PUMPING MAGNETIC LIQUID WITHOUT ANY MOVING PARTS, BUT BY MAGNETIC STRESS

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A magnetic field rotating on the free surface of a ferrofluid layer is shown to induce a considerable fluid motion towards the direction the field is rolling. The measured flow velocity i) increases with the square of the magnetic field amplitude, ii) is proportional to the thickness of the fluid layer, and iii) has a maximum at a driving frequency of about 3 kHz. The pumping speed can be estimated with a two-dimensional flow model.

Introduction. Polarizable fluids can show a macroscopic reaction to external electric or magnetic fields. While for most fluids the influence of a magnetic field is fairly weak, colloidal suspensions of magnetic particles – the so-called Ferrofluids – do show a strong response particularly to static magnetic fields [1]. If these fields are time-dependent, a rich variety of phenomena occurs. The internal rotation of magnetization in an externally rotating magnetic field gives rise to nontrivial effects [2, 3]. A particularly interesting example is the driving of a macroscopic flow by means of an external rotating magnetic field [4], because it should allow for a very fine tuning both of the speed and of the direction of the flow even in microscopic channels [5].

1. Experimental setup. We present a novel magnetic fluid [6] in an open flow geometry designed for a quantitative comparison between the measured flow velocity and its theoretical estimation. It is scetched in Fig. 1a).

A circular Macrolon® duct with a mean diameter d of 100 mm and a square cross-section of 5 mm × 5 mm (and 2.5 mm × 2.5 mm in a second set of measurements) is filled brimful with a magnetic fluid [6] with a viscosity $\eta = 5.4 \cdot 10^{-3}$ Pa s. The orientation of the two coils producing a rotating magnetic field is also indicated in Fig. 1a: one coil is wrapped around this circular channel and provides a magnetic field in the azimuthal direction, and the outer coil produces the vertical component of the field. Both coils are driven with an ac-current with a phase
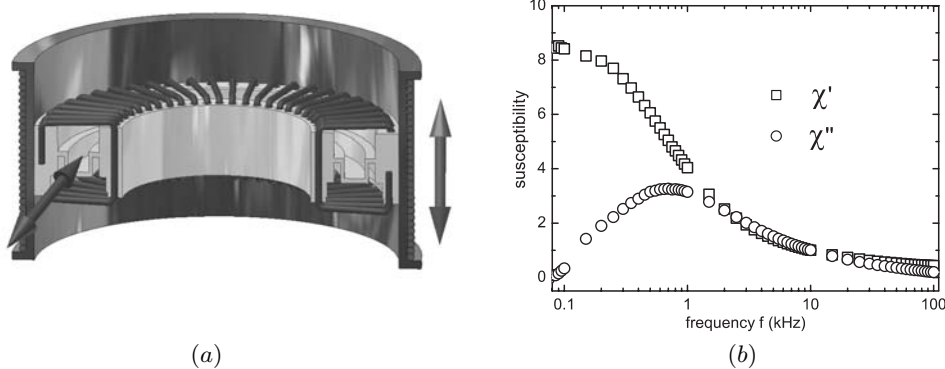

(a)   (b)

Fig. 1. (a) Experimental setup. The arrows indicate the direction of oscillating magnetic fields provided by the coils. (b) The magnetic susceptibility as a function of the frequency of the external alternating magnetic field. Here the real and imaginary parts are denoted by squares and circles, respectively.

difference of 90°, thus producing a rotating field on the free surface of the fluid within the duct.

The characterization of the magnetic susceptibility of the fluid in the duct is of primary importance for the pumping of the fluid, as discussed in this paper. It has been measured as a function of the frequency of an external oscillating azimuthal magnetic field by means of a pick-up coil placed into the liquid. Precisely speaking, the magnetization was determined from the difference of the signal detected by the pick-up coil in an empty and a filled channel, under the influence of an oscillatory azimuthal field. The results are presented in Fig. 1*b*. The measured susceptibilities of this novel cobalt-based fluids are fairly large compared to the more common magnetite-based fluids. In particular, the large imaginary part of the susceptibility is essential for the pumping action described in this paper.

