TURBULENT MHD ROTATION OF LIQUID METAL IN A HOMOPOLAR DEVICE

A. Kapusta[1], *B. Mikhailovich*[1], *P. Terhoeven*[2]

[1] *Center for MHD Studies, Ben-Gurion University of the Negev, Beer-Sheva, Israel (borismic@bgu.ac.il)*

[2] *Moeller GmbH, Hein-Moeller-Str. 7-11, 53115, Bonn, Germany*

MHD homopolar devices, where the interaction of the radial field of current density and an axial magnetic field generates an azimuthal component of the electromagnetic body forces (EMBF) field and sets liquid metal in rotational motion, are in use in various applications [1].

In our work, we use a simple and reliable model of "external" friction [2, 3] that allows us to describe the dynamics of melt flow in such devices in turbulent regimes. Here the non-stationary turbulent flow in the cylindrical cavity of the device can be described in the induction-free approximation by the dimensionless equation in the cylindrical system of coordinates r, ϕ, z:

$$\Delta U_\varphi - \beta^2 U_\varphi - \frac{\mathrm{Re}_\omega}{2\pi}\frac{\partial U_\varphi}{\partial \tau} = -C_1 \mathrm{Ha}^* \mathrm{Ha} \cdot T(\tau)\,, \tag{1}$$

where $U_\varphi = V_\varphi / V_0$, $\quad V_0 = \omega R_1$, $\quad T_0 = 2\pi/\omega$, $\quad \tau = \omega t$, $\quad \Delta = \dfrac{\partial^2}{\partial r^2} + \dfrac{1}{r}\dfrac{\partial}{\partial r} - \dfrac{1}{r^2}$,

$$\beta^2 = \lambda(\tau) + \mathrm{Ha}^2 \theta(\tau), \lambda(\tau) = C_0 \mathrm{Re}_\omega \left\langle \Omega(\tau) \right\rangle / \delta_z,$$

$$\mathrm{Re}_\omega = \omega R_1^2/\nu, \delta_z = Z_1/R_1, \left\langle \Omega(\tau)\right\rangle = \frac{1}{2}\int\limits_0^1 \frac{1}{r}\left|\frac{\partial \left(r U_\varphi\right)}{\partial r}\right| \mathrm{d}r,$$

$$\mathrm{Ha} = B_{\mathrm{ef}} R_1 \sqrt{\sigma/\rho\nu}, \mathrm{Ha}^* = B^*_{\mathrm{ef}} R_1 \sqrt{\sigma/\rho\nu},$$

$$B_{\mathrm{ef}} = \frac{B_0\sqrt{2}}{r_1 z_1}\int\limits_0^{r_1}\int\limits_0^{0.5 z_1} b_z(r,z)\mathrm{d}r\mathrm{d}z, B_0 = \mu_0 j_0 R_0, C_1(r) = -\int\limits_0^1 j_r \mathrm{d}z,$$

$B^*_{\mathrm{ef}} = j_0/\sqrt{2}\sigma\omega\bar{R}$, $\quad \bar{R} = (R_1 + R_2)/2$, $\quad A$ is a relative amplitude of the electric current having the frequency ω, C_0 is an empirical constant. Equation (1) is solved under the following initial and boundary conditions:

$$U_\varphi|_{\tau=0} = U_{\mathrm{st}}; \quad U_\varphi|_{r=1} = 0; \quad U_\varphi|_{r=0} = 0. \tag{2}$$

A homopolar device comprises a cylindrical working cavity partially filled with a liquid metal, which is formed by a cylindrical electrode, a body made of dielectric and a central electrode. Around the body, an annular winding is arranged. The electric current is passed through the central electrode, liquid metal, cylindrical electrode and the winding connected in series. The current induces a radial field of current density j_r in the liquid metal, and inside the annular winding it mainly induces an axial magnetic field B_z. The interaction of the radial field of current density and the axial magnetic field generates an azimuthal component of the electromagnetic body forces (EMBF) field. Under the action of these forces, the melt rotates at a certain angular velocity Ω_{st}.

The current density field in such device was analyzed in [3], where components had the form:

$$j_r = \sum_{k=1}^{\infty} J_1(\delta_k r) \frac{B_k \mathrm{ch}\nu_k(1-z)}{\mathrm{sh}\nu_k}, \tag{3}$$

$$j_z = \frac{1}{\delta_z} \sum_{k=1}^{\infty} J_0(\delta_k r) \frac{B_k \mathrm{sh}\nu_k(1-z)}{\mathrm{sh}\nu_k}. \tag{4}$$

The magnetic field activated in melt by the electric current flowing in a turn of finite thickness was calculated as a result of superposition of magnetic fields excited by annular currents I_0/K:

$$B = \frac{\mu_0 I_0 R^2}{2K} \sum_{k=0}^{K} \left[R^2 + \left(\frac{kh}{K} \right)^2 \right]^{-3/2}. \tag{5}$$

