MHD-CHANNEL FLOW OF LIQUID METAL UNDER INHOMOGENEOUS MAGNETIC FIELD – PART 1: EXPERIMENT

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Introduction. The problem of molten metal control by a spatially inhomogeneous magnetic fields is getting more and more important for industrial applications in metallurgy and crystal growth. This method, also known as electromagnetic brake, is applied for suppression of undesirable jet-like structures of liquid metal flows, which take place, for example, in the process of continuous casting. In an industrial facility a magnetic field is created by an electromagnet mounted in the mould of a slab caster, where it controls the flow of the hot metal. The electromagnetic brake provides the necessary conditions of the mould for obtaining a major reduction of non-metallic inclusions as well as a significantly reduced risk of surface cracks. The process of electromagnetic brake, its advantages and drawbacks are broadly described in special publications. Some of the relevant references are [1, 2, 3].

1. MHD flow under inhomogeneous magnetic field. In the experiment we deal with a flat channel flow confined by the rigid non-conducting walls. The flow is driven by a horizontal pressure gradient. An external steady magnetic field is locally applied with the aim to create an essential gradient of the field in a stream wise direction. In the transversal direction the magnetic field is homogeneous. Since the fluid is incompressible and electrically conducting, the flow is governed by the momentum equation with the Lorentz term:

$$\frac{\mathrm{d}\overline{V}}{\mathrm{d}t} = -\frac{1}{\rho}\overline{\nabla p} + \nu\overline{V} + \frac{1}{\rho}\overline{j} \times \overline{B}; \tag{1}$$

equation of continuity for the vectors of velocity and electrical current density and the Ohm's law:

$$\nabla \cdot \overline{V} = 0, \quad \nabla \cdot \overline{j} = 0, \quad \overline{V} \times \overline{B} = \overline{\nabla\varphi} + \overline{j}/\sigma \qquad (2,3,4)$$

To set the equation (1) into dimensionless form we introduce the following governing parameters. The ratio of inertial and viscous forces is characterized by the Reynolds number $\mathrm{Re} = V_0H/\nu$. The strength of electromagnetic forces in relation to the viscosity and inertia are described by the dimensionless Hartmann $\mathrm{Ha}^2 = B_0^2H^2\sigma/(\rho\nu)$ and Stuart $\mathrm{N} = \mathrm{Ha}^2/\mathrm{Re}$ numbers respectively. Here σ, ρ and ν denote the electrical conductivity, density and kinematic viscosity of the fluid. V_0, H, B_0 are mean flow rate velocity, height of the liquid layer, and z component of magnetic field in the central point of a magnet system.

The current is generated by the difference of an electromotive force inside and outside the magnet, where the magnetic field has maximum or vanish respectively. Interaction of the transversal component of electrical current with the vertical magnetic field drags the flow in its central part. In the vicinity of the sidewalls because of a boundary effect the electrical current changes its direction from the transversal to the flow wise. This transformation increases rotational part of the electromagnetic force. As a result a strong velocity gradient develops in this region. The velocity profile is so-called M-shaped with two expressed maxima near the sidewalls and minimum of velocity in the central part. Such a flow is good investigated theoretically and experimental (see, for example, [4, 5, 6, 7, 8, 9]).

Since the effect of inhomogeneous magnetic field on a MHD flow is essentially rotational, it is convenient to base our estimation on the momentum equation rewritten in the term of vorticity:

$$\rho\frac{\mathrm{d}\omega_z}{\mathrm{d}t} = \rho\nu\Delta_\perp\omega_z + \mathrm{rot}_z(\overline{j} \times \overline{B}) \approx \rho\nu\Delta_\perp\omega + B_z\frac{\partial j_z}{\partial z} - j_x\frac{\partial B_z}{\partial x} \tag{5}$$

Here $\Delta_\perp = \partial^2/\partial x^2 + \partial^2/\partial y^2$.