2. Experimental results. The pump does work: a rotating field produced by the coils leads to a motion of the fluid in the azimuthal direction of the channel [7]. It's velocity is of the order of mm/s and can thus be determined by visual inspection of tracer particles swimming on its surface. The following observation is also a qualitative one: when changing the phase difference between the ac-fields from +90° to −90°, the flow changes its sign. The flow direction is such that the vorticity of the flow field is locally parallel to the rotation vector of the magnetic field, i.e., the fluid flows towards the direction the field is rolling.

For quantitative measurements it is necessary to use particles which are small compared to the channel width (dandruff, diameter about 1 mm). The velocity is defined by taking the time a particle needs to travel a few centimeters. The size dependence of these measurements is taken into account by using the numerically calcultated – roughly parabolic – velocity profiles, and by assuming that a floating particle represents the mean speed with respect to its diameter. A result obtained for a fixed frequency of 1 kHz is presented in Fig. 2*a*, where the maximal flow velocity within the channel is shown as a function of the amplitude G_0 of the driving external magnetic field. The velocity increases proportional to G_0^2, as demonstrated by the solid line, a parabola. Here we follow the nomenclature of Ref. [8], where the external magnetic far field is marked by **G** and the local one by **H**.

Having demonstrated this quadratic dependence of the velocity on the magnetic field, a first approach to collapse data obtained at different fields is to intro-
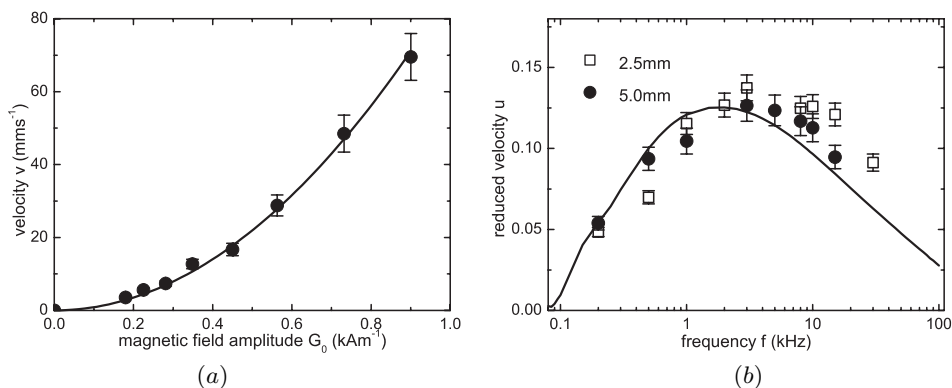

Fig. 2. (*a*)Maximal velocity measured as a function of the field amplitude at a fixed frequency of 1 kHz. (*b*) The reduced velocity *u* as a function of the driving frequency. Solid circles (open squares) are obtained in the 5 mm × 5 mm (2.5 mm × 2.5 mm) duct, respectively. The solid line represents the value expected on the basis of the measurement of the ac-susceptibility.

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duce a reduced velocity by dividing by the square of the external field. Another important influence determining the fluid velocity is the height of the channel: it turns out that the velocity is larger in bigger channels. This leads us to reduce the velocity also by dividing by the height L of the duct. In order to get a dimensionless number, one also has to scale with the viscosity of the fluid. Thus we define

$$u = v_{\text{max}} \frac{\eta}{L\mu_0 G_0^2} \tag{1}$$

as a reduced flow velocity. Its measured values are presented as a function of the driving frequency of the rotating magnetic field in Fig. 2b), for velocities obtained in a $5\,\text{mm} \times 5\,\text{mm}$ and a $2.5\,\text{mm} \times 2.5\,\text{mm}$ duct. Both measurements show the maximum of this velocity in the range of 2–$4\,\text{kHz}$.

