BENDING OF FLEXIBLE MAGNETIC RODS

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Introduction. Filamentary structures carry out important functions inside the cell [1]. Magnetic filamentary structures are used by the magnetotactic bacteria for the navigation purposes [2]. Flexible magnetic filaments created recently in different labs [3, 4] possess interesting static and dynamic properties which are studied on the basis of the flexible magnetic filament model developed in [5, 6, 7]. It predicts the scaling law for the displacement of the end of the filament at which the transversal force is applied with a time t^{α}. The exponent α is field-dependent [7] and its value at zero field 3/4 gives the characteristic frequency dependence of the complex shear modulus $\omega^{3/4}$ observed for networks of filaments [8]. An experimental investigation of the viscoelastic properties of the magnetorheological suspension carried out recently [9] qualitatively corresponds to the predictions of the flexible filament model – both real and imaginary parts have the same order of magnitude and the exponent α is field-dependent. The model [9] based on the analysis of the single chain behaviour in the shear flow has not reproduced this behaviour reasonably well – the calculated imaginary part of the complex shear modulus is much below the one given by the experiment. On the other hand, the obtained field dependence of the exponent α is much stronger $\big(\alpha = 0.69\,(H = 0)$ and $\alpha = 0.18\,(H = 29.7\,\mathrm{kA/m})\big)$ than it follows from the flexible filament model [7]. A satisfactory model of the viscoelastic properties of the magnetorheological suspension should include the features of the single chain behaviour and their network. Here we are studying the response of the flexible magnetic filament to the transversal force applied at its end which models the network properties of the system of the magnetic particle chains.

1. Model. The equations of the flexible magnetic filament model for the tangent angle and tension presented in the dimensionless form are [5, 6, 7]:

$$\vartheta_t = -\left(\vartheta_{lll} + \frac{1}{2}\left(\vartheta_l^3\right)_l\right) - \left(\vartheta_l\Lambda\right)_l - \vartheta_l\Lambda_l + \mathrm{Cm}\left(\sin\left(2\vartheta\right)\right)_{ll} - \mathrm{Cm}\,\vartheta_l^2\sin\left(2\vartheta\right), \quad (1)$$

$$\vartheta_l^2\Lambda - \Lambda_{ll} = -\vartheta_l\left(\vartheta_{lll} + \frac{1}{2}\vartheta_l^3\right) + 2\mathrm{Cm}\,\vartheta_l^2\cos\left(2\vartheta\right) + \mathrm{Cm}\left(\vartheta_l\sin\left(2\vartheta\right)\right)_l, \quad (2)$$

where ϑ is the tangent angle, Λ is the compressional stress and Cm is the magnetoelastic number, which characterizes the ratio of the magnetic and elastic forces.

The boundary condition for the first equation of the set compatible with the condition of the fixed end at $l = -1$ reads [7]($\mathbf{K}$ – body force):

$$\mathbf{K}\left(-1\right) = 0, \quad (3)$$

This end is taken to be clamped

$$\vartheta\left(-1\right) = 0. \quad (4)$$

The boundary conditions on the free end are (F_a is an applied force):

$$\vartheta_l\,(+1) = 0 \tag{5}$$

and

$$\mathbf{F}\,(+1) = F_a \mathbf{e}_y. \tag{6}$$

In the case of small deformations of the rod, equations (1) and (2) can be considerably simplified. In this case, since $\Lambda = 0$ up to the second order small term, equation (1) reads

$$\vartheta_t = -\vartheta_{llll} + 2\mathrm{Cm}\,\vartheta_{ll}. \tag{7}$$

Introducing $\vartheta = y_x$ for the displacement of the rod y after integrating equation (7) once we have

$$y_t = -y_{xxxx} + 2\mathrm{Cm}\,y_{xx}. \tag{8}$$

The solution of equation (8) with the boundary conditions corresponding to a semi-infinite rod

