END EFFECTS AND EFFICIENCY IN A NON COUPLED MHD ALTERNATE FLOW

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Introduction Numerous works were devoted to the end effect in a flat MHD channel flow to analyse the entrance and exit effect in the magnetic zone ([1], [2]). In steady flow, this effect is characterized by the well-known M-shape velocity profile. When the flow is pulsating at a high frequency, the formation of the M-profile is suppressed if the inertia forces of the fluid are greater than the electromagnetic forces. This paper assumes this last hypothesis and supposes the velocity constant all over the cross-section of the flow. Of course this is not true very near the walls where the velocity vanishes. But this region is extremely reduced for a high Hartmann number [3].

1. Position of the problem. The work is focused on an oscillating flow in an MHD channel of rectangular cross-section and thin conductive top and bottom walls [Molokov [4]]. The length L_B of the MHD generator part of the channel (Fig. 1) is limited and the other dimensions, width $2l$ and thickness $2a$, are assumed such as $2l \gg 2a$. The end effects at the both extremities of the generator are taken into account (Fig. 1).

The lateral walls are constituted by perfectly conducting electrodes on a length L_B. The top and bottom walls thickness of the channel, of electrical conductivity σ_w, is e_w. An incompressible liquid metal of density ρ, magnetic permeability μ_0 and electrical conductivity σ flows in the channel along the x-direction with a velocity amplitude u_0 and pulsation ω imposed along the channel by an oscillatory pressure gradient. The fluid flows as a solid body with a perfect electric contact at the walls and at the electrodes. The oscillating pressure gradient, the motive term, is supposed obtained by thermo-acoustic effect that has the potential of producing mechanical power from heat source with no moving part in a confined container. The principle is based on the use of a temperature gradient imposed at both extremities of a stack of plates properly disposed in a closed tube to

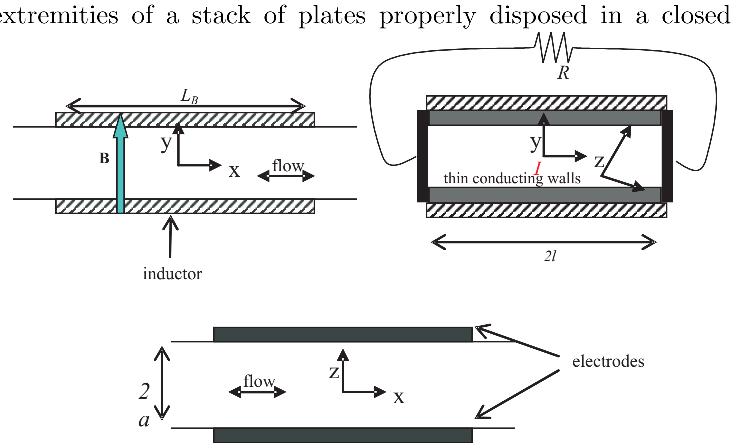

Fig. 1. Views of the MHD channel, the electromagnetic inductor creates a vertical magnetic field; the resulting current lines are collected by the electrodes.

create spontaneously a standing wave in the tube as in Swift's work ([5] to [10]). The oscillating velocity in the MHD channel, subjected to a constant external magnetic field $\mathbf{B}_0 = B_0 \cdot \mathbf{e}_y$ imposed by an inductor, generates an AC electric current collected by two electrodes placed in $z = \pm l$; the electric current supplies a load assimilated here to a resistance R. The present study takes end effects into account, it assumes also the walls of the channel electrically conducting and is established for a small magnetic Reynolds number Rm .