In this estimation we consider the vertical component of vorticity $\omega_z = \mathrm{rot}_z\overline{V}$ and neglect the horizontal components of magnetic field and spatial variation of B_z in y and z directions. Neglecting vertical structure of the flow, let average the equation (5), integrating that in z direction:

$$\frac{\rho}{H}\frac{\mathrm{d}}{\mathrm{d}t}\int\limits_H \omega_z \mathrm{d}z = \rho\frac{\mathrm{d}\widetilde{\omega}_z}{\mathrm{d}t} \approx \rho\nu\Delta_\perp\widetilde{\omega} + \frac{B_z}{H}\cdot j_z\big|_{\delta_H} - \widetilde{j}_x\frac{\partial B_z}{\partial x} \tag{6}$$

Here the averaged values are marked by tilde. Taking into account that thickness of the Hartmann boundary layer $\delta_H = H/\mathrm{Ha}$ is much smaller compared with H, the properties of this layer can be used as boundary conditions for the main core flow. For example, the value $j_z\big|_{\delta_H}$ in (6) is the component of electrical current penetrating into the core flow from the Hartmann boundary layer. To estimate the orders of terms in equation (6) we introduce the followings scales: V_v, l_v, $V_\mathrm{v}/l_\mathrm{v}$ – typical velocity, length and vorticity scales of the flow; $\hat{j}_z$, $\hat{j}_\perp$ – typical electric current density for the its vertical and horizontal components. Turnover time $l_\mathrm{v}/V_\mathrm{v}$ is an appropriate temporal scale. Now switching to a dimensionless form, the equation (5) reads:

$$\frac{\mathrm{d}\widetilde{\omega}_z}{\mathrm{d}t} \approx \underbrace{\frac{\nu}{V_\mathrm{v}l_\mathrm{v}}\Delta_\perp\widetilde{\omega}_z}_{1} + \underbrace{\Big(\frac{l_\mathrm{v}^2 B_0}{\rho V_\mathrm{v}^2}\frac{\hat{j}_z}{H}\Big)B_z\cdot j_z\big|_{\delta_H}}_{2} - \underbrace{\Big(\frac{l_\mathrm{v}^2 B_0}{\rho V_\mathrm{v}^2}\frac{\hat{j}_\perp}{l_B}\Big)\widetilde{j}_x\frac{\partial B_z}{\partial x}}_{3} \tag{7}$$

Typical value of the horizontal components of electrical current density $\hat{j}_\perp$ can be estimated by taking curl from the Ohm law (4):

$$\mathrm{rot}_z\bar{j} = \sigma\cdot\mathrm{rot}_z(\overline{V}\times\overline{B}) \approx= \sigma\cdot V_x\frac{\partial B_z}{\partial x} \tag{8}$$

Considering that the spatial scale of an electrical current is defined by the scale of velocity l_v, we can write according to equation (8) the following scale for the current:

$$\hat{j}_\perp \approx \sigma V_\mathrm{v} B_0\frac{l_\mathrm{v}}{l_B} \tag{9}$$

The component of electrical current parallel to magnetic field generated by the Hartmann boundary layer is proportional to vorticity of the core flow $\widetilde{\omega}_z$ and thickness of this layer δ_H [10, 11, 12]:

$$j_z\big|_{\delta_H} \sim \sigma\widetilde{\omega}_z B_z\delta_H$$

Consequently, the scale of the z component of electrical current is

$$\hat{j}_z \approx \sigma V_\mathrm{v} B_0\frac{\delta_H}{l_\mathrm{v}} \tag{10}$$

Taking into account (9), (10) we can rewrite the dimensionless equation (7) in the following form:

$$\frac{\mathrm{d}\widetilde{\omega}_z}{\mathrm{d}t} \approx \underbrace{\frac{1}{\mathrm{Re}}\frac{H}{l_\mathrm{v}}\Delta_\perp\widetilde{\omega}_z}_{1} + \underbrace{\frac{\mathrm{Ha}}{\mathrm{Re}}\frac{l_\mathrm{v}}{H}\cdot B_z j_z\big|_{\delta_H}}_{2} - \underbrace{\frac{\mathrm{Ha}}{\mathrm{Re}}\frac{l_\mathrm{v}}{H}\Big(\frac{l_\mathrm{v}}{l_B}\Big)^2\cdot\widetilde{j}_x\frac{\partial B_z}{\partial x}}_{3} \tag{11}$$

According to this estimation the viscous (term 1) is negligible, while the Reynolds number remains large.