The distribution of azimuthal forces along the radius is approximated by the function defined in regions I ("quasi-solid" flow with $0 \le r \le r_i$) and II ($r_i < r \le 1$) and having the form:

$$C_1(r) = A_{a\mathrm{I}} \cdot r \cdot \theta_i(r) + A_{a\mathrm{II}} \cdot \theta_{\mathrm{II}}(r)/r\,, \tag{6}$$

where $\theta_i(r)$ is the Heaviside's step function in the respective region. In this case, we can describe the stationary velocity of rotating flows in regions I and II, respectively, by the following equations:

$$LU_{\varphi\mathrm{I}} - \beta^2 U_{\varphi\mathrm{I}} = -A_{a\mathrm{I}} \cdot \mathrm{Ha} \cdot \mathrm{Ha}^*\, r\,, \tag{7}$$

$$LU_{\varphi\mathrm{II}} - \beta^2 U_{\varphi\mathrm{II}} = -A_{a\mathrm{II}} \cdot \mathrm{Ha} \cdot \mathrm{Ha}^*\, r\,. \tag{8}$$

These equations are solved under the following boundary conditions:

$$\begin{gathered} U_{\varphi\mathrm{I}}|_{r=0} = 0; \quad U_{\varphi\mathrm{II}}|_{r=1} = 0; \quad U_{\varphi\mathrm{I}}|_{r=r_i} = U_{\varphi\mathrm{II}}|_{r=r_i}; \\ \left.\frac{\mathrm{d}(rU_{\varphi\mathrm{I}})}{\mathrm{d}r}\right|_{r=r_i} = \left.\frac{\mathrm{d}(rU_{\varphi\mathrm{II}})}{\mathrm{d}r}\right|_{r=r_i}. \end{gathered} \tag{9}$$

The solution of the problem has the form:

$$U_{\varphi\mathrm{I}} = C_1 I_1(\beta r) + A_{a\mathrm{I}} \cdot \mathrm{Ha} \cdot \mathrm{Ha}^* r/\beta^2\,, \tag{10}$$

$$U_{\varphi\mathrm{II}} = C_2 I_1(\beta r) + C_3 K_1(\beta r) + A_{a\mathrm{II}} \cdot \mathrm{Ha} \cdot Ha^*/\beta^2 r\,, \tag{11}$$

where the constants C_1, C_2, C_3 can be found by solving a system of equations satisfying the boundary conditions and continuity and smoothness conditions at the boundary between regions I and II.

The angular velocity in a "quasi-solid" flow $\langle\Omega\rangle$ is defined from the algebraic equation:

$$\langle\Omega\rangle^2 + Q\langle\Omega\rangle - Q^* = 0\,, \tag{12}$$

where

$$Q = \frac{\mathrm{Ha}^2 \delta_z}{C_0 \mathrm{Re}_\omega}; \qquad Q^* = \frac{A_{a\mathrm{I}} \mathrm{Ha} \cdot \mathrm{Ha}^* \delta_z}{C_0 \mathrm{Re}_\omega}.$$

Its solution determines $\langle\Omega\rangle$ as a function of parameters:

$$\langle\Omega\rangle = \frac{Q}{2} \left(\sqrt{1 + \frac{4Q^*}{Q^2}} - 1 \right). \tag{13}$$

Further we examine Eq. (1) in region I using a quasi-solid profile of the azimuthal velocity. In this case, $\Delta U_{\phi \mathrm{I}} = 0$, and the equation determining $U_{\phi \mathrm{I}}$ is reduced to the following form, where $T(\tau)$ is the electric current temporal dependence:

$$\frac{\partial U_{\varphi \mathrm{I}}}{\partial \tau} + \frac{2\pi}{\mathrm{Re}_\omega} \beta^2(\tau) U_{\varphi \mathrm{I}} = 2\pi \cdot A_{a\mathrm{I}} \frac{\mathrm{Ha} \cdot \mathrm{Ha}^*}{\mathrm{Re}_\omega}\, r\, T^2(\tau) \tag{14}$$

with the initial condition:

$$U_\varphi|_{r=0} = U_{\mathrm{st}}\,. \tag{15}$$

We seek the solution of problem (14)–(15) in the form:

$$U_{\varphi \mathrm{I}} = \Omega(\tau) \cdot r. \tag{16}$$

Substituting (16) into Eqs. (14)–(15), we obtain:

$$\Omega(\tau) + \frac{2\pi c_0 k_r}{\delta_z} \Omega^2(\tau) + \frac{2\pi \mathrm{Ha}^2}{\mathrm{Re}_\omega} T^2(\tau)\Omega(\tau) = \frac{2\pi A_{a\mathrm{I}} \cdot \mathrm{Ha} \cdot \mathrm{Ha}^*}{k_r \mathrm{Re}_\omega} T^2(\tau) \tag{17}$$

with the initial condition:

$$\Omega(\tau)|_{r=0} = 1. \tag{18}$$

The solution of this problem for different forms of $T(\tau)$ makes it possible to compute the evolution of liquid metal free surface level.

As a result of this work, the distribution of electromagnetic fields is described, and the dynamic characteristics of the homopolar device are analyzed on the basis of the "external" friction model.

REFERENCES

1. Y. Gelfgat, O. Lielausis, E. Sherbinin. *Liquid Metal under the Action of Electromagnetic Forces* (Zinatne, Riga, 1976).

2. A. Kapusta, L. Levitsky. *Magnetohydrodynamics*, vol. 27 (1991), no. 3, pp. 353–355.

3. H. Branover, A. Kapusta, B. Mikhailovich and I. Zilberman. In *Proc. Fifth International* Pamir *Conference on Fundamental and Applied MHD* (Ramatuelle, France, 2002), vol. I, pp. 169–171