2. Formulation of the problem. Under the following conditions

$$\mathrm{Rm} = \mu_0 \sigma u_0 a \ll 1, \qquad v_r = \frac{\omega a}{u_0} \ll 1,$$

which mean that the induced magnetic field can be neglected and that the transportation mechanism is controlled by the fluid velocity. In these conditions the electric field can be chosen in the form:

$$\mathbf{E} = -\nabla\Phi$$

and taking into account the conservation of the current density $\nabla \cdot \mathbf{j} = 0$ provides that the electrical potential Φ satisfies a classical Laplace equation: $\Delta\Phi = 0$. This formulation assumes the velocity constant even at the extremities of the channel that supposes the electromagnetic force lower than the inertia forces, and consequently small values of the interaction parameter.

$$N = \frac{B_0^2 \sigma a}{\rho u_0} \ll 1$$

The dimensionless current density between the two electrodes located in $z = \pm l$ is controlled by the following equation:

$$j_z^* = -\frac{l}{a}\frac{\partial \Phi^*}{\partial z} + u^* B_{0y}^*$$

a part of this current closes in the top and bottom walls, the rest in the load. Besides the electrodes the current lines can close only in the walls. Due to the form of the fluid velocity $u = u_0 \cdot \cos(\omega t) \rightarrow u^* = \cos(t^*)$, it is supposed that all the variables in the form $X^* = X_m^* + X_1^* \cdot \cos(t^*)$. The imposed magnetic field has a constant value all along the channel and is decreasing at the both extremities. The calculation will be done by supposing the following law of decay of the imposed magnetic field. This law depends on the typical length δ, which is one of the parameters of the study

$$B_{0y}^* = \frac{1}{2}\left[1 - \tanh\left(\frac{x^* - L/2a}{\delta^*}\right)\right]$$

In the following, the influence of magnetic field attenuation distance δ^* will be studied for three different values: $\delta' = 2.5$, 5 and 10.

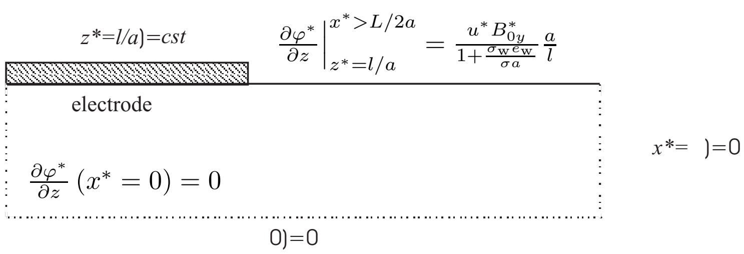

Fig. 2. Boundary conditions in the domain of study.

3. Boundary conditions. The boundary conditions are presented in Fig. 2. The following hypothesis is assumed.

- Each electrode is at a constant potential. The potential difference between the electrodes is imposed as one of the parameters thats depend on the load resistance.
- By symmetry, the axis $z = 0$ is taken as the origin of the potential.
- When $x \to \infty$, the potential leads to zero, and in addition $\frac{\partial \varphi^*}{\partial z}(x^* = 0) = 0$
- Outside the electrode zone ($x^* > L_B/2a$), the z-component of the current density for $z^* = l/a$ can only close in the top and bottom walls, thus:

$$\int_0^a j_z \mathrm{d}y = -\int_0^{e_\mathrm{w}} j_{wz} \mathrm{d}y \quad \text{Leading to} \quad \left.\frac{\partial \varphi^*}{\partial z}\right|_{z^*=l/a}^{x^*>L/2a} = \frac{u^* B_{0y}^*}{1 + \frac{\sigma_\mathrm{w} e_\mathrm{w}}{\sigma a}} \frac{a}{l}$$

This condition supposes of course the continuity of the electrical field at the fluid/wall interface. All the boundary conditions expressed in Fig. 2 are given in the dimensionless form. As it can be seen they take into account the conductance ratio, $C = \sigma_\mathrm{w} e_\mathrm{w}/\sigma a) = r/r_\mathrm{w}$ of the internal conductance of the fluid over the internal conductance of the walls

