretical and experimental stability studies of four-eddies MVF showed that it is unstable at parameters exceeding some critical values. The analysis of the experimental trajectories of the hydrogen bubbles motion over the liquid metal surface, and the calculated velocity field behaviour has revealed the following stages of instability caused by the increase of the initial magnetic field. In the first stage, the eddies located on the diagonal periodically change their intensity and the surface experiences oscillations. In the second stage, the oscillations increase and the structure of MVF periodically changes from four to three almost equal eddies due to a rectangular configuration of the layer. The third stage is characterized by non-regular oscillations of MVF and the appearance of free surface long-wave oscillations along the $OX$-axis (in Fig. 1*a*). To determine the critical value of the magnetic field in the experiment, we treated the signals registered by the conductive anemometer $U(t)$ during a fixed time period . We calculated the oscillation energy $W = \sum_k A_k^2$ of the signal $U'(t) = (U(t) - \overline{U})/U_a$, where $\overline{U}$ is the time-averaged value of $U(t)$, $U_a$ amplitude of oscillation, and $A_k$ is the Fourier-spectrum. Fig. 2*b* illustrates the dependence of the energy $W$ on the

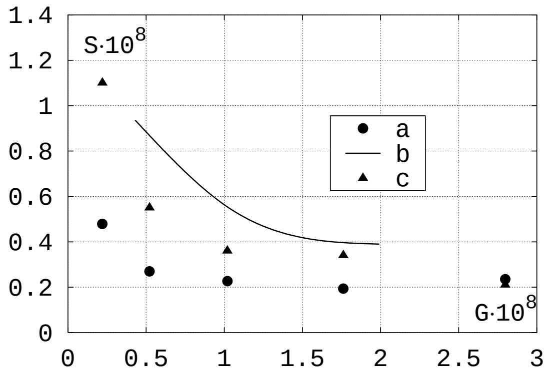



*Fig. 3.* Neutral curve for a layer with the free surface: (a) – experiment, (b) – calculations. (c) – Neutral curve for a layer with the solid top boundary.

initial magnetic field $B^0$, which is used to estimate the stability threshold (for free and solid top surfaces). Based on these measures and calculations, the neutral curves have been constructed for layers with free (Fig. 3, a,b) and solid (Fig. 3, c) top surfaces.

**Acknowledgements.** The work is supported by RFBR under grant 04-01-08024, by State contract IP-04-03, by Award No. PE-009-0 of the U.S. Civilian Research & Development Foundation for the Independent States of the Former Soviet Union (CRDF).

**REFERENCES**

1. I. Kolesnichenko, S. Khripchenko. Surface instability of the plane layer of conducting liquid. *Magnetohydrodynamics*, vol. 39 (2003), no. 4, pp. 427–434.
2. V. Almukhametov, S. Khripchenko. Electrovortex flow mechanisms for an electrolyzer containing a solid anode. *Magnetohydrodynamics*, vol. 23 (1987), no. 3, pp. 317–319.
3. I. Kirko, V. Almukhametov, S. Khripchenko. Physical modelling of unstable behaviour of electrolyte-metall interface in powerfull aluminum electrolysers. *Dokl. Acad. Nauk USSR*, vol. 302 (1988), no. 4, pp. 845–847 (in Russian).