3. Interpretation. The pumping action can be understood as a manifestation of the magnetic stress acting on the magnetized fluid, as summarized in [9]. This assumption explains all qualitative features of the observation: as long as the magnetization is proportional to the magnetic field (which has been measured to be the case for our fluid, within a precision of 5% for fields up to about $1500\,\text{Am}^{-1}$), the stress must be proportional to G^2 as demonstrated in Fig. 2a). If the frequency approaches zero, the magnetization and the field are parallel to each other, the tangential stress is zero and thus the motion of the fluid stops. For finite frequencies the velocity is proportional to the χ'' component of the susceptibility, which must increase linearly (to lowest order) with the frequency. For higher frequencies, the imaginary part of the susceptibility χ'' has a maximum at about $1\,\text{kHz}$, which explains that the maximal pumping velocity is observed around that frequency.

For the quantitative calculation we solve the Laplace equation for the velocity field including the no-slip condition for the fluid at the bottom and the side walls of the duct. At the upper surface of the fluid the magnetic stress provides the boundary condition

$$\eta \frac{\partial v}{\partial z} = \frac{\mu_0}{2} (M_z H_x - M_x H_z), \tag{2}$$

because Eq. (65) of Ref. [9] is applicable to our geometry.

Both field components H_x and H_z and the corresponding magnetization M_x and M_z are not constant in our case, but depend both on time and space. By numerical computations of the internal field $\mathbf{H}$ and the resulting magnetization $\mathbf{M} = \chi\mathbf{H}$ we get for the maximum fluid velocity in the middle of the upper surface

$$v_{\text{max}} = \chi'' \frac{\mu_0 G_0^2}{2} \frac{L}{\eta} \alpha \left(\frac{1 + N_{\text{eff}}\chi'}{(1 + N_{\text{eff}}\chi')^2 + (N_{\text{eff}}\chi'')^2} \right). \tag{3}$$

Here N_{eff} denotes an effective demagnetization factor and α is a reduction factor due to the given geometry depending on the aspect ratio $a = L_y/L_z$ of the rectangular channel. In our case $a = 1$ and we obtain $\alpha \approx 0.369$.

In our square cross-section, the magnetization – and the ensuing stress at the surface of the fluid – is not homogenous. We thus extract an effective demagnetization factor N_{eff} by fitting Eq. (3) to the numerically obtained velocity for different values of χ. We obtain $N_{\text{eff}} \approx 0.656$, which seems realistic when compare to $N = 0.5$ for the case of a circular cylinder.

From Eqs. (1), (3) we finally get a theoretical estimation for the reduced velocity

$$u = \frac{\alpha}{2} \frac{\chi''(1 + N_{\text{eff}}\chi')}{(1 + N_{\text{eff}}\chi')^2 + (N_{\text{eff}}\chi'')^2} \tag{4}$$

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The limiting case of an infinitly wide channel ($N = 1, \alpha = 1$) is included in this formula.

The reduced velocity obtained from Eq. (4) is presented in Fig. 2*b* as a solid line, with the values of $\chi\prime$ and $\chi\prime\prime$ taken from the measurements presented in Fig. 1*b*. The agreement between the measured velocities and the solid line is on a 20% level. This can partly be attributed to the limited accuracy of the measurement procedure, which is indicated by the error bars in Fig. 2*b*. The systematic deviations between the data and the theoretical curve are believed to reflect the precision of the simplifying assumptions going into the consideration presented above. For example, the influence of the shape of the meniscus of the fluid adds a three-dimensional complication to the problem, whose influence on the maximal pumping speed is hard to estimate. Moreover, it should be noted that magnetic fluids are not perfectly stable both in their magnetic susceptibility and their viscosity, which might add to the small mismatch between the expectation based on the measurement of the susceptibility and the measured velocity. Finally, taking into account the small amplitude of the magnetic field, any magnetoviscous effects have been neglected for the calculations.

In summary, the pump presented here works well and has an interesting potential especially in small geometries, where a mechanical driving of the flow is not possible. More important, it does seem safe to conclude that the ansatz of a magnetic stress driven motion captures the essence of this pump on a quantitative level.

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