The *mean flow*, which has commensurable scales $l_\mathrm{v} \approx l_B \approx H$, is governed by the *third term*. Just the gradient of magnetic field creates an electromagnetic force that essentially transforms the flow. This force is proportional to the Stuart number N. One can see that the third term in (11) vanishes if the size of a flow structure l_v is much smaller than the scale of magnetic field l_B, ($l_\mathrm{v}/l_B \ll \mathrm{Ha}^{-0.5}$). Consequently, for the relatively small-scale *velocity fluctuations* the *second term* in (11) can play an essential role. This term is responsible for the so-called Hartmann friction, which is caused by an absolute value of magnetic field and takes place even if the flow scale is negligible in relation to a gradient of the field. The effect of Hartmann friction is inversely proportional to the dimensionless parameter named $\mathrm{Rh} = H/l_\mathrm{v})\mathrm{Re}/\mathrm{Ha}$, which is a ratio of inertia and shear stress created by the Hartmann boundary layer. In the present notation the Rh parameter contains an indefinite value l_v, that is a spatial scale of a certain flow structure. For an experimental practice one can redefine this parameter as a ratio of mechanical works of inertial forces and shear stresses in the Hartmann boundary layer. Thus the corresponding modified Rh number could be written as follows:

$$\mathrm{Rh_m}(x) = \frac{H\cdot\rho V_0^2}{\int\limits_{-\infty}^{x}\rho\nu\frac{V_0}{\delta_H}\mathrm{d}x} = \frac{V_0 H/\nu}{B_0 H\sqrt{\sigma/\rho\nu}}\left(\int\limits_{-\infty}^{x/H}\frac{B_z(x/H)}{B_0}\mathrm{d}\Big(\frac{x}{H}\Big)\right) = \frac{\mathrm{Re}}{\mathrm{Ha}}\left(\int\limits_{-\infty}^{x/H}\frac{B_z(x/H)}{B_0}\mathrm{d}\Big(\frac{x}{H}\Big)\right)^{-1}$$

The dynamic pressure ρV_0^2 was taken as an appropriate estimation for the work of inertial force. For this parameter we use a stream wise distribution of magnetic field B_z in the central part of our test section.

In the same manner we modify the Stuart number $\mathrm{N_m}(x)$ as a ratio of mechanical works performed by electromagnetic and inertial forces.

$$\mathrm{N_m}(x) = \frac{\int\limits_{-\infty}^{x}\sigma V_0 B_z^2(x)\mathrm{d}x}{\rho V_0^2} = \frac{\sigma B_0^2 H}{\rho V_0^2}\int\limits_{-\infty}^{x/H}\Big(\frac{B_z(x/H)}{B_0}\Big)^2\mathrm{d}\Big(\frac{x}{H}\Big) = \mathrm{N}\cdot\int\limits_{-\infty}^{x/H}\Big(\frac{B_z(x/H)}{B_0}\Big)^2\mathrm{d}\Big(\frac{x}{H}\Big)$$