$$y|_{x=-\infty} = 0, \qquad y_{xx}|_{x=0} = 0, \qquad (y_{xxx} - 2\mathrm{Cm}\,y_x)|_{x=0} = -F_a \tag{9}$$

and the initial condition $y|_{t=0} = 0$ can be easily found by a Laplace transformation. For the displacement of the end of the rod in this case we have

$$y(0,t) = \frac{Fa}{2\mathrm{Cm}^{3/2}}\,\varphi\left(2\mathrm{Cm}\,\sqrt{t}\right), \tag{10}$$

where for $\tau \geqslant 0$

$$\varphi(\tau) = \tau^{3/2}\sum_{n=0}^{+\infty} a_n \tau^n\,, \qquad a_n = \frac{(-1)^n}{\Gamma\left(\frac{7}{4}+\frac{4}{2}\right)}\sum_{k=0}^{n}\frac{(-1/2)k}{2^k k!} \tag{11}$$

and moreover we have an asymptotic approximation

$$\varphi\,(\tau) \sim \sqrt{\frac{2}{\pi}}\tau\left(1 - \frac{1}{4\tau^2} + \frac{13}{32\tau^4} - \frac{183}{128\tau^6}\right), \quad \tau \to +\infty\,. \tag{12}$$

Hence, if $t \ll \dfrac{1}{4\mathrm{Cm}^2}$, then

$$y(0,t) \sim \frac{\sqrt{2}\,F_a\,t^{3/4}}{\Gamma\left(\frac{7}{4}\right)}, \tag{13}$$

also if $t \gg \dfrac{1}{4\mathrm{Cm}^2}$, then

$$y\,(0,t) \sim \sqrt{\frac{2}{\pi}}\,F_a\,\sqrt{\frac{t}{\mathrm{Cm}}}. \tag{14}$$

The function

$$2\mathrm{Cm}^{3/2} y\left(0, \frac{\tau^2}{4\mathrm{Cm}^2}\right) = F_a\,\varphi\,(\tau) \tag{15}$$

does not depend on Cm.

The exponents 0.75 and 0.5 found in the frame of the linear model for small and high Cm values correspondingly agree with those found by numerical analysis in [7].

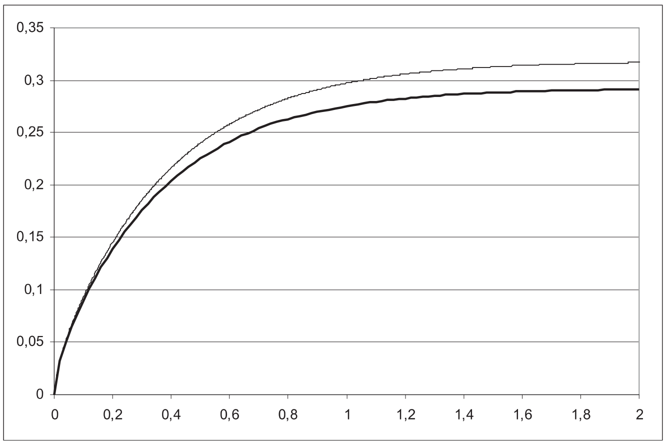

Fig. 1. Displacement of the end of the rod. Cm = 1, $F_a = 0.45$. Tight line – non-linear model, bold line – linear model.

2. Numerical simulation results. The validity of the linear analysis may be compared with the numerical simulation results obtained on the basis of a full flexible magnetic filament model. The numerical algorithm is described in [5].

In Fig. 1 the time dependence of the displacement of the free end of the filament is compared with that of a full non-linear model. Good agreement for small time is observed between the linear model and the results given by the full set of non-linear equations. As one can see for the values of the parameters given, the nonlinear model predictsa bigger displacement at the stationary state. The comparison of the shapes of the filament calculated in the frames of linear and non-linear models for the values of the parameters given in the caption of Fig. 1 is shown in Fig. 2 for different times of the filament deformation. We can see that the overall agreement of the shapes for these particular values of the parameters is reasonably good. The influence of different parameters on the steady state of the filament as obtained on the basis of the linear model and the full set of the nonlinear equations is shown in Fig. 3. We can see that at larger Cm values the discrepancy between the linear and non-linear models increases and becomes about 20% at Cm = 10 in comparison to 10% at Cm = 1.