4. Results and discussion. The solution of Laplace equation $\Delta\varphi^* = 0$, respecting the above boundary conditions, are researched by a numerical procedure using the commercial code Fluent. The grid is composed of regular rectangles and is sufficiently tight to take into account the strong potential variations at the end of the electrode ($x = L_B/2$). The convergence results from the comparison with the analytic solution are obtained for a very high aspect ratio and very long channel. The numerical results give distribution of the current density electromagnetic forces and the efficiency of the generator. This last results can be calculated by the ratio of the electric power furnished by the process, RI^2 and the introduced mechanical power P_{EM}, sum of the electric power and the Joule losses:

$$(\mathbf{J} \wedge \mathbf{B}) \cdot U_0 = P_{EM} = \overline{RI'^2} + 8a \int_0^\infty \int_0^l \left(\frac{\overline{j_x^2}}{\sigma} + \frac{\overline{j_z^2}}{\sigma} \right) \mathrm{d}x \cdot \mathrm{d}z + 8a \int_0^\infty \int_0^l \left(\frac{\overline{j_{\mathrm{w}x}^2}}{\sigma_\mathrm{w}} + \frac{\overline{j_{\mathrm{w}z}^2}}{\sigma_\mathrm{w}} \right) \mathrm{d}x \cdot \mathrm{d}z$$

Using that definition the efficiency n is defined by:

$$\eta = \frac{\frac{L_B}{l} \varphi^{*2} \left({}^{l}\!/\!_{a} \right) \left(\frac{1-K}{K} - C_\mathrm{w} \right)}{\iiint_V (\mathbf{j}^* \wedge \mathbf{B}^*) \cdot \mathbf{u}^* \cdot \mathrm{d}x \cdot \mathrm{d}y \cdot \mathrm{d}z}, \quad K = \frac{1}{1 + \frac{r}{R} + \frac{r}{r_\mathrm{w}}} \quad \text{being the load factor.}$$

Fig. 3 gives the evolution of the efficiency versus the aspect ratio for an insulating wall. It can be seen that the efficiency strongly decreases when the aspect ratio decreases. Consequently, a good power extraction can be obtained only for an aspect ratio larger than 8. Fig. 4 proposes a typical distribution of the electromagnetic forces along the channel. The pick of the force corresponds to the extremity of the channel and is responsible for the M-shape velocity profile for the low interaction parameter.

The objective of this work is to identify the effect of the aspect ratio for an alternate MHD generator working at a sufficiently high frequency to neglect the influence of the strongly non-uniform distribution of both the electric current and the induced forces on the velocity profile, which is assumed to be constant. The numerical procedure used to take into account the wall conductivity and the assumed small value of the magnetic Reynolds number allowing to neglect the induced magnetic field.

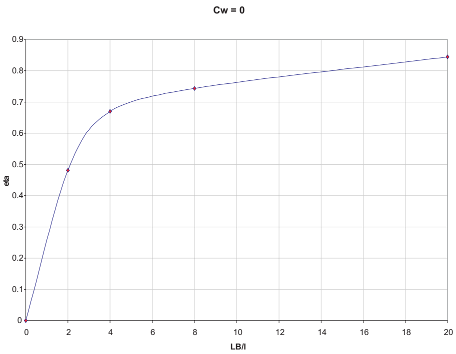

Fig. 3. Maximum efficiency versus the aspect ratio for $\delta^* = 2.5$, $C_{\mathrm{w}} = 0$. The more the aspect ratio, the more the MHD channel is near an infinite channel, then the efficiency tends toward the maximum value of an infinite channel.

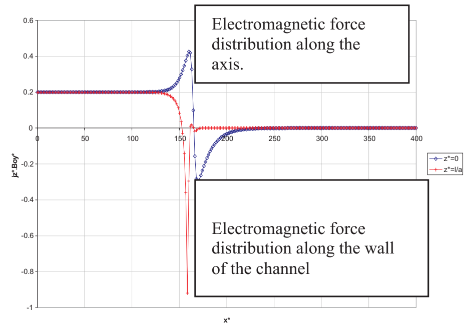

Fig. 4. Graph of the x-component of electromagnetic forces on the axis of the channel and along the wall for a magnetic field attenuation distance $\delta^* = 2.5$, $L_B/2l = 8$, $C_{\mathrm{w}} = 0$.

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