2. Experimental setup. The test section of experimental facility includes a Plexiglas channel with a rectangular cross section $L = 100$ mm width and $H = 20$ mm height. A longitude of the channel is half a meter. A honeycomb was installed in the entrance part of the channel to ensure homogeneous initial velocity profile. A transversal steady inhomogeneous magnetic field, was created by a pair of permanent magnets with the following dimensions: 30 mm along the flow and 100 mm in transversal direction. The magnets were located at the top and the bottom casing walls of the channel. The magnitude of magnetic field was $B_0 = 0.504$ T in the central point of the gap between the poles. The distribution of magnetic field was essentially inhomogeneous along the flow in x-direction. For example, the vertical component of magnetic field B_z varnished on the distance of two gauges from the magnet center. A eutectic alloy of gallium, indium and tin $\mathrm{Ga^{68}In^{20}Sn^{12}}$ was used as a model fluid. This alloy has the following physical properties: melting points of $+10.5^\circ$C, density $\rho = 6360$ kg/m^3, electrical conductivity $\sigma = 3.46\cdot10^6$ Ohm^{-1} and kinematic viscosity $\nu = 3.4\cdot10^{-7}$ m^2/s. A channel flow of the model liquid was driven by an electromagnetic pump. Inside the region of applied magnetic field the electric potential distribution was measured by a moveable potential probe.

3. Transformation of mean velocity. For a quantitative characterization of the M-shaped flow we introduce a simple parameter, which is a dimensionless thickness of the side boundary layer. The definition of this parameter is illustrated in Fig. 1. An inflectional tangent to M-shaped velocity profile on the inner shear layer separates an y-intercept with a length marked in Figure by s. Dealing with a dimensionless velocity profile we can consider value s as an appropriate scale for the thickness of the side boundary layer. Since the length s is proportional to an inclination of the tangent and, consequently, to the derivative $\partial V_x/\partial y$, one can say that the thickness of the boundary layer is proportional to the z component of vorticity $\omega_z = \partial V_y/\partial x - \partial V_x/\partial y$.

Here we assume that y component of velocity V_y is much smaller than the V_x. As it was estimated above, the vorticity is caused by a rotational part of the electromagnetic force, which is proportional to the Stuart number. In Fig. 1*b* we plot the s parameter against the modified Stuart number. Four experimental series are obtained at the fixed distances from the magnet and variable Reynolds number. During two other runs the flow rate was maintained constant but the potential probe was moved in the middle horizontal plane. It turns out that if the dimensionless coordinate x/H does not exceeds value -0.5, the experimental points are placed along the one curve fitted by the following power law: $s(\mathrm{N_m}) = 2.34\mathrm{N_m}^{-0.37}$. It means that before the coordinate $x/H = 0.5$ the M-shape velocity profile develops as **self-similar**. After the mentioned coordinate the experimental points occupy the certain curves in dependence on the Reynolds number. In this region the mean velocity profile can be considered as almost developed.

4. Transformation of velocity fluctuations. According to the methodology of potential velocimetry, we interpret the measured RMS fluctuations of electric potential as fluctuations of velocity so far as those two characteristics are interdependent. In Fig. 2*a* the relative RMS values of fluctuations measured by the potential probe in the central