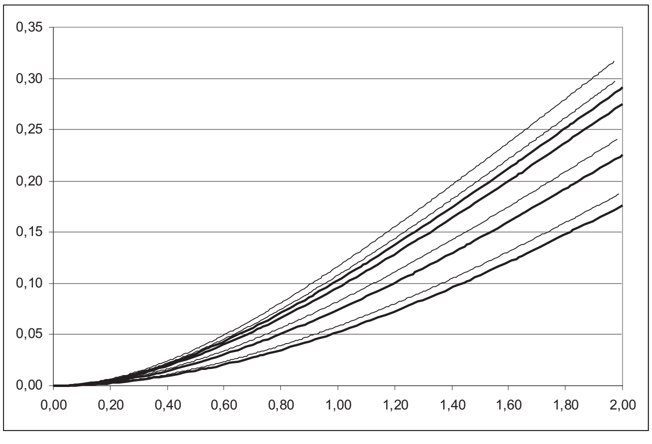

Fig. 2. Configuration of a rod Cm = 1, $F_a = 0.45$. ¿From lower to upper $\tau = 0.3$, 0.5, 1, 2. Tight line – non-linear model, bold line – linear model.

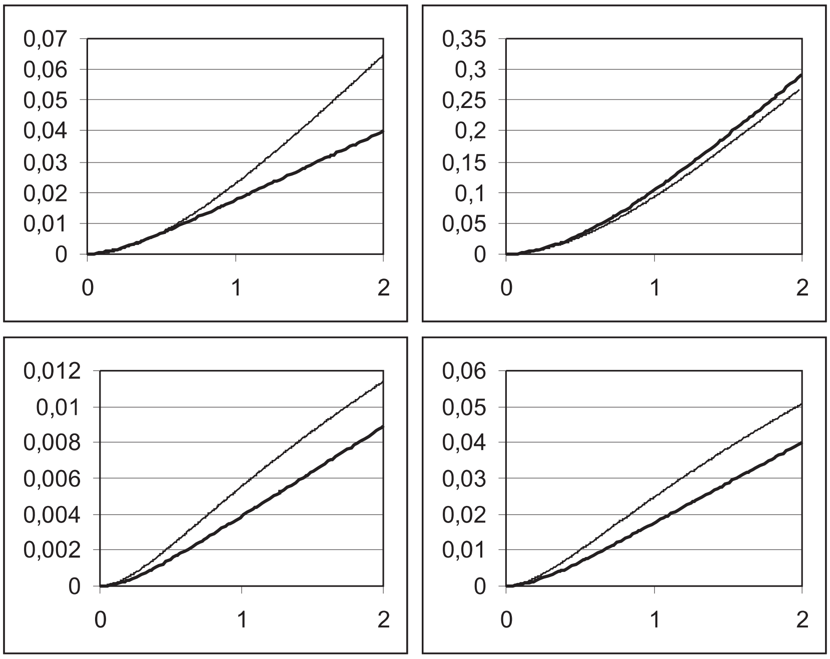

Fig. 3. Configuration of a flexible magnetic rod. Tight line – non-linear model, bold line – linear model. Left top – Cm = 1, $F_a = 0.1$; right top – Cm = 1, $F_a = 0.45$; left bottom – Cm = 10, $F_a = 0.1$; right bottom – Cm = 10, $F_a = 0.45$.

3. Conclusions. We have shown here that the scaling law [7], describing the magnetic filament deformation curves in the network, follows to the obtained from the analysis of the linear model. The properties of the network of the magnetic particle chains determine the viscoelstic properties of the magnetorheological suspensions. The full model of the viscoelastic properties of the magnetorheological suspension should take into account both the single chain properties and their networking.

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