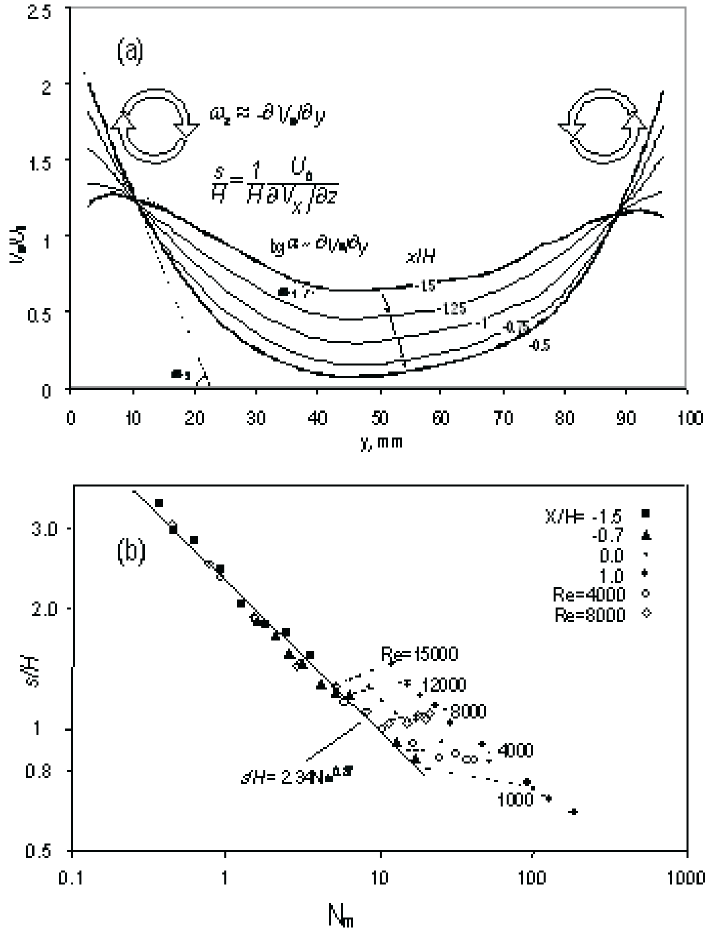

Fig. 1. Transformation of mean velocity profile. (*a*) Stream wise component of velocity V_x at the different distances x/H with respect to the magnet center, Re $= 4000$, $U_0 = 8$ cm/s – mean flow rate velocity, ω_z – vertical component of vorticity, s/H – dimensionless thickness of the side boundary layer, $H = 20$ mm – height of the flow. (*b*) Dependence of the side boundary layer thickness s/H on the modified Stuart number $\mathrm{N_m}$.

part of the flow are plotted against the Reynolds number. Twelve experimental series cover a range of $1 < x/H < 1.5$ before and past the magnet. The measurement points are situated on the axes (semi height and semi width) of the channel. Before the magnet system a level of perturbations varies in a range of 4–8.5 % in dependence on the Reynolds number. At the small Reynolds numbers, the perturbations are large scale and generate a relatively high level of velocity fluctuations. For example, at the Re $= 2000$ an expressed maximum of fluctuation occurs on the coordinates $-1 < x/H < -0.5$. This regime is a result of oscillations generated by an unstable frontal boundary layer, which develops before the streamlined area of the applied magnetic field and contains a critical point. In the range of Reynolds number $2000 <$ Re < 4000 the intensity of fluctuation decreases. It means that flow around the frontal critical points becomes more regular and the spatial scale of the generated velocity perturbations decreases. Past the coordinate $x/H > -0.5$ the dependence of velocity fluctuations from the Reynolds number becomes monotonic. At the Re < 4000, the intensity of fluctuations does not exceed 0.6 %. That is well pronounced electromagnetic braking effect. With the growth of Reynolds number the intensity of fluctuations increases. It is remarkable, that at the fixed values of Reynolds number, the fluctuations decay moving downstream. It means that in the central part

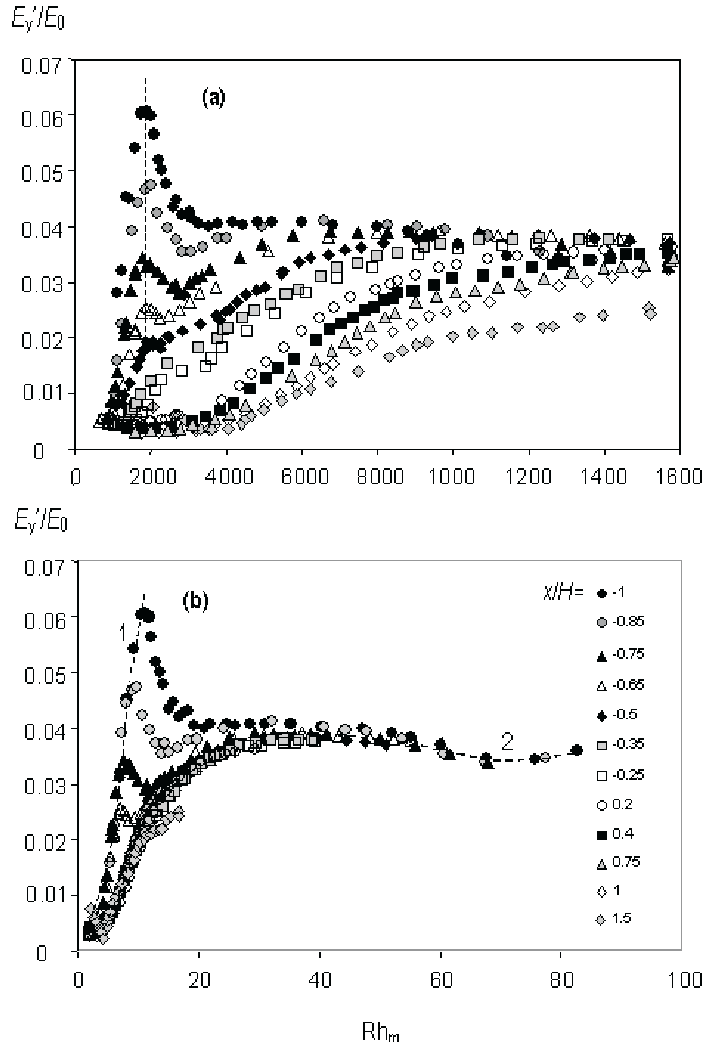

Fig. 2. Intensity of electric potential fluctuations (E'_y/E_0 in the region of applied magnetic field in the central part of the channel ($y = 0$) versus the (a) Reynolds number and (b) the modified $\mathrm{Rh_m}$ number. x/H – dimensionless distance from the center of the magnet. The negative and positive values correspond to the space before and past the magnet respectively. E_0 – strength of electric potential corresponding to the mean flow rate velocity.

of the flow the velocity fluctuations are not fed by the energy but lose it due to the Hartmann friction. Basing on this consideration we try to systemize the experimental data plotting the value of velocity fluctuations against the modified $\mathrm{Rh_m}$ number (see Fig. 2*b*). There are two principal groups of the experimental points in this Figure. The series corresponding to $x/H = -1.5 \ldots -0.65$ form the first group. Those curves have an expressed maximum of the velocity fluctuations caused, as mentioned above, by instability of the frontal boundary layer. In Fig. 2*a* all those maxima are outlined by the dashed line. At the further growth of the $\mathrm{Rh_m}$ parameter the experimental points attract to the curve 2 through a transitional regime. The curve 2 completely combines six experimental runs corresponding to the coordinates $x/H = -0.2 \ldots 0.5$. One can say that for the mentioned conditions the $\mathrm{Rh_m}$ is a universal parameter.

5. Summary. Statement of the studied problem serves as a physical model for the industrial process where structure and intensity of flow are controlled by magnetic field. The investigated M-shaped flow is considerably inertial. We manage to recognize a transitional regime of the mean flow transformation. On the distance characterized by the range of $x/H < -0.5$ the side boundary layer develops as self similar. The process of development is governed by the modified Stuart number.

An influence of applied magnetic field on the velocity fluctuations is also sufficient. As the flow is subjected to magnetic field, the intensity of pulsations reduces in the center 2-fold compared with the initial flow. Maximum of the intensity of velocity fluctuations, that constitutes value 8.5%, is much more pronounced ahead the magnet system. At the growing Reynolds number a relative value of the intensity of velocity fluctuations decreases and tends to a constant value 3.5% at $\mathrm{Re} > 5000$. The electromagnetic braking of velocity fluctuations is governed by the Hartmann friction. In the region of a relatively high magnetic field ($180 < \mathrm{Ha} < 400$) the vortical structures have expressed two-dimensional properties and the process of velocity fluctuations decay is governed by the modified $\mathrm{Rh_m}$.

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